Airport Runway Capacity and Delay: Some Models for Planners and Managers

Occasional Paper

This Paper seeks to provide an understanding of runway capacity and of the factors on which it depends. The analysis is incorporated in a set of easy-to-use models for the estimation of delays to aircraft under any given pattern of demand. These models are intended to provide the planner with simple tools for the rapid assessment of the impact on delay of changes in demand such as might be achieved by regulatory or pricing policies. In addition the capacity analysis itself forms the basis for a rapid preliminary assessment of the impact of alternative runway configurations, new aircraft types or altered separation standards.

Airport Runway Capacity and Delay:

Some Models for Planners and Managers

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FOREWORD

The assessment of airport capacity and the associated planning of new facilities is a complex, difficult and often expensive exercise. The traditional approach has involved the development and use of large, complex models to simulate conditions under alternative scenarios. The very complexity of these models and their requirement for large amounts of data has limited the options to be examined.

The work reported in this paper represents a different philosophy in that it concerns the development of relatively simple models for the analysis of one aspect of airport operations—the capacity of a runway system to service aircraft of different types under known demand conditions. The models allow the planner to assess the impact on aircraft delays of changes in the demand for take-off/landing movements, runway configurations, different aircraft types, separation standards, etc. The models are based on queueing theory and do not require large amounts of data or repeated simulation runs.

The Bureau wishes to acknowledge the contribution made by Ms J. Ascione to the development and programming of the models, and also the useful and informative discussions with Mr G. Challinor and Mr P. Daly of Airways Operations Division during the development of the models.

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SUMMARY

An important aspect of airport performance is the capacity of its runway system to service the demands imposed by aircraft movements.

The paper seeks to provide an understanding of runway capacity and of the factors on which it depends. The analysis is incorporated in a set of easy-to-use models for the estimation of delays to aircraft under any given pattern of demand. These models are intended to provide the planner with simple tools for the rapid assessment of the impact on delay of changes in demand such as might be achieved by regulatory or pricing policies. In addition the capacity analysis itself forms the basis for a rapid preliminary assessment of the impact of alternative runway configurations, new aircraft types or altered separation standards.

In this work runway capacity is defined as the maximum attainable service rate and depends only on the physical and operational characteristics of the runway system and the aircraft which use it. A detailed analysis of runway operations shows how the great complexity of these operations can be taken into account by the consideration of the following five factors:

- air traffic control separation standards;
- aircraft characteristics;
- runway configuration;
- movement mix; and
- air traffic control operational strategies.

Other factors which influence runway capacity, eg safety, weather, noise, etc, can be shown to act through one or more of these five factors.

An important point emerging from the analysis is that the time for which an aircraft movement occupies the runway system, thereby delaying a following movement, depends on the types of both movements. There is therefore no well defined service time to be associated with a particular movement type, rather, an *intermovement time* can be assigned to every *pair* of movements on the basis of the separation rule appropriate to the members of that pair.

It follows from the definition that runway capacity is the inverse of the *average* intermovement time during a continuously busy period. The first three of the above five factors together determine the values of the intermovement times, while the averaging process is governed by the last two which describe which movements occur and in what order.

The actual measurement of delay due to runway congestion is not simple. Direct measurement, as for example, the comparison of actual with scheduled movement times, fails to distinguish runway delay from that due to other causes. An additional difficulty is that scheduled times already include an allowance for expected delay. An indirect statistical method is described and applied to assess the delays to domestic RPT aircraft using Sydney (Kingsford Smith) Airport (KSA) during 1979.

Delay models have been developed and are described in this paper. An analytical approach is used in preference to simulation because of its simpler model structure, reduced data requirements and its avoidance of the need for repeated computer runs associated with Monte Carlo techniques.

The models are based on (time dependent) queueing theory which is a quantitative description of the passage or flow of 'customers' (aircraft) through a facility (the runway system) which provides a service (take-offs and landings). The system is described in terms of the demand and service processes which determine the values of parameters in a set of equations whose solution permits the calculation of such performance measures as queue length and waiting times (delays).

The techniques of queueing theory are well established, but have not generally been considered capable of providing a realistic description of the operational complexities of different aircraft movement types on multiple, interacting runways. The problem is overcome in this work by segregating the capacity and delay analyses. The capacity analysis takes account of the operational complexities and the results can be incorporated into the delay models as a description of the service process.

A simple random (Poisson) demand function is assumed, described by an average demand rate (for each movement type) which may be varied throughout the day to reflect the peaks, troughs and mix variations in the demand profile in a real situation.

A wide variety of performance measures can be provided. For planning purposes the most generally useful are total delay, average delay and marginal delay (the additional total delay due to one extra user).

The models have been implemented as computer programs whose principal input requirements are:

- the matrix of intermovement times; and
- the demand profile for each movement type.

An example is provided of the use of the models in the context of policy assessment.

CHAPTER 1—INTRODUCTION

In fulfilling its role as the interface between the air and surface phases of an overall transportation task the modern airport provides a wide variety of facilities to meet the demands associated with the movement of aircraft, surface vehicles, passengers and freight. The concentration of these facilities in a relatively small area, and the consequent focussing of both the air and surface movements on that area, have made airport capacity one of the key issues to be addressed if the growing demand for air transportation is to be met.

This paper is concerned with one aspect of airport capacity—the ability of the runway system to meet the demand for aircraft movements. As the demand approaches the capacity of the runways, congestion increases, and costly delays are experienced by aircraft on the ground and in the air. An estimate of these delay costs will be essential to any assessment of the performance of existing facilities and to any cost benefit analysis relating to the provision of increased capacity.

Runway congestion is not, of course, the only source of delays; in particular the capacities of loading gates and passenger check-in facilities have often been strained. The runways, however, and their associated problems can be considered to have a certain primacy because the resources of land and capital devoted to them are very much greater than those devoted to other components of the airport system. In addition, in the context of major urban airports, the availability of land is often a restriction on the expansion of runway facilities, whereas this is not usually a problem for other components. Finally, the provision and performance of the runways may be of wider interest in the Australian context because they are publicly owned while other components of the airport system may be privately owned or operated.

In the long run the construction of additional runways or new airports may be inevitable if a growing demand for these facilities is to be met. Such 'high capital' solutions however, typically have lead times of the order of a decade and, in the meantime, the only solution to the congestion problem is to make more efficient use of the existing facilities. Indeed, the discovery of a more efficient way to use the existing facilities throws new light on the congestion problem and may revise the estimates of the need for more capacity or of its timing.

The concept of an economically efficient use of runway capacity has received considerable attention since 1968 (Carlin and Park 1969), when runway congestion first became serious in the United States, and a variety of regulatory and pricing policies (the so called 'low capital' solutions) intended to achieve efficient use of runways have been proposed (Carlin and Park 1969, Odoni and Vittek 1976). It is not the purpose of the present paper to assess the merits of these proposals but rather to provide the tools which will be essential to such an assessment.

The aim of the paper is to provide an understanding of the factors which determine runway capacities and then to present a set of easy-to-use models which provide estimates of delay as a function of demand for arbitrary runway configurations. Hitherto, the prediction of runway delay has usually involved the use of complex and expensive simulation programs (Ball 1976, Atack 1978, Newell 1979). The models presented here are intended to provide the planner with *simple* tools for the ready assessment of the impact on delay of altered demand such as might be achieved by regulatory or pricing policies.

In addition, the runway capacity analysis provides the basis for a rapid initial examination of the likely effects of alternative runway configurations, new aircraft types or altered separation standards and of the interactions between them.

CHAPTER 2—DEMAND, CAPACITY AND DELAY

The purpose of the airport is to meet a *demand* for aircraft landing and take-off facilities. These facilities (in particular the runways) have a certain *capacity* and are provided and operated at a certain cost. This paper is not concerned with the initial investment costs, nor with what are normally considered operational and maintenance costs, but only with the cost represented by *delay* arising from the routine operation of the system under varying levels of demand. This delay cost will be an important component of any efficiency assessment or cost benefit study.

The three key terms here are capacity, demand and delay, and it is important to have a clear understanding of what these terms mean. In the following paragraphs they are given precise definitions.

CAPACITY

Until the early 1970s considerations of runway capacity and delay were based on models developed in the early 1960s by the Airborne Instruments Laboratory (AIL) under contract to the United States Federal Aviation Administration (FAA 1968). Although these models have been superseded, it is important to be aware of their definition of capacity which, although unsatisfactory, was widely adopted.

The AIL defined 'practical runway capacity' as:

> 'the number of aircraft operations during a specific time interval corresponding to a tolerable level of average delay'.

This definition is based on the intuitively appealing idea that the capacity of a facility ought to be a measure of the level of service which it is designed routinely to provide. It does, however, raise a number of problems. The specification of a 'tolerable' delay (usually 4 minutes) is clearly arbitrary and can lead to apparent anomalies such as airports frequently operating over capacity (presumably with 'intolerable' delays). The main difficulty, however, is that the definition tied capacity to the details of demand and delay in an extremely complicated way. As the purpose of the analysis was to investigate just this dependence of delay on capacity and demand, the definition was unsatisfactory.

In the work that followed the congestion 'crisis' of the late 1960s, which forms the basis of the modern analysis of runway capacity (FAA 1973), the early definition was abandoned. In this work the (hourly) runway capacity was defined as:

> 'the maximum number of aircraft operations that an airfield can accommodate during an hour when there is a continuous demand for service'.

In this definition the capacity is a service rate (in the sense of queueing theory) and is given by the reciprocal of the weighted average of the service times of the aircraft which use the runways during the hour in question. It is independent of the demand (except in so far as the mix of aircraft types is important) and is unaffected by the level of delay being experienced. In practice it is readily measured by counting aircraft movements during congested periods.

The relation between the old and the new definition of runway capacity is illustrated in Figure 2.1 which shows schematically the dependence of average delay on demand for runway usage. The curved band indicates a possible range of this dependence with variations in the distribution of usage throughout the day. The horizontal band indicates a range of arbitrarily set 'tolerable' delays. The intersection of the two bands

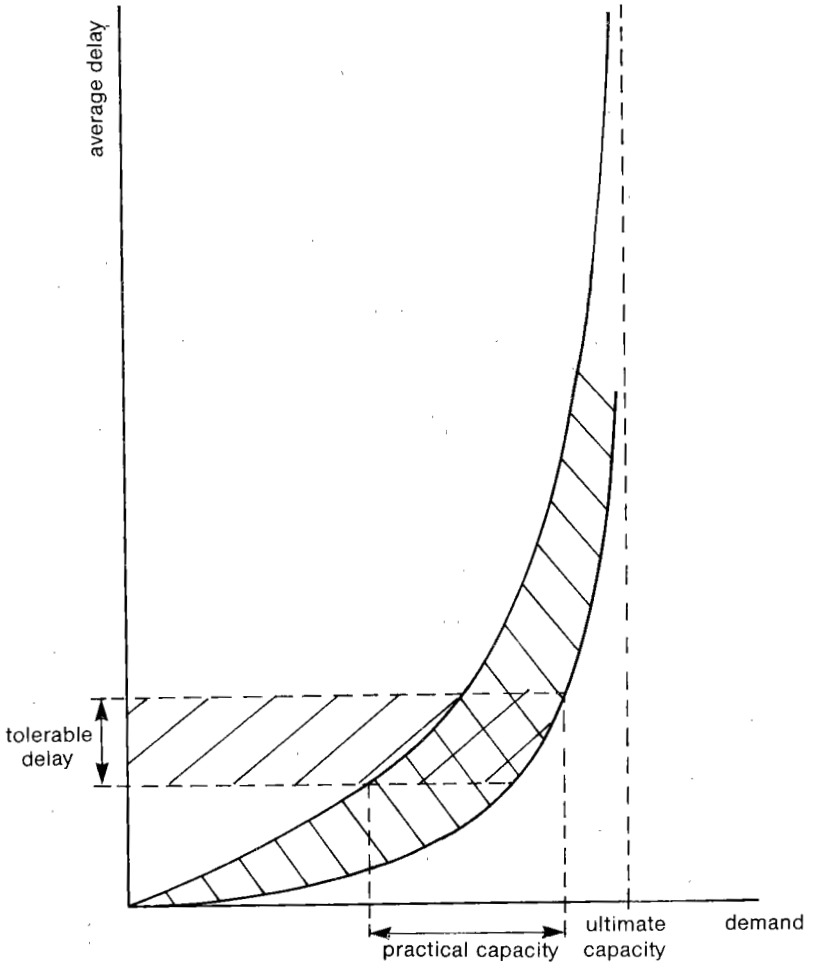


Figure 2.1
Definitions of capacity

defines a range of corresponding 'practical' runway capacities. It is clear that the concept is not a precise one.

By contrast, it is found that there is a fairly well defined maximum rate at which aircraft can be handled during congested periods. This rate does depend on weather conditions and on the mix of aircraft types but is not otherwise affected by variations in demand. This value has been variously called the 'ultimate capacity' or 'throughput', but will be referred to in this report simply as the 'runway capacity'.

On the basis of this definition a series of charts was produced for the estimation of runway capacity under a wide range of conditions (FAA 1976). The details of these charts and their use will not be considered here as they were developed for American conditions which are not identical with the Australian ones, and in any case include assumptions and limitations which it will be necessary to alter. The essential features of the capacity analysis, however, are important and these will be discussed in detail in Chapter 3.

DEMAND

Demand is specified as a rate, the number of requests (for runway use) per unit time. Its most important characteristic is that it is not constant, but varies from near zero during slack periods to levels which may temporarily be well above the airport runway capacity. Furthermore, the rates of change of demand can be rapid in the sense that the airport system may not have time to reach a steady state (of congestion) before the demand changes. Long delays may be experienced, for instance, during periods of low demand due to the backlog of waiting aircraft from an earlier busy period.

It is important therefore to specify the demand as a function of time throughout the period of interest. In most cases the period of interest, from the point of view of delay analysis, will be a single day which may be considered in isolation from adjacent days because the fall off in demand during the night (sometimes due to a noise curfew) is sufficient to dissipate any backlog of congestion. The day is divided up, usually into 24 blocks of one hour duration, during each of which the number of requests for runway use is specified. This sequence is known as the demand profile and its shape is characteristic of the different types of day, weekdays, weekends, public holidays and so on.

During the year the daily demand profile also exhibits weekly and seasonal variations which must be taken into account in any overall analysis of the costs of delay. However, because of the separability of days (due to the low night-time demand), these longer term variations are readily taken into account simply by summing the results for the appropriate numbers of different types and levels of daily demand. In this paper, therefore, the essential unit of time for the delay analysis is the single day, and the longer term variations will not be considered except in Chapter 4 where the delay profiles for each day of the week will be examined.

As will be discussed in Chapter 3 an important factor in the determination of runway capacity is the mix of movements of different types which make up the demand. Movement types are specified by type of aircraft, whether landing or taking off, and possibly the runway used. A full specification of the daily demand will therefore require a demand profile for each movement type.

The specification of a hypothetical demand (such as a forecast) presents no conceptual difficulties. Surprisingly however, the measurement of a real demand can be problematic. The usual method is to count the actual number of aircraft movements in successive hours. That this is not the real demand (the number of requests for runway use) is readily seen by considering what would be observed when demand exceeds capacity. Clearly what is measured is the demand as modified by the more or less congested airport system. Nevertheless, for moderate delays, the actual flow of traffic can be a reasonable approximation to the demand.

Attempts to estimate demand from the published schedules of regular public transport (RPT) aircraft are also subject to error. One difficulty is that current schedules already contain an allowance for expected delay. Another is that delays in other parts of the airways system will alter the time at which an aircraft makes its demand on the runway system.

These problems are considered further in Chapter 4 where the issue is the isolation of the component of delay due to runway congestion and in Chapter 5 where the details of the random fluctuation in demand must be taken into account.

DELAY

Delay is quite straightforwardly defined as:

> 'the difference between the time it would take an aircraft to be served without interference from other aircraft and the actual time it takes the aircraft to be served' (FAA 1973).

Delay is the principal measure of the degradation of service caused by congestion and may be considered as the common currency against which other service degradations are exchanged. Thus, for instance, as congestion increases, the separation standards can be thought of as a means for converting an unacceptable increased risk of collision into the less serious degradation represented by delay.

Runway congestion is not, of course, the only source of delay in the airways system. En route, unfavourable winds and bad weather can increase journey times. On the ground, passenger terminal and loading gate congestion both lead to delays. One of the most difficult problems in the practical measurement of delay has been to identify the delays due to different sources and in particular to isolate the component of delay due to runway congestion. This problem is dealt with in Chapter 4.

The calculation of delay is a complex problem which has been approached in a variety of ways with mixed results. The AIL work (FAA 1968) used analytical relations based on steady state queueing theory for movements on a single runway. The limitations of this approach were recognised and procedures for applying corrections for overload periods and other special cases were specified. This use of ad hoc correction procedures can be legitimate when the purpose is to take account of minor perturbations, but cannot be justified in this case as a vehicle for introducing one of the principal determining factors (varying demand including overload conditions) into a theory (steady state) from which it had been specifically excluded. In addition the procedures were complicated and difficult to use.

The complexities of the problem were recognised in the work commissioned by the FAA in the early 1970s and there was developed an all embracing computer simulation model of the airside components of the airport system. The model was used to produce a series of delay charts for use in conjunction with the capacity charts with which they were published (FAA 1976).

At about the same time but in the slightly different context of air traffic control system capacity, Arthur D. Little Inc., under contract to the FAA, developed the theory of time dependent queues (FAA 1970, Koopman 1972). This analytical approach allows the important fluctuations in demand to be taken into account while avoiding many of the problems associated with simulations.

The relative merits of the different approaches to the calculation and prediction of delay are discussed in Chapter 5 where the time dependent queueing techniques are adopted and extended.

CHAPTER 3—RUNWAY CAPACITY

In the definition given in Chapter 2, runway capacity was related only to the physical and operational characteristics of the runway system and of the aircraft which use it. This chapter describes in detail how the capacity is related to the various factors on which it depends.

FIVE MAIN FACTORS

Although runway capacity may be affected by a very large number of diverse factors it can, in an operational sense, be treated as being determined by just five main factors. This is not an approximation in the sense that certain minor factors are ignored, but rather a classification of the principal mechanisms *directly* affecting runway capacity. Any other factor can be treated operationally as acting through one or more of these mechanisms.

Separation standards

These are the separations (which may be specified in terms of time or distance) imposed between aircraft manoeuvring on the runways or in the adjacent airspace. The fundamental rule is that an aircraft landing or taking-off must have the runway ahead of it free of other aircraft. In order to ensure that this rule is never violated certain other separations must be maintained so that, in the event of an accident, a following aircraft may take timely evasive action. In poor visibility, when aircraft operate under Instrument Flight Rules (IFR), air traffic control (ATC) is responsible for the maintenance of the separations which are formalised and precisely laid down. In good visibility aircraft may operate more flexibly under Visual Flight Rules (VFR) with pilots assuming partial responsibility for obeying the single occupancy rule. The actual separations achieved then tend to be smaller than under IFR and subject to greater variation.

In addition a wake turbulance separation (WTS) may have to be imposed to protect lighter aircraft from the dangerous vortices in the wake of a preceding heavier aircraft.

The details of the separation standards are discussed in Appendices I and II.

Aircraft characteristics

The most important ones are weight, speed and instrumentation. Weight is relevant to the wake turbulance separation while speed governs the times required to fly certain standard separations. The ability to operate in conditions of poor visibility depends on the aircraft being equipped with the appropriate instrumentation which operates in conjunction with the complementary, ground based, equipment installed on certain runways. Both weight and speed determine runway occupancy times; and all three may be required to determine whether an aircraft can use a particular runway.

Runway configuration

This is essentially the information contained in a plan of the runway lay-out. The most important points are the separations between runways and the location of intersections and exit taxiways. Runway lengths and strengths and information about neighbouring obstructions are also necessary if they are limiting for any aircraft types.

Movement mix

A movement will be defined by the type of aircraft, whether it is landing or taking off and the runway on which it occurs. The movement mix is the set of proportions (positive fractions which sum to one) of all movements represented by each type.

Air traffic control strategies

This covers such discretionary policies as the choice of runway operating mode, the granting of priority to certain movement types and the decision to alternate landings and take-offs or to treat aircraft on a first come, first served basis.

The claim that these five factors cover everything which may affect runway capacity is of course open to challenge; it may however be illustrated in the case of a number of particular aspects of airport operations.

Safety. This is of course a prime consideration in airport operation, and the separation standards are designed to achieve safe operations. As circumstances change and new risks arise the separations may be altered. This is what occurred following the introduction of wide bodied heavy jets in the late 1960s and led to the adoption of the wake turbulance separations (FAA 1977).

Weather. This is taken into account via the separation standards which depend on visibility conditions, as well as via the runway configuration if strong winds or lack of instrumentation prevent the use of certain runways. In addition, if certain aircraft types cannot operate in adverse weather conditions this will be reflected in the movement mix.

Noise. Regulations designed to reduce noise exposure generally forbid certain movement types on certain runways. They will therefore be taken into account via the runway configuration or the movement mix.

In the same way other issues as diverse as changes in demand and the introduction of new instrumentation can be shown to be covered by these five main headings. The rest of this chapter is devoted to showing how runway capacity depends on these factors.

RUNWAY OPERATIONS

For simplicity the main features of the analysis will be described first for landings and take-offs by aircraft of the same type on a single runway under IFR. This will provide a framework for a discussion of the greater complexity due to multiple runways and of the more flexible operations under VFR.

Figure 3.1 is a schematic representation of the runway with its approach and departure paths. The arrival and departure fixes may be of the order of 30-50 miles from the airport while the final approach path, in line with the runway from the approach gate to the runway threshold, may be 6-10 miles long. An arriving aircraft follows some more or less curved path from an arrival fix to the approach gate where different paths merge. It then follows the straight glide path defined by the radar beams of the instrument landing system (ILS), and touches down after passing over the threshold at about 50 feet. After touch down the aircraft decelerates to a speed at which it can turn off at a suitable exit. During periods of runway congestion the aircaft may be delayed, but the lost time may actually be spent far from the airport in path stretching detours either before or after the arrival fix or in specified holding patterns nearer the approach gate.

A departing aircraft travels along the taxiways to a holding point near the runway threshold where it may have to wait before being given clearance to line up on the runway prior to departure. After receiving take-off clearance the aircraft accelerates down the runway, lifts off and follows a common departure path for only a short distance before turning towards one of a number of departure fixes.

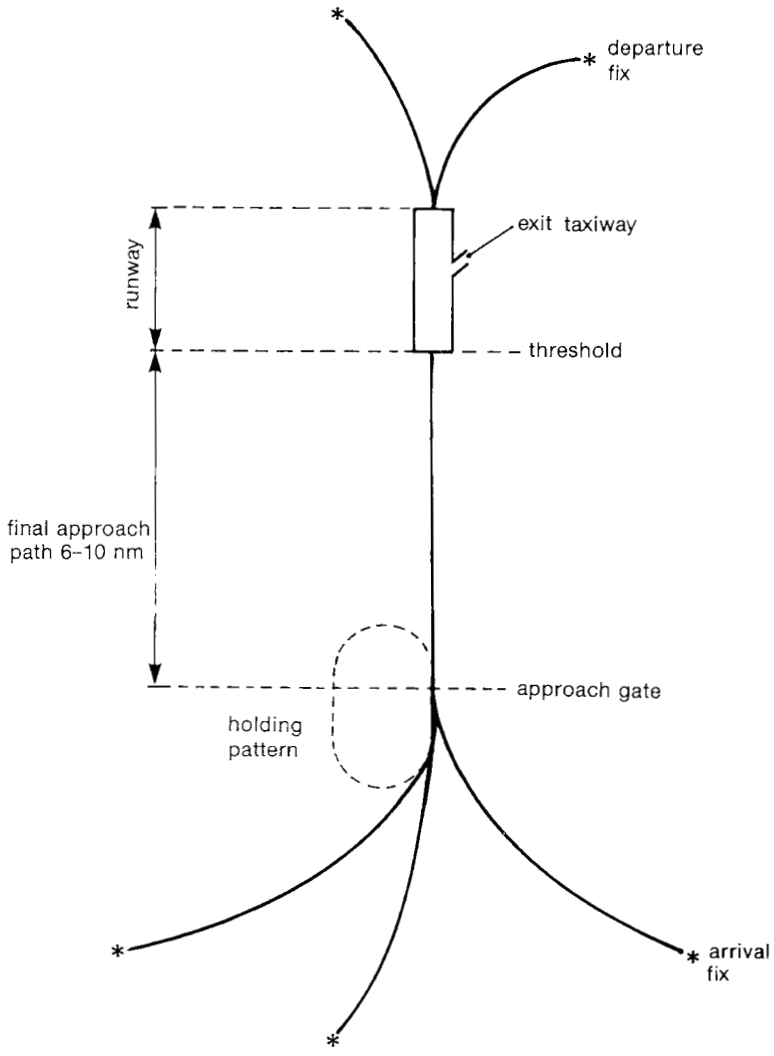


Figure 3.1
Runway approach and departure paths

Time-distance charts

If attention is confined to aircraft on the runway and on the approach and departure paths in line with it, a graphical representation of a series of movements can be obtained on a time-distance chart as shown in Figure 3.2.

In Figure 3.2 the ordinate represents distance along the runway and its extensions while the abscissa represents time. Physical movement in a straight line along the runway and its extensions is represented on the time-distance chart by a curve whose slope represents the speed of movement. The ends of the curve represent the times and positions at which aircraft enter or leave the region of interest.

A sequence of five movements is shown in the figure. It is assumed that the movements occur during a continuously busy period in the sense that no pair of adjacent movements could be closer together without violating the separation rules.

Movement 1 is a landing. The aircraft approaches the runway at constant speed and crosses the threshold at time t_1. After landing it decelerates and turns off the runway at the fourth exit at time t_1'.

Movement 2, another landing, follows movement 1, constrained by the requirement that successive arrivals be separated by at least distance S_{AA} (the arrival-arrival separation) on the approach path. It crosses the threshold at time t_2. Note that the runway is unoccupied between t_1 and t_2.

Movement 3 is a take-off. The time t_3, at which it starts from the threshold and accelerates down the runway, is determined by the time at which the previous landing (movement 2) vacates the runway.

Movement 4 is another take-off, constrained not to leave the threshold until the time t_4, a time S_{DD} (the departure-departure separation) after the previous take-off.

Movement 5 is another landing which must not be closer than distance S_{DA} (the departure-arrival separation) to the threshold at the time, t_4, at which the previous departure (movement 4) started its take-off run. In fact, arrivals on the final approach path have priority over departures; if movement 5 had been closer than S_{DA} at time t_4, movement 4 would not have been cleared for take-off until movement 5 had landed and vacated the runway.

Intermovement times

In this account the times t_1, t_2, t_3, t_4 and t_5 at which the aircraft cross the threshold have been singled out for attention. These are the notional times at which the movement can be considered to 'occur'. In fact, of course, it occurs over a period of time. A landing, for instance, might plausibly be said to end when the aircraft vacates the runway—but when did it start? This clearly depends on the type of the previous movement. Similarly a take-off might be considered to start when it leaves the threshold, but the decision as to when it is complete and no longer restraining the following movement depends on the type of that movement and the applicable separation rule.

The important point to be emphasised is that the time for which an aircraft movement 'occupies' the runway system, thereby delaying a following movement, depends on the types of *both* movements. There is therefore no well defined service time to be associated with a particular movement type, rather, an *intermovement time* can be assigned to every *pair* of movements on the basis of the separation rule appropriate to the members of that pair.

This is why it is convenient to define a time at which the movement 'occurs'. The choice of the event (usually crossing the threshold) on which to base this time is arbitrary, so long as it is well defined and the same for each movement of a given type. The intermovement time for a pair of movements is just the interval between the times at which the movements occur and is determined by the relevant separation rule.

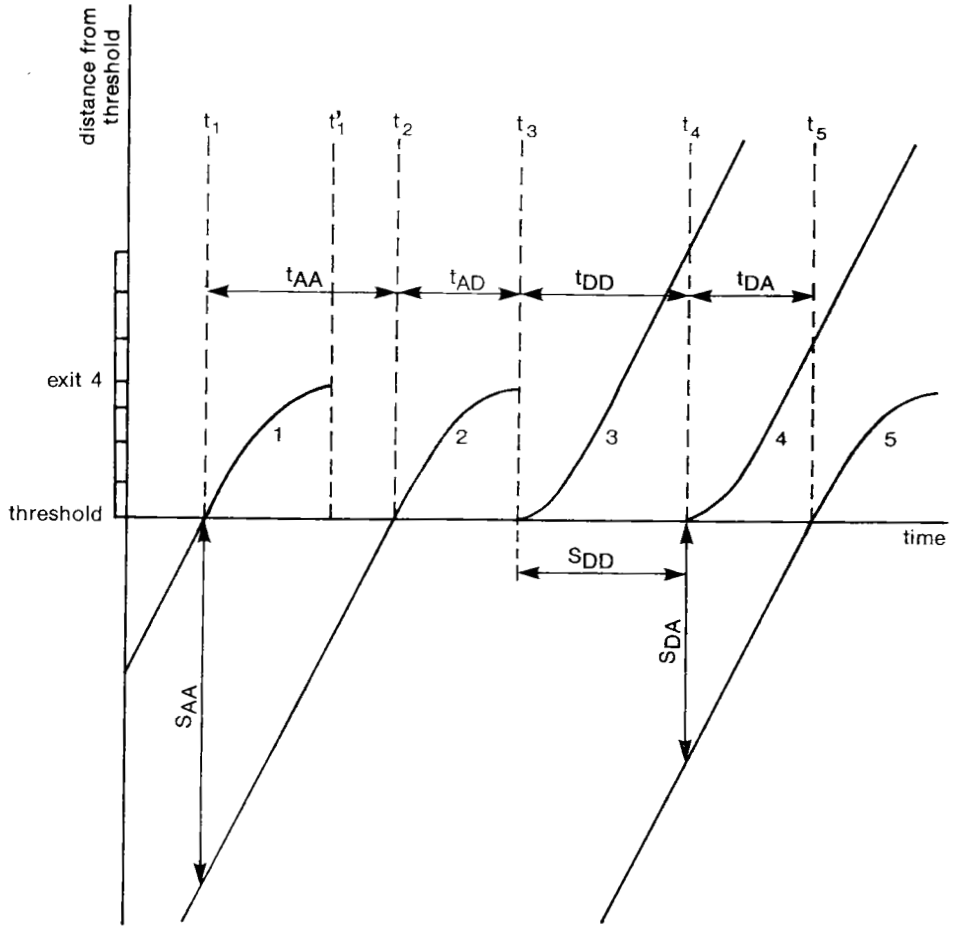


Figure 3.2
Intermovement times on a single runway

In the example above there were only two types of movement, arrivals (A) and departures (D) and consequently four intermovement times, t_{AA}, t_{AD}, t_{DA} and t_{DD}. It is convenient to record them in 2 x 2 *intermovement time (IMT) matrix* in which the rows correspond to the type of the first movement of a pair and the columns to the type of the second movement. For example:

		SECOND	
		A	D
FIRST	A	t_{AA}	t_{AD}
	D	t_{DA}	t_{DD}

Aircraft and movement types

In addition to arrival and departure, it will normally be necessary to take account of aircraft characteristics in distinguishing between movement types. However, a compromise is obviously required between a very precise categorisation of aircraft types and the need to keep the number of movement types down to manageable levels for modelling purposes. Two characteristics of aircraft which have an important effect on intermovement times are weight and speed. Weight is the dominant factor and acts via the wake turbulence separation which must be imposed to protect following aircraft from the dangerous turbulence in the wake of preceding (usually heavier) aircraft. For the purpose of specifying these wake turbulence separations, aircraft are divided into three main weight classes and it is appropriate to adopt this division for the capacity analysis.

Speed determines the runway occupancy times as well as the travel times over certain specified separation distances on the approach and departure paths.

An independent division of aircraft according to both weight and speed could lead to a large number of types. Fortunately, for civilian aircraft using the major airports there is a rough correlation between weight and speed, the heavier aircraft tending to be the faster. For the capacity analysis it is sufficient to distinguish heavy, medium and light aircraft and to adopt an appropriate mean speed for each class.

Multiple runways

So far the discussion has concerned movements on a single runway. If two or more runways are in use the intermovement times between movements on different runways will, of course, depend on separation rules which take the details of the runway configuration into account. Time-distance charts can again be used to help visualise the sequence of movements and the intermovement times. Figure 3.3 illustrates the case of two intersecting runways as shown in the inset. The two halves of the figure, in which movements on each runway are plotted, are coupled by a common time axis, ie a vertical section represents the same instant in each half of the figure. On the graph for each runway the position of the intersection with the other runway is indicated by the dashed line parallel to the time axis.

A sequence of four movements is shown.

Movement 1 is an arrival on runway 1. It crosses the threshold at time t_1 and the intersection at time t_2.

Movement 2 is a departure on runway 2. It cannot start its take-off run until the time, t_2, at which the previous movement on runway 1 has passed the intersection. It, in turn, passes the intersection at time t_3 allowing:

Movement 3, a departure on runway 1, to start its take-off run. It passes the intersection at time t_3.

Movement 4 is an arrival on runway 2. It must not be closer than distance S_{DA} to its threshold when the previous movement passes the intersection.

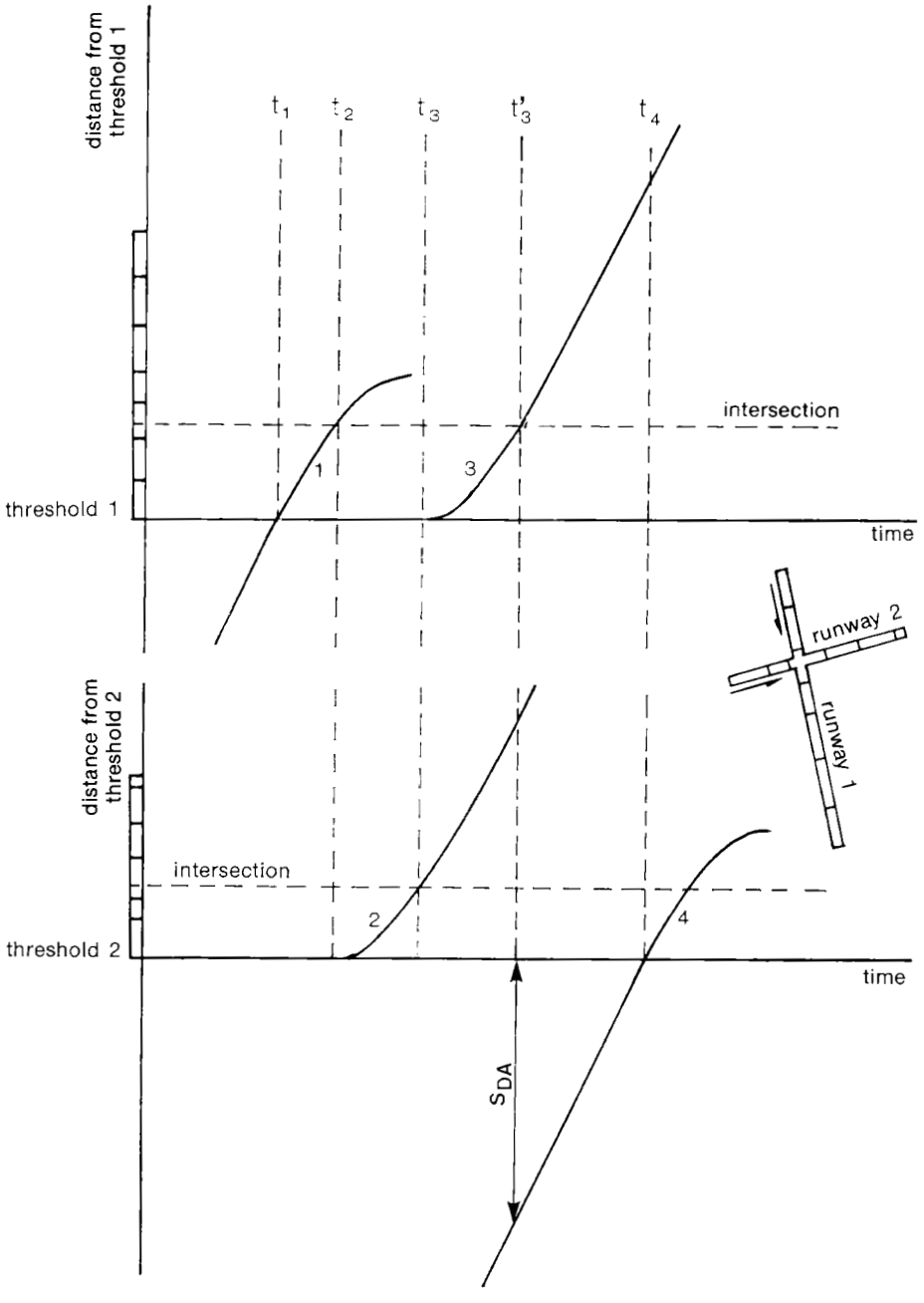


Figure 3.3
Intermovement times on two intersecting runways

In this example there are four movement types, counting arrivals and departures on both runways. By continuing the above analysis all 16 possible movement pairs could be considered and the 4 x 4 intermovement time matrix built up. The matrix would most conveniently be laid out as in Figure 3.4.

Interaction between runways

In Figure 3.4 the different movement types have been ordered so as to keep together all those on the same runway. The matrix can then be divided into a number of blocks associated with particular runways and pairs of runways. Thus the upper left and lower right (diagonal) blocks describe movement pairs on the *single* runways 1 and 2 respectively while the upper right and lower left (off-diagonal) blocks describe the interactions *between* the two runways.

The degree of interaction between runways is indicated by the values of the elements in the off-diagonal blocks. If they are, on average, of the same order as those in the diagonal blocks the interaction is strong (as in the above case of intersecting runways) and, as will become clear, relatively little capacity advantage is provided by the second runway. On the other hand, if the off-diagonal block elements are small, or zero, the runways operate essentially independently (as, for example with widely spaced parallel runways) and the capacities of the single runways may be added.

This brief discussion illustrates the main features of the analysis. In Appendix I, a 12 x 12 intermovement time matrix is developed for the principal runway configuration at Kingsford Smith Airport. The 12 movement types considered are arrivals and departures of heavy, medium and light aircraft on each of the two intersecting runways.

INTERMOVEMENT TIMES AND CAPACITY

In the above discussion, the intermovement times were defined in the context of a continuously busy period during which successive aircraft movements follow each other as closely as possible (successive aircraft being separated by their intermovement times). According to the definition given in Chapter 2 the runway capacity is equal to the number of aircraft movements during a continuously busy hour. As the busy hour is made up of the sum of the intermovement times, the runway capacity is dependent on which movements occur (the movement mix) and in what order (the air traffic control strategies).

The definition of movements used in this report includes aircraft type, whether landing or taking off and runway used. Two points should be made about this definition. In previous work the aircraft mix has usually been specified separately from the ratio of arrivals to departures. This implies that the arrival-departure ratio is the same for all aircraft types, which is not generally the case. In the long term, of course, arrivals must equal departures if the airport is not to fill up or become empty. In the shorter term, however (periods of the order of an hour), the arrival-departure ratio for different aircraft types can vary widely and can have a profound effect on capacity.

The second point concerns the inclusion of the runway in the specification of movement type. The aircraft type, and whether it is arriving or departing, is given unambiguously by the demand during the period of interest. The choice of runway, however, is dependent on such things as weather, pilot preference, and air traffic control strategies. To some extent the choice is constrained by the limitation of certain movements to certain runways because of runway length, strength, instrumentation or noise regulations, but there remains an assignment which can be made only on the basis of experience and a knowledge of the actual runway usage at the airport during busy periods.

The capacity calculation

It is convenient to label the movement types with an index i. In the example above, involving arrivals and departures on two runways, i would take the values 1, 2, 3 or 4 to

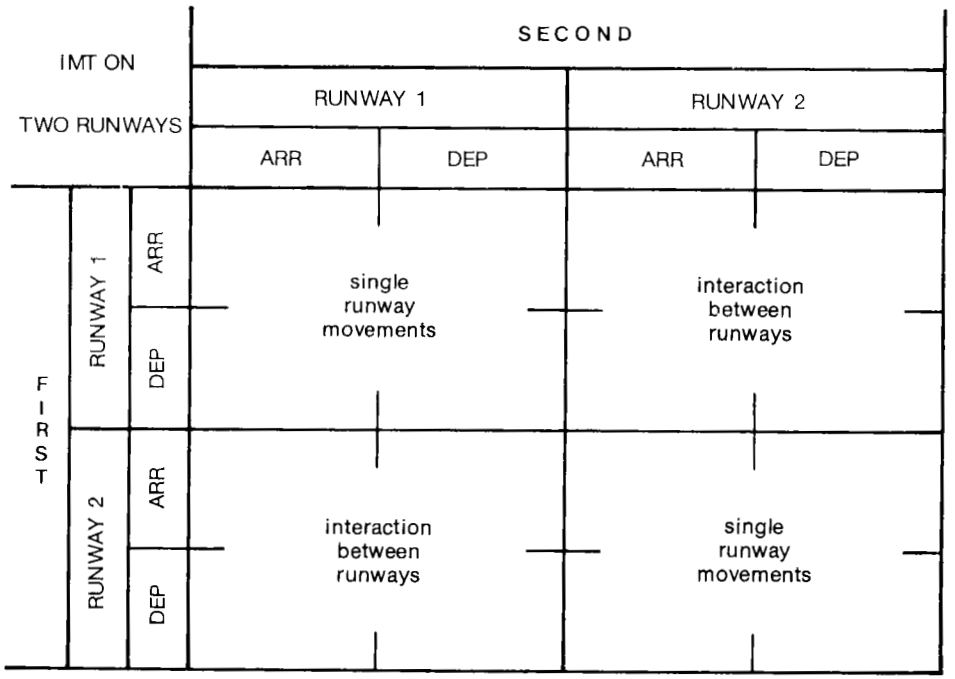


Figure 3.4
Intermovement time (IMT) matrix for multiple runways

represent the four movement types. In the numerical example below i takes the values A or D (arrival or departure). The proportion of movements of type i can be denoted p_i, and the movement mix is then defined by the set of quantities p_i for all values of i. For example $(p_A, p_D) = (0.4, 0.6)$ represents a mix with 40 per cent arrivals and 60 per cent departures.

The elements of the intermovement time matrix can be denoted t_{ij}, the time between a movement of type i followed by one of type j, and the proportion of such pairs (i, j) of adjacent movements can be denoted pij. These values can be recorded in a *pair probability matrix* (p_{ij}) similar to the intermovement time matrix (t_{ij}). The p_{ij} values are determined by the movement mix and by any air traffic control sequencing procedures or priority rules as discussed in the next section.

With this notation the average intermovement time for a sequence of movements during a busy period is given by:

$$T = \sum_{ij} p_{ij} t_{ij}$$

The capacity during the busy period is given by:

$$C = 1/T$$

Usually capacity is expressed in movements per hour while intermovement times are given in seconds. In which case:

$$C = 3600/T$$

A numerical example

The calculation can be illustrated by the simple case of a single runway being used for arrivals and departures by a single type of aircraft. There are only two movement types, arrivals (A) and departures (D). Assume an intermovement time matrix:

t_{ij} (sec)	A	D
A	120	55
D	65	80

and a movement mix:

$$(p_A, p_D) = (0.5, 0.5)$$

ie equal numbers of arrivals and departures.

The pair probabilities pij depend on the sequencing procedures. If the sequence of arrivals and departures is random and aircraft are dealt with on a first come first served basis it is easy to show that:

$$p_{ij} = p_i p_j$$

and the pair probability matrix becomes:

p_{ij}	A	D	
A	0.25	0.25	(first come first served)
D	0.25	0.25	

The average intermovement time is then

$$T = \sum_{ij} p_{ij} t_{ij} = 80 \text{ sec}$$

and the capacity is

$$C = 3600/T = 45 \text{ movements/hour.}$$

Two other sequencing procedures are easy to deal with. If arrivals were given priority (and were available) all the arrivals could be dealt with before all the departures. Then apart from one change over pair, (A, D), there would only be (A, A) and (D, D) pairs. The pair probability matrix would become:

p_{ij}	A	D	
A	0.5	0	(arrival priority)
D	0	0.5	

This leads to an average intermovement time,

$t = 100$ sec

and a capacity,

$C = 36$ movements/hour.

The other possibility is to alternate arrivals and departures. This would result in a pair probability matrix

p_{ij}	A	D	
A	0	0.5	(alternate)
D	0.5	0	

which leads to an average intermovement time

$T = 60$ sec

and a capacity

$C = 60$ movements/hour.

Thus different sequencing procedures clearly have a considerable effect on capacity. Compared with the capacity under the first come first served procedure, a policy of priority for arrivals reduces capacity by 20 per cent while alternation of arrivals and departures increases it by 33 per cent.

APPLICATIONS OF THE ANALYSIS

While the above discussion has been deliberately simplified in order to emphasise the main ideas, it provides a valuable framework for the consideration of problems relating to runway capacity.

Some of the contexts in which the analysis may be useful in the early stages of assessment are as follows.

- The planning of new or extended runway configurations. The intermovement time matrix provides a clear presentation of the information about interactions between runways and their effect on capacity.
- Modifications to the separation rules.
- The introduction of new technology. Innovations such as the microwave landing system or wake turbulence monitoring equipment will permit reduced separation in certain circumstances.

In Chapter 5 this capacity analysis is incorporated in a simple delay model where it provides the basis for a realistic description of runway operations taking into account the effect of fluctuations in the mix of aircraft.

LIMITATIONS OF THE ANALYSIS

In analysing the operations of complex systems it is important to be aware of the limitations to any particular depth of analysis. It will be apparent in the above discussion that many approximations and aggregations have been made. These are of

two types. The first is that in which the approximation could be improved or removed entirely within the existing framework of the analysis. For these a judgement must be made as to whether the additional effort or complexity is justified by the increased accuracy of the results. An example, in which the effort is justified is the 'triplet correction', described below, where errors of the order of 10 per cent can be eliminated relatively easily. The extension to 'quartet' and higher order corrections is not, however, considered justified.

Another approximation which could be progressively improved within the present framework is the aggregation of aircraft into a given number of categories. In Appendix I, three weight categories are considered sufficient although nothing but computational and data gathering effort prevents the use of more or different categories.

The second type of approximation is that which is inherent in the analysis and which could only be improved by a more or less radically different approach. An example is the use of the first come first served policy, adopted because of the difficulty of modelling any more complicated sequencing policy. This is probably the dominant systematic error and tends to reduce the calculated capacities.

The overall error in the absolute value of these calculated capacities is difficult to analyse but can be observed empirically to be in the range 5-15 per cent. This is considered adequate for the analysis whose aim is not so much the prediction of absolute values as the correct description of the variation of capacity with changes to its determining factors.

The triplet correction

It has been assumed that the time at which a movement occurs is restricted only by the time of the previous movement, and not by earlier ones. This was the justification for considering only pairs of movements. In fact the assumption fails for any triplet of movements (i, j, k) for which

$$t_{ik} > t_{ij} + t_{jk}$$

In this case, clearly, movement k is restricted by the next to last movement i rather than by movement j. When aircraft are treated on a first come, first served basis, a relatively simple correction to the capacity calculation can be applied and is described in Appendix III. For typical intermovement time matrices and movement mixes it results in a significant correction which reduces capacity by about 5-10 per cent.

It is, of course, possible to consider higher order corrections in which even earlier movements have an effect. These have been ignored because their effect on the final capacity is generally less than 1 per cent.

Air traffic control strategies and the p_{ij} matrix

The 2 x 2 arrival and departure example illustrated the advantages to be gained by choosing an appropriate strategy which avoids, so far as possible, sequences of movements having large intermovement times. In this very limited example it was possible to specify precisely the (p_{ij}) matrix corresponding to each pure strategy. In more realistic cases, with larger numbers of movement types, both the specification of the (p_{ij}) matrix for the pure strategy, and the actual application of the strategy, become very much more difficult.

Attempts to analyse the effects of priority rules lead to similar problems. In practice, of course, priority rules and pure strategies can never be applied rigidly, but are continually modified by experienced operators as circumstances demand. The incorporation in the analysis of such deviations from the rules is impractical.

In fact the overall result of the complexity of the system, together with the random fluctuations in the demand, is frequently a service pattern which is not very different from first come first served, at least from the point of view of capacity calculations. As noted above this is the easiest strategy for which to specify the (p_{ij}) matrix. Since some sequencing *does* occur the capacities derived will tend to be pessimistic, but the errors are not great and to some extent are offset by other approximations in the theory.

CHAPTER 4—THE MEASUREMENT OF DELAY

In view of the importance of delay as the principal and most publicly visible measure of the degradation of service due to congestion, it is, at first sight, surprising to find that no routine measurements of delay are made.

In fact there are considerable methodological difficulties in making direct measurements of delay on the basis of information currently recorded on control tower flight strips and airline trip records.

This chapter discusses some of these difficulties and then describes a statistical approach which makes use of the airline trip records in an indirect way.

THE DIFFICULTY OF DIRECT MEASUREMENT

As was pointed out in Chapter 2 the attribution of delay to the different components of the airways system is a difficult exercise. A landing ten minutes behind schedule may be due to runway congestion, but it may equally be due to head winds en route or even to a late departure at the airport of origin. In order to identify that component of delay due to runway congestion, it would be convenient to be able to identify a time and position at which the aircraft makes its demand on the runway system and before which no delay due to runway congestion is experienced. Unfortunately this is not possible, even though aircraft arriving at an airport do pass over a well defined arrival fix and are required to report to the control tower at that time. The reporting time is recorded on the control tower flight strip for that aircraft although its actual position when the report is made may vary over 20-30 miles. At that time an estimate of the aircraft's landing time is made and recorded on the flight strip. During congested periods aircraft may be required to lose time by following more or less circuitous routes and the estimated landing times take such manoeuvres into account. Indeed, if congestion is serious, delays may be absorbed well before the arrival fix and in extreme cases departure from the airport of origin may be delayed.

Departure delays are, in principle, more readily measured because they occur within the confines of the airport. An aircraft can be considered to be ready to leave and to make its demand on the runway system as soon as loading is complete. If clearance to start engines is then requested and received promptly, the time and the take-off (wheels off) time, which are recorded on the control tower flight strips and by the airline, can form the basis of an analysis of departure delay. Unfortunately, there is no guarantee that the engine start time follows promptly on the completion of loading. If departure delay due to runway congestion is expected and there is no pressure for an early release of the loading gate, the pilot may prefer to absorb part of the delay at the loading gate before starting the engines. Once again, as with arrivals, the comparison of an actual with a scheduled departure time is not helpful because it takes no account of the source of delay.

A further difficulty associated with the use of schedules is that they already contain an allowance for expected delay. In the case of arrivals, average delays measured with respect to schedules reflect only the increase since the schedule was established. Indeed, in a static situation, average delays might appear to decline as schedules take more fully into account the actual duration of flights.

THE STATISTICAL ESTIMATION OF DELAYS

In the absence of a satisfactory direct method of measuring delay Carlin and Park (1969) employed a statistical technique in their study of the three major New York airports. The method is applicable, in principle, to both arrivals and departures although different systematic difficulties arise in each case.
In the present context it is applied to domestic jets using KSA on interstate routes.

Arrivals

The method makes use of records of the actual flight times (wheels off to wheels on) for arrivals at the airport of interest, in this case KSA. These flights are by aircraft of different types, coming from different origins and arriving at different times of day. Each flight time is assumed to be the sum of three components: the average undelayed time which depends only on the route and the aircraft type; the average delay due to congestion which is expected to reflect the daily demand profile and accordingly depends on the time of day of the aircraft's arrival at KSA, and a random term which represents all other influences which cause an actual flight time to deviate from the average. Formally the model is:

$$F_{ijk} = T_i + D_j + R_{ijk}$$

T_i is the average undelayed time for the i-th route/aircraft combination. For example i might represent DC-9 aircraft on the Melbourne to Sydney route. D_j is the average delay experienced by arrivals at KSA during the j-th period during the day, for example between 7 pm and 8 pm. R_{ijk} then represents all other influences on the actual flight time of the k-th instance of a DC-9 arriving at Sydney from Melbourne between 7 pm and 8 pm. F_{ijk} is then the actual duration of this flight.

The technique used is regression of the F_{ijk} against two sets of dummy variables, one representing the route/aircraft combinations and one representing the time of day. The calculation yields estimates of the T_i and D_j as coefficients of each set of dummy variables.

Departures

In principle, a similar analysis can be applied to departures with taxi-out time taking the place of flight time and the taxi path from terminal to departure threshold taking the place of the flight route from the city of origin.

In fact the similarity of taxiing performance of the aircraft considered allows the distinction between aircraft to be ignored. Furthermore, as the vast majority of domestic jet take-offs are on runway 16 for noise reduction reasons, it is also possible to ignore the contribution of the taxi path to the variation in taxi-out time. The route/aircraft component and hence the first set of dummy variables can therefore be eliminated from the model and the taxi-out time assumed to be composed only of a fixed average plus the congestion delay and the random contribution.

A serious systematic problem arises, however, in the application of this analysis to departures. An essential assumption is that the time whose variation is being studied necessarily contains the congestion delay. In the case of arrivals the assumption is valid except in the extreme case of a flight departure being delayed due to congestion at the destination. In the case of departures, however, the taxi-out time does not necessarily include the delay. On the data records used in this analysis the taxi-out time is defined as the time between the engine start and the aircraft becoming airborne. As pointed out above, part of the delay may be absorbed before engine start and would not be detected by this analysis.

Data source

Data on the actual flight and taxi times were provided by Ansett Airlines of Australia who kindly made available copies of the trip-records for all flights by their aircraft

arriving and departing KSA during 1979—some 29 000 records. These records and the details and limitations of the regression technique are described in Appendix IV.

Patterns of demand and delay

The delays D_j have been described as depending on time of day because there are fairly well defined patterns of busy and slack periods throughout the day. These patterns also have a weekly cycle in which there is a characteristic demand (and hence delay) profile for each day of the week.
Thus weekdays exhibit a morning and evening peak with the Monday morning and Friday evening peaks being particularly pronounced. Fridays tend to have 10-15 per cent more movements than the mid week average while Saturdays and Sundays have 15-20 per cent fewer movements and a flatter profile.

In order to bring out these daily and weekly patterns the Ansett trip records were sorted according to day of week and the regression model applied to the data for each day separately. The results are the mean delay profiles averaged over the year for 'typical' days. Certain public holidays and some adjacent days are clearly not typical in their pattern of movements, and a total of 37 such 'atypical' days were excluded from the analysis.

RESULTS

The average delay profiles for arrivals (A) and departures (D) for each day of the week are shown in Figure 4.1. The error bars give the standard error of the estimate of the mean delay. It should be emphasised that the standard error is *not* a measure of the standard deviation of the delay to individual aircraft. In queueing systems this standard deviation is large, typically of the same order as the mean.

The delays resulting from these calculations are not absolute, but *relative* to the delay occurring during a specified reference period. This delay can be assumed small if the reference period is chosen when demand is slack; the delays resulting from the regression calculations then approximate the absolute delays during successive periods. For arrivals the reference period is 0700-0800 on Sunday morning; for departures 1500-1600 on Sunday afternoon.

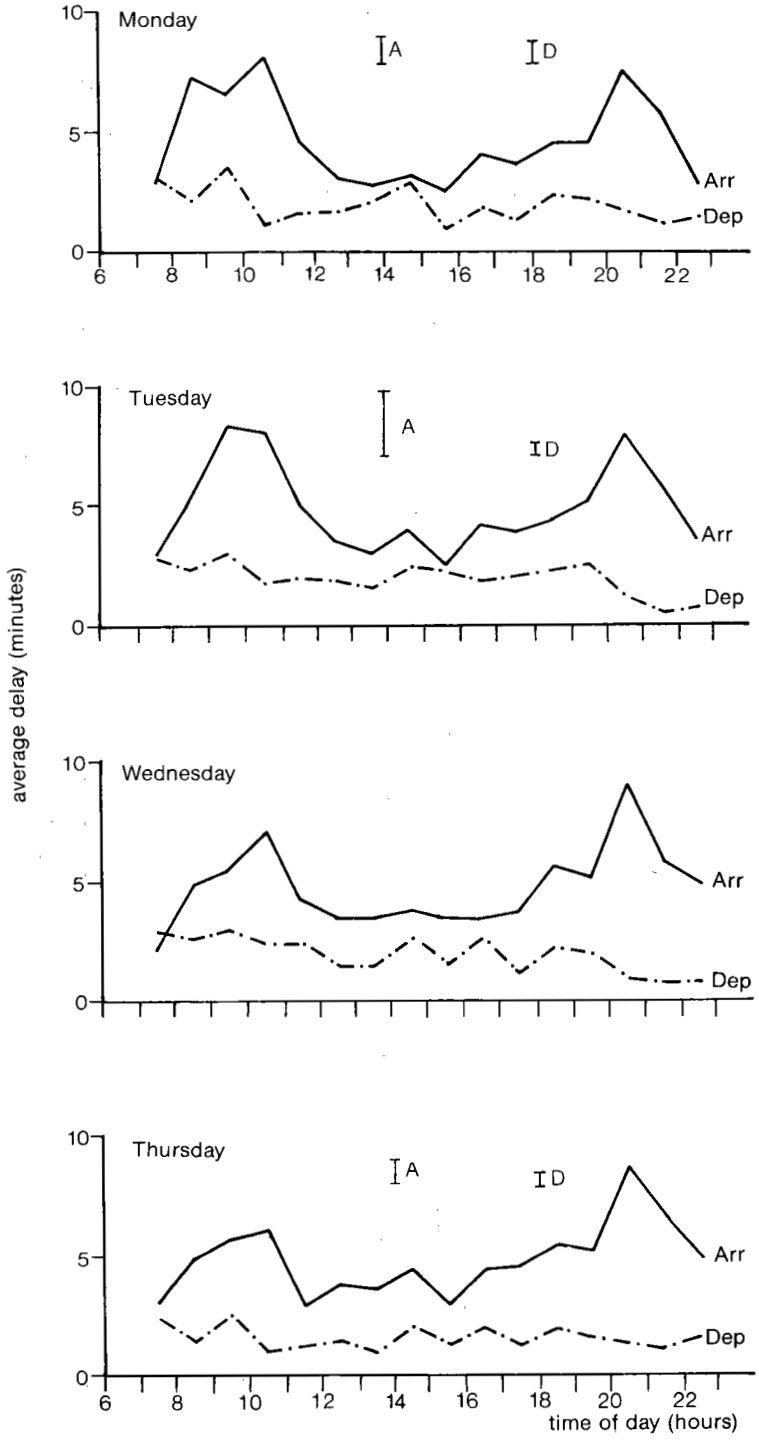
Arrivals

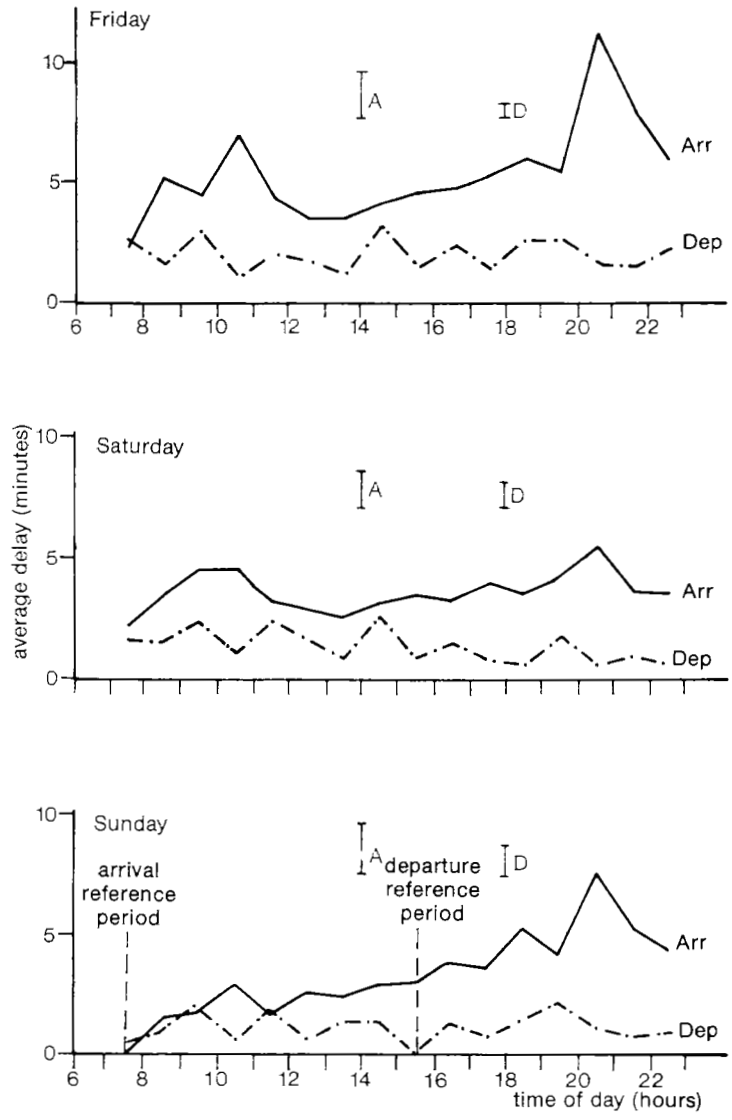
These results constitute the first estimate of the level and distribution of average delay for regular public transport arrivals over an extended period (the calendar year 1979). Qualitatively, they are as expected. The weekdays exhibit a morning and evening peak, reflecting the demand at these times, with the evening peak tending to grow relative to the morning peak from Monday to Friday.

The maximum average delay of about 11 minutes occurs during the Friday evening 2000-2100 peak, due largely to traffic 'going away for the week-end'. Delays on Saturday and most of Sunday are low reflecting the smaller number of movements on those days. The Sunday evening and broad Monday morning peaks correspond to traffic returning from the week-end for the start of the business week.

Departures

The departure delay profiles are not informative. The variations with time is hardly more than the standard error of the estimate and there is no sign of the strong congestion delay peaks found for the arrivals. This is due to the possibility of aircraft absorbing a large part of the delay during congested periods before engine start. This delay is therefore lost to the analysis.


Monday
Tuesday
Wednesday
Thursday
A
D
Arr
Dep
10
5
0
6
8
10
12
14
16
18
20
22
average delay (minutes)
time of day (hours)



A-arrivals

D-departures

I=standard error on estimate of mean

Figure 4.1
Average relative delay profiles at Kingsford Smith Airport by day of week

CHAPTER 5—DELAY MODELLING

MODELLING APPROACH

The mathematical modelling of real systems can be approached from two opposing points of view which might be described as 'model everything' and 'model as little as possible'.

The 'model everything' approach generally implies computer simulation as only the very simplest of real systems are completely amenable to mathematical analysis. The principal advantage of this approach is usually claimed to be its realism; in so far as the model *is* a complete description of the real system *any* question which can be asked about the real system can be asked of the model. It is therefore not necessary to specify in advance the problem to be studied or to frame a narrow set of questions. Indeed, interest may subsequently shift to new aspects of the system but, so long as all components of the system are faithfully represented, the model will still be applicable.

This feature has its negative side. The automatic inclusion of 'everything' (or of everything that seems relevant) does not necessarily provide an insight into the relative importance of the different factors modelled or help to identify causal relationships. Complex series of carefully designed experiments with the model may still be required to obtain an understanding of the system. The systematic variation of the large quantities of data required to specify all the factors modelled can be very time consuming.

A further problem, associated particularly with computer simulation, is the use of Monte Carlo sampling techniques to represent stochastic processes. A given run of the model then provides results which constitute a sample from a distribution. Many runs (large samples) may be required to provide confidence in the statistical means and standard deviations of the performance measures of interest.

The alternative 'model as little as possible' approach is based on the assumption that there is a limited range of questions of interest which it is the model's task to answer. The model is then developed with these questions in mind, and only those features of the real system which directly affect the aspects of interest are explicitly modelled. The choice of what to model and what to leave out obviously presupposes a good understanding of the system and a significant part of the modelling effort may be spent acquiring the understanding to make this choice. Furthermore many of the features which are not modelled explicitly must nevertheless be taken into account in some aggregated way.

The limitations of this 'model as little as possible' approach are evident. The model is tailored to a particular range of interests and may no longer be appropriate if these interests change. The simplifications implicit in the aggregation of many features of the real system inevitably introduce systematic errors. In many cases different systematic errors may tend to cancel each other and in general their signs and approximate magnitudes will be known; nevertheless these errors do place limits on what can be expected of the absolute values of the results.

However, this is not always a a major limitation. Most modelling endeavours, including those which adopt the 'model everything' approach, are intended to provide a means for *comparing* the relative merits of policies or configurations of the system. What is important then is not so much the absolute values of the quantities studied, whose magnitudes will generally be known from experience of the real system, as their dependence on the factors which are varied.

The advantages of the 'model as little as possible' approach are the reduced effort devoted purely to model building (and programming), a simpler model, and the reduced quantity of input data required. If, in addition, the decision to adopt this approach has reduced the problem to one amenable to analytic techniques, other advantages may be available. In particular, stochastic processes may be treated by well-known analytic techniques which yield means, variances and other statistical information from a single run.

In this work the second approach has been adopted.

TIME DEPENDENT QUEUEING MODELS

A set of delay models has been developed for the assessment of the performance of runway systems in terms of congestion and delay for any given pattern of demand. The models are analytic ones based on queueing theory (Wagner 1969) which is a quantitative description of the passage or flow of 'customers' through a facility which provides a 'service'. The system is described in terms of such features as the demand and service processes and the queueing disciplines, the objective being to calculate measures of performance such as queue lengths and waiting times. The random fluctuations characteristic of queueing systems are specifically taken into account giving results which are long term averages and variances of the measures of interest. This is to be contrasted with the simulation approach where the output would represent the delay or congestion on a particular hypothetical day.

The essence of the approach is to define a set of quantities $P_n(t)$ which are the probabilities at time t that there are n 'customers' in the system. The variation with time of the $P_n(t)$ is then described by a set of differential or difference equations whose coefficients depend on the parameters which specify the demand and service processes. A knowledge of the $P_n(t)$ allows the measures of performance of the system to be calculated.

Early applications of queueing theory were limited to *steady state* solutions of the equations, a fact which made the approach almost useless as a description of airport systems where one of the main characteristics is the rapid variation of the demand. However, modern computing facilities permit the equations to be solved numerically in the general case and in particular to take account of varying demand and service rates (Koopman 1972).

THE SINGLE QUEUE APPROXIMATION

The description of the aircraft/airport system as a single channel, single server queue is consistent with the capacity analysis given in Chapter 3. That analysis takes account of much of the complexity due to different aircraft types and multiple interacting runways, and the definition of capacity corresponds exactly to that of a service rate as used in queueing theory. A single-queue model, with the service process based on this capacity analysis, will therefore give an adequate description of the aggregate performance of the system. Subsequently an approximate disaggregation of the results will allow the total delay to be allocated among the different movement types.

The size of the system

If the demand rate exceeds the runway capacity for any lengthy period the queues of waiting aircraft may grow without limit. However, the calculations cannot be performed with an unlimited number of variables (the $P_n(t)$) and equations, and so an arbitrary limit N is placed on the number of aircraft which can be in the system at once. The interpretation of this device depends on whether the aircraft are landing or taking off. During very congested periods it is realistic to expect that no more than a certain number of aircraft would be stacked, waiting to land, and that subsequent arrivals would be diverted to other airports. The interpretation is less clear in the case of aircraft taking off, as any turned away are nevertheless still at the airport waiting to take off. The

problem can be avoided in most cases by choosing N sufficiently large for the probability, $P_n(t)$, that the system is full, to be very small. In any case, situations in which the queue lengths regularly approached a reasonably large limit, N, would be of little interest to a planner as they would be accompanied by unacceptably long delays.

The demand process

The demand process describes the fluctuations in the sequence of requests for use of the runways and is specified by the distribution of times between successive requests. The Poisson process describes a 'random' sequence of requests occurring at a specified average rate and with a negative exponential distribution of inter-request times. It is known to be a good description of many real randomly changing flows. It has the added advantage of being easy to treat mathematically.

The time of each request is taken to be the time at which the movement would have 'occurred' (in the sense described in Chapter 3) if there had been no delay due to other movements. This definition follows from the special features of the concept of service time as applied to runway movements and will be discussed further in the next section.

The varying demand for runway use is usually given in the form of a demand profile, or histogram, as in Figure 6.1 which shows the average demand rate (for all movement types) during successive periods throughout the day. The approximation inherent in the abrupt changes between constant levels can be reduced by shortening the time periods in the demand profile. Periods shorter than one hour are not usually justified by the available information on demand or by the overall accuracy of the analysis.

In the models a Poisson demand process is assumed with the average rate in each period given by the total of the demand rates for each movement type. Even when planned schedules suggest that more regular demand might be expected, the Poisson process will still be the best description of the demand process, if delays and uncorrelated events in other parts of the airways system cause disturbances to the scheduled request times of the same order as the intervals between these requests. This will be the case, particularly during congested periods when delays are greatest. The approximation is poorer during slack periods, but these contribute little to the overall delay.

The service process

The operational description of aircraft movements on the runway system given in Chapter 3 and the concepts introduced there form the basis for an appropriate description of the service process.

The service process is specified by a probability distribution for the service times and the reciprocal of the mean service time is identified with the airport runway capacity. The service times in question are identified with the intermovement times defined in Chapter 3 although this involves an important modification of the concept of service time. The issue has been discussed in Chapter 3 but it is as well to re-emphasise the point in the present queueing context. The important point about service is the constraint it imposes on the following customer rather than what happens to the customer currently being served. Normally, of course, the two are closely related. In the conventional single server queue the service time of a customer *is* the constraint on the following customer who cannot begin his service until the previous service is complete. This is not the case in the airport context. Referring to Figure 3.2, movement 2 is constrained by movement 1 to cross the threshold at time t_2 even though the earlier aircraft has completed its (runway) service and vacated the runway significantly before t_2. Similarly in Figure 3.3, the aircraft of movement 2 commences its take-off run at time t_2 while the previous aircraft (movement 1) is still being 'served' on the intersecting runway—but beyond the intersection. Thus the role of constraining the time of occurrence of a following movement, normally played by the service times, is played in the airport context by the intermovement times. Their probability distribution is given by the (t_{ij}) and (p_{ij}) matrices.

An example of such a probability distribution is given in Figure 5.1 where the histogram gives the probability that an intermovement time fall within a given 20 second interval. The distribution is based on the (t_{ij}) matrix for the main runway configuration at Kingsford Smith Airport under visual meteorological conditions as presented in Appendix I, and on a movement mix representative of a moderately busy period. The distribution has a mean intermovement time of 70.9 seconds, corresponding to a runway capacity of 50.8 movements/hour, and a standard deviation of 39.7 seconds. An important parameter for the classification of service time distributions is γ, defined as the ratio of the variance to the square of the mean; in this case $\gamma = 0.3$.

A general distribution such as that shown in Figure 5.1 is difficult to treat analytically. In terms of the parameter γ however, it is clearly intermediate between the limiting cases of a fixed service time ($\gamma = 0$) and a 'random' service time ($\gamma = 1$) described by a negative exponential distribution. These two distributions are analytically more tractable and have formed the basis for the majority of queueing theory analyses. Steady state results are, however, available for intermediate distributions ($0 < \gamma < 1$) for which it can be shown that average queue lengths (and hence average delays) are proportional to $(1 + \gamma)$. In this case therefore average queue lengths double in passing from fixed ($\gamma = 0$) to random ($\gamma = 1$) service times. Although this linear dependence of average queue length on γ does not apply in the time dependent case, increasing randomness in the service time distribution is still expected to result in longer average queue lengths. The results for the intermediate distribution ($0 < \gamma < 1$) should therefore be bounded by those for the fixed ($\gamma = 0$) and random ($\gamma = 1$) distribution. By examining the behaviour of the queue and the resulting delays under the two limiting assumptions, for which the calculations are relatively straightforward, the sensitivity to the service time distribution can be estimated, and the results appropriate to the actual distribution approximated (with sufficient accuracy for planning purposes) by interpolating between the two on the basis of the parameter γ. This is illustrated by an example in Figure 5.2 where the average delay profiles resulting from the limiting assumptions about the service time distribution are plotted, together with the interpolated estimate of the actual profile. (The circumstances leading to these delays are the initial conditions of the example analysed in Chapter 6).

In Appendix V the equations describing the queueing system under the two limiting assumptions for the service time distribution are presented and aspects of their solution discussed. The formulae relating the system performance measures to the $P_n(t)$ obtained from the equations are also presented.

MEASURES OF PERFORMANCE

The immediate results of the queueing calculations are the quantities $P_n(t)$ which permit a wide variety of measures of performance of the runway system to be evaluated. The choice of the most appropriate measure in any particular case will depend on the user and on the issues being addressed.

Average or expected delay

The average delay to be expected by aircraft using the runway system at a particular time of day is the most publicly visible measure of the airport's performance. It is the delay which must be allowed for in airline schedules and which the individual user must take account of in deciding to use the airport. The average delay fluctuates throughout the day with its peaks lagging slightly behind the peaks in the demand profile (because the queues build-up during the demand peaks and remain as backlogs to be served after the demand has fallen to more normal values).

Total delay

The overall economic impact of congestion will be reflected in the total delay suffered by all users during the day. To be really useful in this context, however, this overall total should be disaggregated (see next section) into separate totals for the different movement types which may be subject to very different costs of delay.

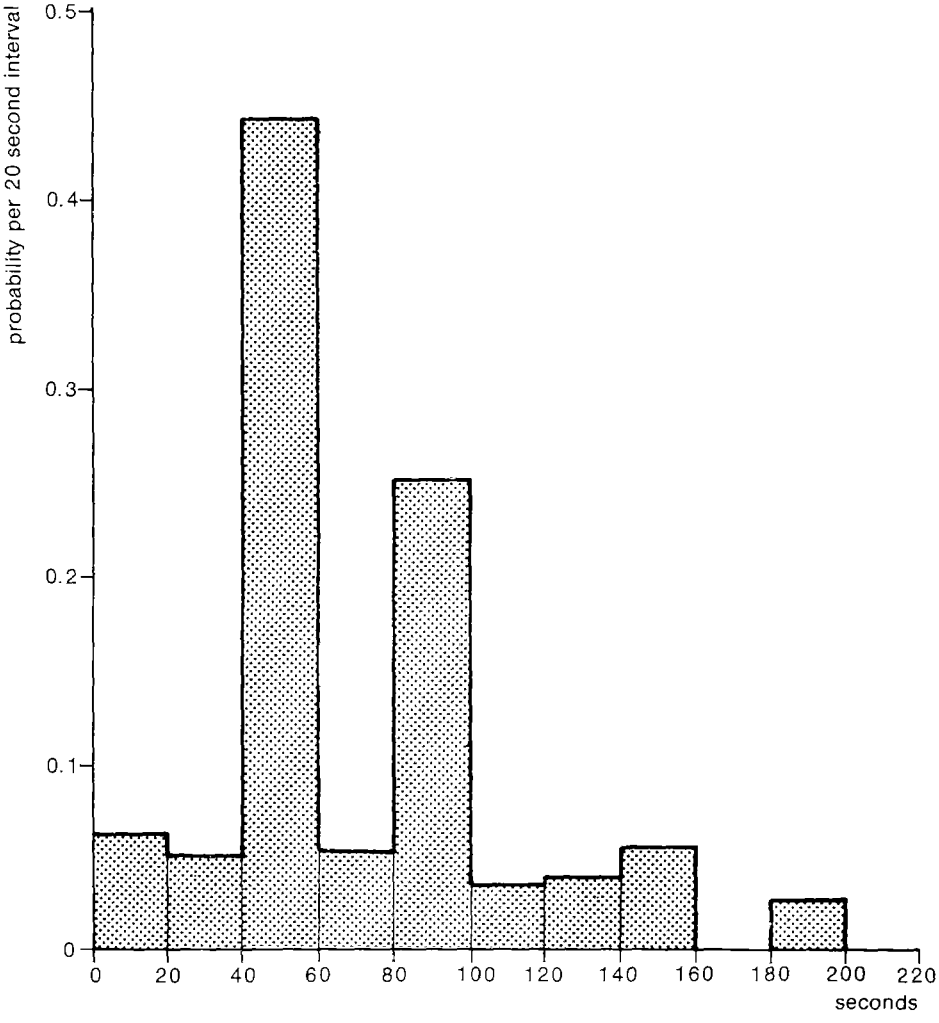


Figure 5.1
Example of distribution of intermovement times for Kingsford Smith Airport runways 16 and 07 under VFR

Marginal delay

An additional (marginal) user of a congested facility not only suffers the average delay associated with the level of congestion at that time, but, by his (additional) presence in the system, aggravates the congestion and increases the delay to following users. This occurs because the following users are all pushed back one space in the queue and the effect persists until the queue is dissipated. Although the increase in the average delay caused by the marginal user may be small (of the order of the intermovement time) it is suffered by a large number of following users and therefore the contribution to the total delay may be very large. The marginal delay at any time is the increase in total delay due to one additional user at that time. The marginal delay fluctuates throughout the day with its peaks slightly ahead of the demand peaks (because this is where the largest number of following users are delayed by the marginal user). Figure 5.3 compares the marginal and average delay profiles (under the initial conditions of the example in Chapter 6) and illustrates the very high additional delays attributable to the marginal user during peak periods even though the average delays remain moderate.

The marginal delay is relevant to the issue of airport charges. During congested periods marginal delays are typically many times greater than average delays, but only the latter are experienced by users and taken into account in their decision to use the airport—and thereby contribute to the congestion. Proposals to base airport charges on marginal delays have been made as such charges would oblige potential users to take account of the delay costs they impose on others.

Distribution of delay

One of the characteristics of queueing systems is the very broad distribution of delay, typically with a standard deviation of the same order as (or greater than) the mean. In practice, this means that, even while average delays remain modest, a significant proportion of users may be suffering very long delays. In some cases, therefore, a description of delay, more complete than that provided by the average delay, may be useful. This is most conveniently provided by the cumulative distribution function which gives the probability that the delay will be less than any particular value. If the complete function is not required, spot values may be useful (for example, the probability that the delay will not exceed 20 minutes).

The delay models now provide the average, total and marginal delays as these seem to be the most generally useful. However, the distribution of delay and indeed other measures are readily calculated from the $P_n(t)$.

DISAGGREGATION OF THE RESULTS

The queueing models describe a sequence of undifferentiated aircraft movements on the runway system, and the delay results are therefore aggregates for all movement types. This is appropriate for the average delay at a particular time of day as all movements suffer the same average delay if they are treated on a first come, first served basis.

As mentioned above, however, total delay during the day must be disaggregated according to movement type if it is to be useful in an economic analysis. This is because the peaks in the demand profiles for each movement type will not, in general, coincide with the peaks in the aggregate demand or delay profiles. The allocation of delay among the different movement types must therefore be done period by period in proportion to the prevailing movement mix and accumulated to give a separate total delay for each movement type. The process is an approximate one but the errors are small so long as the average delays are small compared with the demand update period (usually one hour).

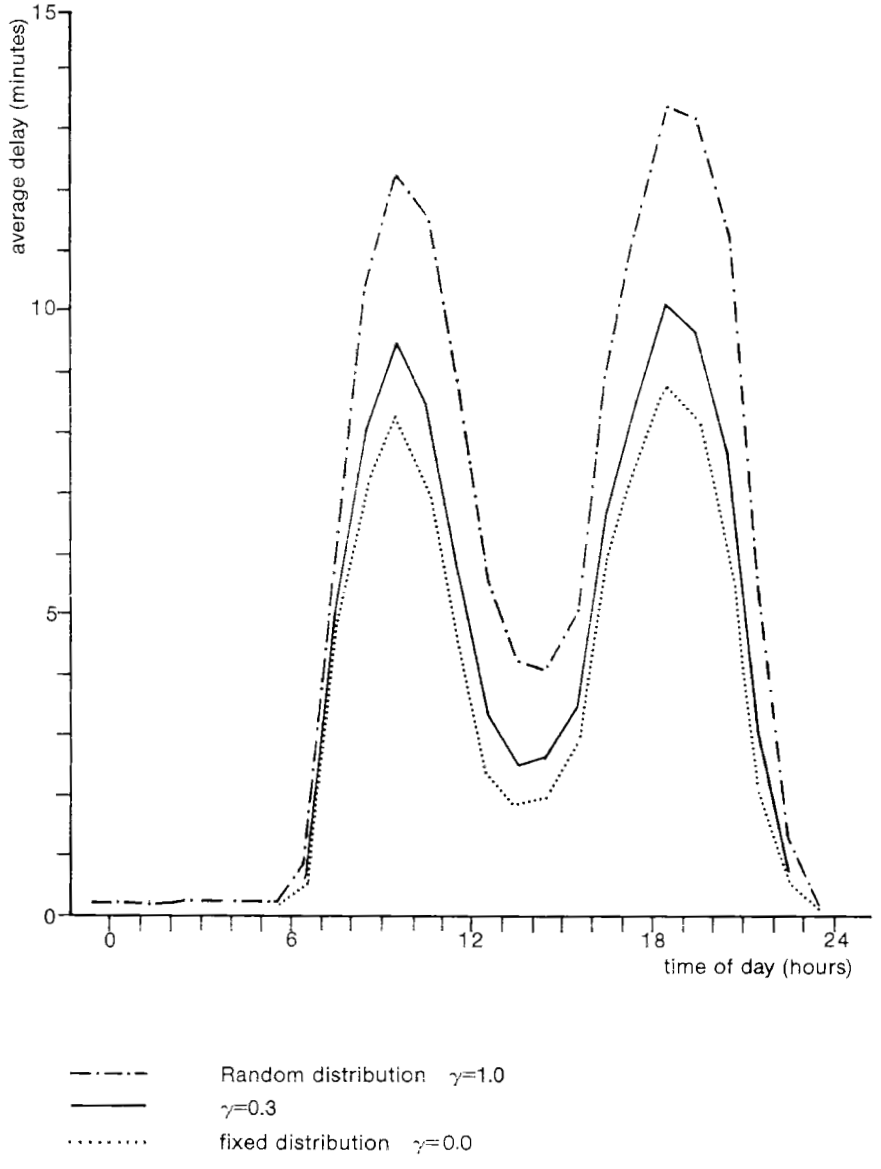


Figure 5.2
Dependence of average delay on intermovement time distribution: Interpolation betweeen limiting cases. (Circumstances as described in chapter 6)

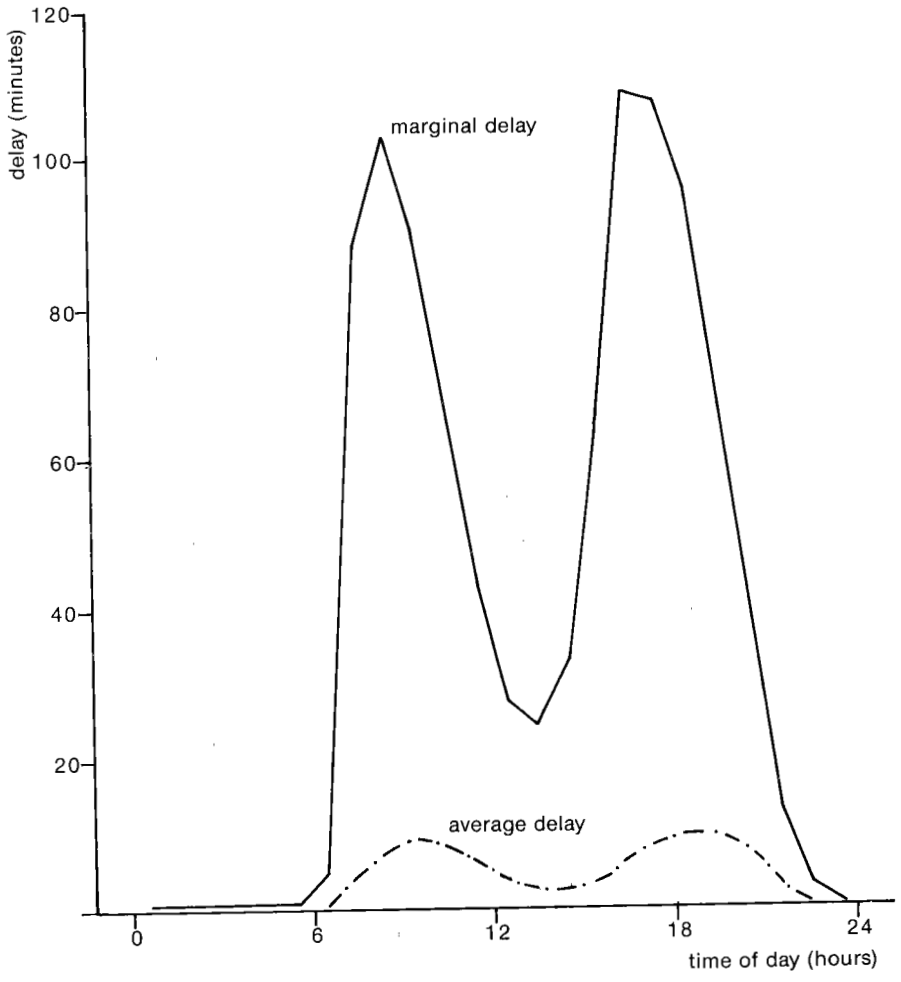


Figure 5.3
Comparison of marginal and average delays.
(Circumstances as described in chapter 6)

IMPLEMENTATION OF MODELS

The models described in this chapter have been implemented as computer programs designed to assist planning and policy assessment. These programs, their input requirements and the form of the output are described in Appendix VI. In the next chapter an example is given of their use in the operational assessment of a range of policies.

CHAPTER 6—POLICY ASSESSMENT AND THE DELAY MODELS

In this chapter the use of the delay models in the context of policy assessment is illustrated with the aid of a hypothetical example. A congested situation, resulting from high demand at Sydney (Kingsford Smith) Airport, is described and analysed with the aid of the models. A range of policies intended to relieve the congestion is assumed to be under consideration. The nature of these policies is not specified, except for the assumption that they result in predictable changes to the demand for use of the runways. The actual determination of the altered demand would be the task of a separate study and is not considered here. The policies are therefore represented by the demand profiles to which they give rise and which are the subject of the delay analysis. The presentation of the resulting system performance (congestion) measures as a function of the demand (and of the corresponding policy) then constitutes the information on which a policy assessment may be based.

Only operational measures of performance, in particular delays, which are the principal outputs of the models, will be considered. For many purposes this may be sufficient. An economic analysis would be more complex and would require that the differing costs of delay experienced by different classes of user be taken into account. These considerations are outside the scope of this paper.

THE EXAMPLE

A congested situation

Figure 6.1 shows the hypothetical demand profile for all movement types, with that portion representing light aircraft movements shown shaded. This profile was obtained by scaling the average hourly demand at Sydney (Kingsford Smith) Airport for Fridays during 1979 (obtained from an analysis of noise monitoring records) to give a total of 725 movements per day[1]. Also shown in the figure is the runway capacity for KSA runways 16 and 07 under VFR as calculated and used by the delay models. The fluctuations in capacity reflect the hourly variation of the mix of movements. The known capacity for this runway configuration, about 50 movements per hour, is obtained during most of the day, with significant deviations only during slack periods when the small numbers of movements distort the mix.

Two peak periods are, somewhat arbitrarily, defined to occur between 0700 and 1100 in the morning and between 1600 and 2100 in the evening (as will be seen below, this choice corresponds approximately to the hours during which the average delay exceeds the 'tolerable' level of 4 minutes per movement).
During these peak periods light aircraft movements account for about 30 per cent of the total demand.

One measure of the congestion problem due to this demand is illustrated by the upper line (labelled '1.0') in Figure 6.2 where the average delay per movement (during each hour) is plotted against time of day[2]. The average delay reaches 9 and 10 minutes during the morning and evening peaks, but it should be remembered that these averages include significant numbers of longer delays.

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1. Note that this profile is *not* intended to be a forecast. In reality, a growth in demand would be expected to result in a smoother profile, with broader peaks, as demand is displaced from the increasingly congested peak periods to adjacent less congested periods. No smoothing has been applied to this profile.

2. Figures 6.2 and 6.3 are plotted from the tabulated computer output described in Appendix VI.

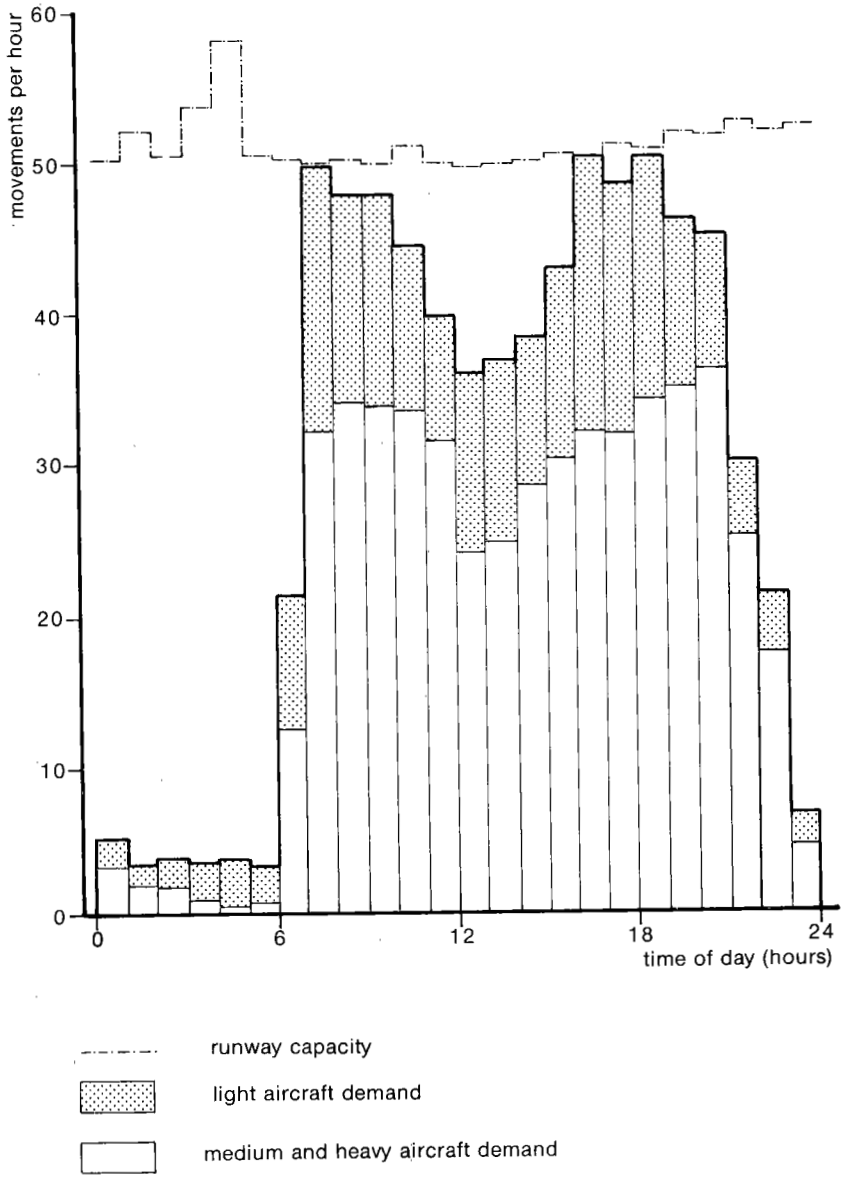

Figure 6.1
Hypothetical demand profile for all movements

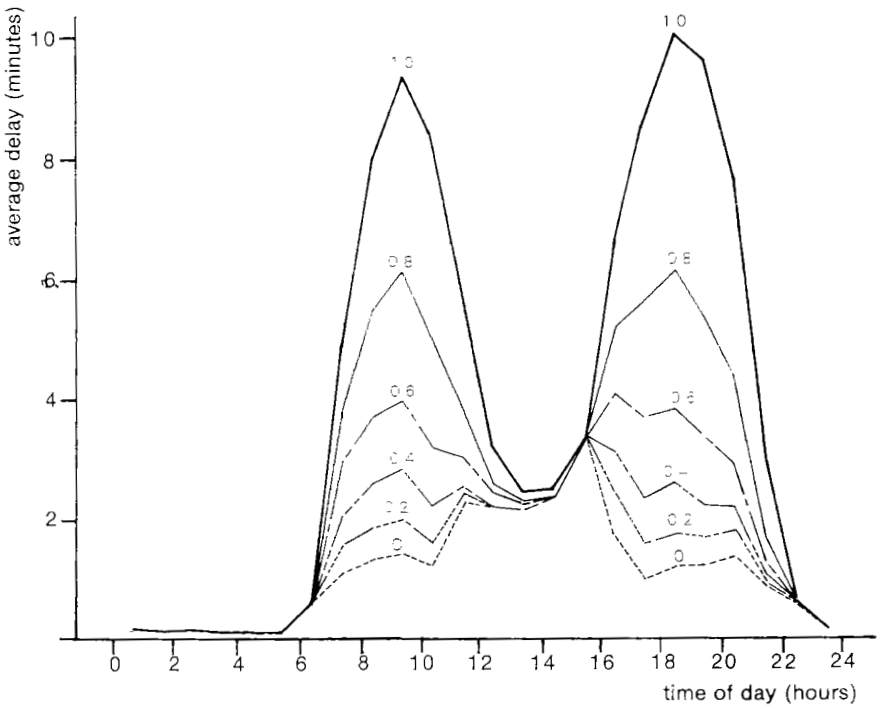

Figure 6.2
Dependence of average delay profiles on light aircraft movements during peak periods

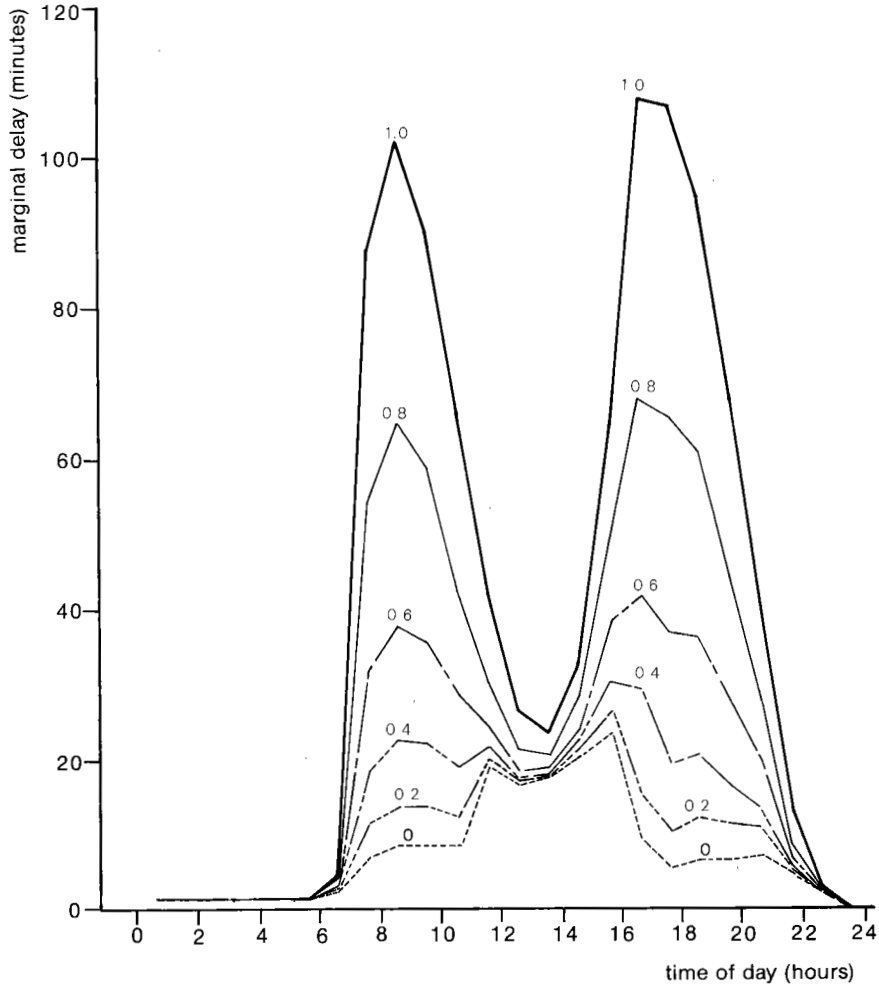

Figure 6.3
Dependence of marginal delay profiles on light aircraft movements during peak periods

The marginal delays associated with this demand profile are shown in Figure 6.3 (upper line, labelled 1.0), and are seen to rise to over 100 minutes during the worst hours of each peak period.

Policy options

The situation described above and illustrated in Figures 6.1-6.3 constitutes a serious congestion problem. It is not the purpose of this paper to propose, or to argue the rationale for particular solutions to the problem, but rather to illustrate an essential facet of the assessment of any proposed solution.
In this example it is assumed that the assessment is of some (unspecified) policy whose effect on the original demand is to limit the demand for light aircraft movements during the periods of peak delays to a fixed proportion of the original demand for these movements.

In what follows, this proportion is varied between zero (total exclusion of light aircraft movements) and one (the original demand) to provide a picture of the dependence of the resulting delays on the severity of the measures adopted.

It must be emphasised that this is a simplified description of what would be required for the complete assessment of a range of policies. In reality, the delay analysis would be performed in conjunction with a demand analysis, which would require such information as the price elasticity of demand for light aircraft movements during the peak hours and the degree to which these movements could be replaced by movements of other types or at different times. In the assessment below, it is assumed that the demand profiles, which form the input to the delay models, result from just such a demand analysis of proposed pricing or regulatory policies. No effort has been made to represent the likely compensating increase in demand for light aircraft movements during off-peak periods. For the purposes of illustrating the use of the delay models, however, this is not important.

The assessment

Figure 6.2 shows the average delay profiles resulting from progressive exclusion of light aircraft movements during the peak periods in steps of 20 per cent of the initial demand for these movements. Total exclusion (labelled 0.0) reduces the average delay during these periods to below 1.5 minutes, but these values are now exceeded by the delays during the off-peak periods whose demand is assumed not to have been affected by the new policies.

The 0.6 profile (40 per cent of light aircraft demand excluded) is of special interest as the peak average delay has been reduced to about 4 minutes, which has often been referred to as the limit of 'tolerable' delay (FAA 1968). In Chapter 2, in the context of the definition of runway capacity, the notion of a 'tolerable' delay was criticised because of the conceptual and computational problems to which it gave rise. That criticism does not apply here, however, as policy assessment *is* the appropriate context in which to make (and defend) judgements about the tolerability of particular levels of delay.

In this operational assessment it is sufficient merely to note the main features of the marginal delay profiles shown in Figure 6.3. Total exclusion reduces the marginal delay during peak periods to below 10 minutes, but this value is now exceeded during the off-peak periods. The maxima of the 0.6 profile, for which the corresponding average delays are 'tolerable', are still about 40 minutes.

At this stage of policy assessment, decision criteria must be adopted, as for instance on the 'tolerability' of particular levels of delay. It may be noted however, that even without such decision criteria one aspect of the hypothetical policy being examined is clearly inappropriate: total exclusion of light aircraft movements during the peak periods reduces delay to values below those occurring during off-peak periods. If it is considered really necessary to reduce delays to very low values, it is obviously

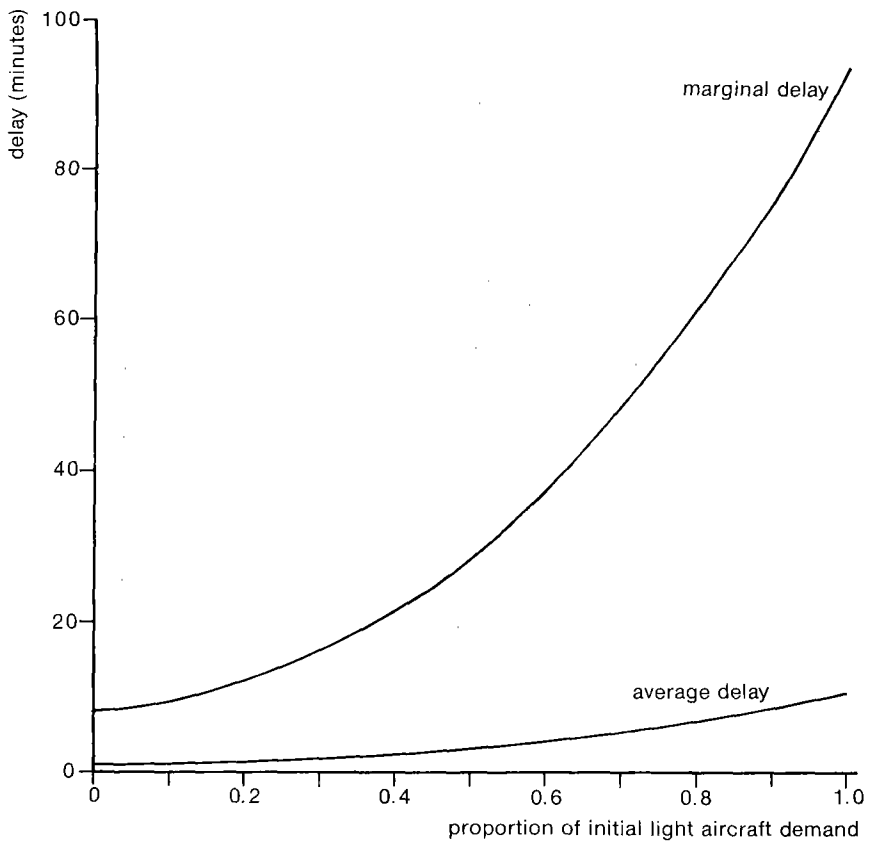

Figure 6.4
Dependence of average and marginal delay for the hour 1800-1900 on light aircraft demand during peak periods

important, at least when considering the more severely restrictive policies, to extend the peak periods to include those hours which initially had only moderate delays. Without such a broadening of the peak periods there would be no justification for excluding more than 60 per cent of light aircraft movements (the 0.4 profile).

It may also be useful to plot certain performance measures as functions of the demand (and the corresponding policy). This is done in Figure 6.4 for the marginal delay and the average delay during the hour 1800-1900 (initially, the hour with the highest average delay). The figure emphasises the great differences between the two measures of delay, that experienced by the individual user, and that occasioned to other users by his presence in the system.

Finally, it should be emphasised that the example discussed in this chapter was chosen purely to illustrate the use of the delay models in the context of policy assessment and is not an endorsement of any particular policy. The models provide technical information about the expected delays under given conditions of demand. The economic, social, and political assessment of the policies which determine the demand and of the consequent delays are outside the scope of this paper.

CHAPTER 7—CONCLUSION

The problem of providing estimates of delays due to runway congestion at busy airports has been addressed. For planning and policy evaluation purposes these estimates do not have to be highly accurate but they must take account of the essential operational features of the aircraft/runway system. The most important of these features are the dependence of runway capacity on the aircraft movement mix and the rapid variability of the demand.

Runway capacity is most appropriately defined independently of delay, and is the maximum rate at which aircraft movements can be handled by the runway system. It can be calculated from an intermovement time matrix (t_{ij}) and a movement mix vector (p_i). The intermovement time matrix results from information about runway layout, aircraft performance and air traffic control separation rules, while the movement mix vector describes the mix of movement types at any one time.

The capacity analysis, which takes account of much of the complexity of the runway operations, has been incorporated in a set of easy-to-use models, based on queueing theory, which provide estimates of delay due to runway congestion as functions of the level and distribution of demand. Apart from control parameters the only input consists of demand profiles for each movement type and the corresponding intermovement time matrix.

With the aid of these models, policies designed to relieve congestion by demand manipulation can be readily assessed both operationally, in terms of their effectiveness in reducing delay, and economically, if the costs of delays to different classes of users are available.

APPENDIX I—THE INTERMOVEMENT TIME MATRIX

In Chapter 3 the intermovement time matrix (t_{ij}) was introduced and its use in the calculation of runway capacities was illustrated by a very simple example. In this Appendix the matrix is derived for the principal runway configuration at Sydney (Kingsford Smith) Airport (KSA). It is derived first for aircraft operating under Instrument Flight Rules (IFR) where the separation standards are well defined, and then modified for the more flexible operations under Visual Flight Rules (VFR) where responsibility for separation rests partially with the pilots.

The intermovement time matrix is determined by the runway configuration, the aircraft characteristics, and the separation standards.

RUNWAY CONFIGURATION

The runway layout at KSA is shown in Figure I.1. The principal mode of operation, and the one analysed here, is that in which both runways are in use, operating in the 16 (southwards) and 07 (eastwards) directions. The important parameters in the analysis are the distances along each runway from the threshold to the intersection with the other runway and to the exit taxiways. These distances (in metres) are:

	Runway 16	*Runway 07*
Exit 1	550	570
" 2	1130	860
Intersection	1335	1050
Exit 3	1650	1210
" 4	2040	1390
" 5	2685	1860
" 6	3350	2380
" 7	3960	2530

AIRCRAFT CHARACTERISTICS

A compromise is required between a very precise categorisation of the aircraft and the need to keep the number of movement types down to manageable levels.

Weight is the dominant characteristic which affects intermovement times through the wake turbulence separations. It is therefore appropriate to recognise aircraft types corresponding to the weight classes heavy, medium and light defined for the purposes of wake turbulence separation (WTS) as discussed in Appendix II.

Certain performance data are also required. These are the speed on final approach and the characteristics of deceleration on landing and of acceleration during take-off. Fortunately, as mentioned in Chapter 3, further subdivision of the aircraft types is not necessary, and it is sufficient to adopt a set of mean performance characteristics to represent aircraft of each type.

Aircraft Type	*Weight Range*	*Approach Speed*
Heavy (H)	over 136000 kg	145 kn
Medium (M)	7000-136000 kg	130 kn
Light (L)	under 7000 kg	90 kn

This division corresponds roughly to placing the international airliners (B-747, DC-10, B-707 and L-1011) in the heavy class, the domestic interstate jets (B-727 and DC-9) and the F27 in the medium, and commuters, charters and other general aviation in the light

class. The main qualification of this division is the placing of the F27 which, as mentioned in Appendix II, is in the lower subdivision of the medium class which has been ignored. In addition its performance characteristics correspond more nearly to those of the light class.

Take-off performance of each aircraft type is given in Figure I.2 in the form of a time-distance plot assuming the aircraft starts its take-off run at the threshold. The plot is derived from the tabulated input to the Runway Congestion Model (Atack 1978) as used by the Department of Transport.

Landing performance is given in a similar way in Figure I.3 for runway 16 and Figure I.4 for runway 07. The curves are based on a simple model of aircraft deceleration similar to that used in the Runway Congestion Model. Landing aircraft touch down and roll without braking until 5 seconds after crossing the threshold. Maximum braking is then applied for 10 seconds, after which a reduced constant braking is applied so that the speed falls to the exit speed at the earliest possible exit. The required data are the positions of the exit taxiways with respect to the threshold, the aircraft's threshold speed, the deceleration at maximum braking and the exit speed.

SEPARATION STANDARDS

The *rule* to be used in calculating the time separation between two movements is determined by whether they are arrivals or departures (landings or take-offs) and on the same or different runways. There are eight such combinations and eight rules. The aircraft characteristics then enter as variables on which the rules operate. In this section the rules are derived from the separation standards specified in the Airways Operations Instructions, Volume 2 (Department of Transport, latest issue), and each rule is used to obtain the submatrix (in seconds) which it contributes to the overall intermovement time matrix.

The following assumptions are made:

- As discussed in Chapter 3 intermovement times are measured between characteristic events assigned to each movement type:
  - — for landings, the event is crossing the threshold of the runway on which the aircraft lands; and
  - — for take-offs, the event is the start of the take-off run; in general this will be at the runway threshold, but light aircraft departures starting beyond the runway intersection are considered below.
- Under IFR, aircraft approaching a runway to land follow a common approach path which extends back 8 nm in line with the runway to the entrance gate.
- Landing aircraft touch down shortly (within a few hundred metres) after the threshold and are not airborne at the runway intersection. Aircraft taking off rotate before, and are airborne at, the intersection. These assumptions determine the need for WTS at the intersection.

Arrival-arrival, same runway

Aircraft must maintain a minimum separation on the common approach path. This separation is normally 3 nm but is increased to the WTS (distance) when required as discussed in Appendix II. If the following aircraft is the faster the separation is limiting at the threshold and the IMT is the time for the following aircraft to fly this distance. If the following aircraft is the slower the separation is limiting at the entrance gate and the IMT is given by:

$$\frac{d}{v_2} - \frac{d-s}{v_1}$$

where d is the length of the common approach path (8 nm), s is the separation (3 nm or the WTS as given in Appendix II) and v_1 and v_2 are the approach speeds of the leading and following aircraft. This submatrix is the same for each runway:

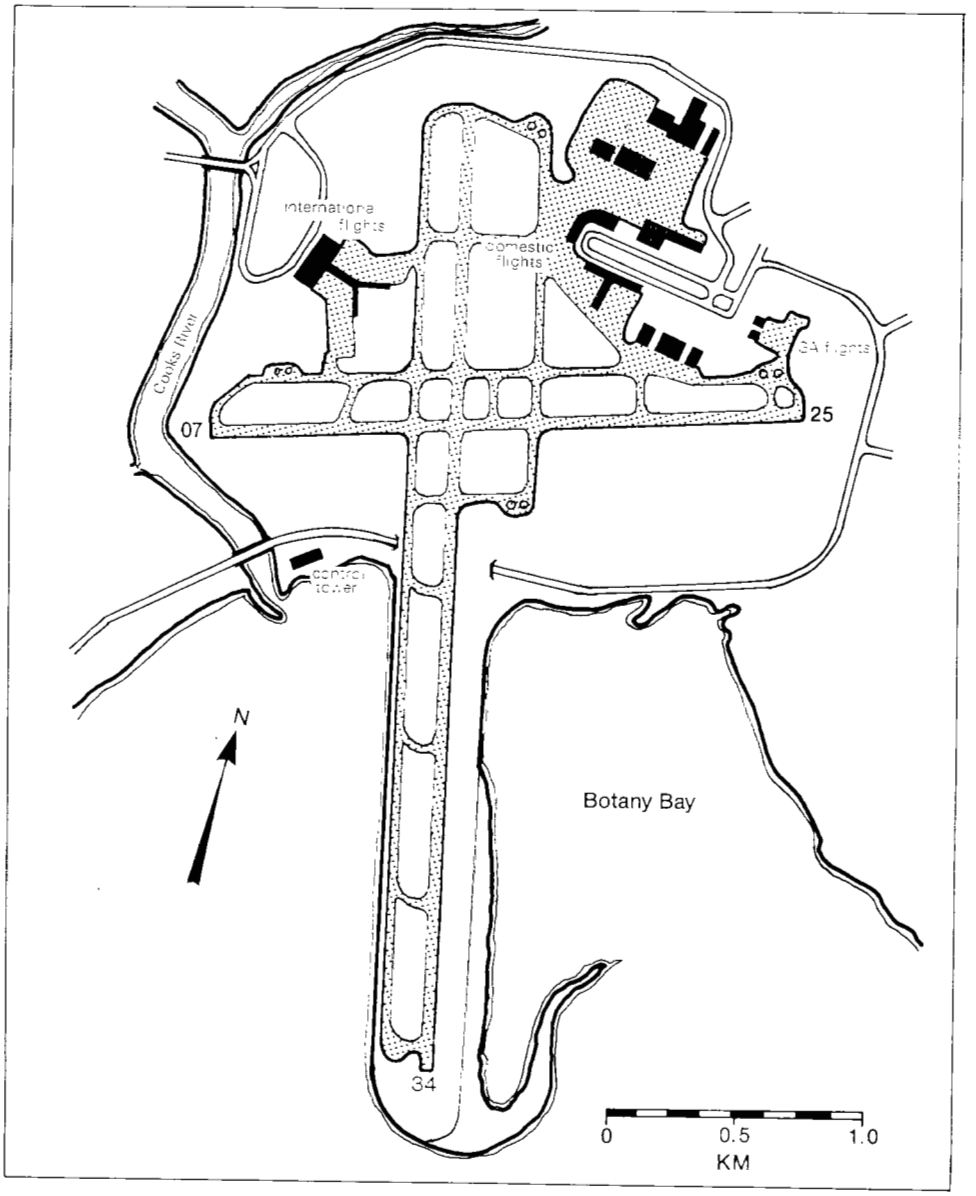

Figure I.1
Kingsford Smith Airport Sydney

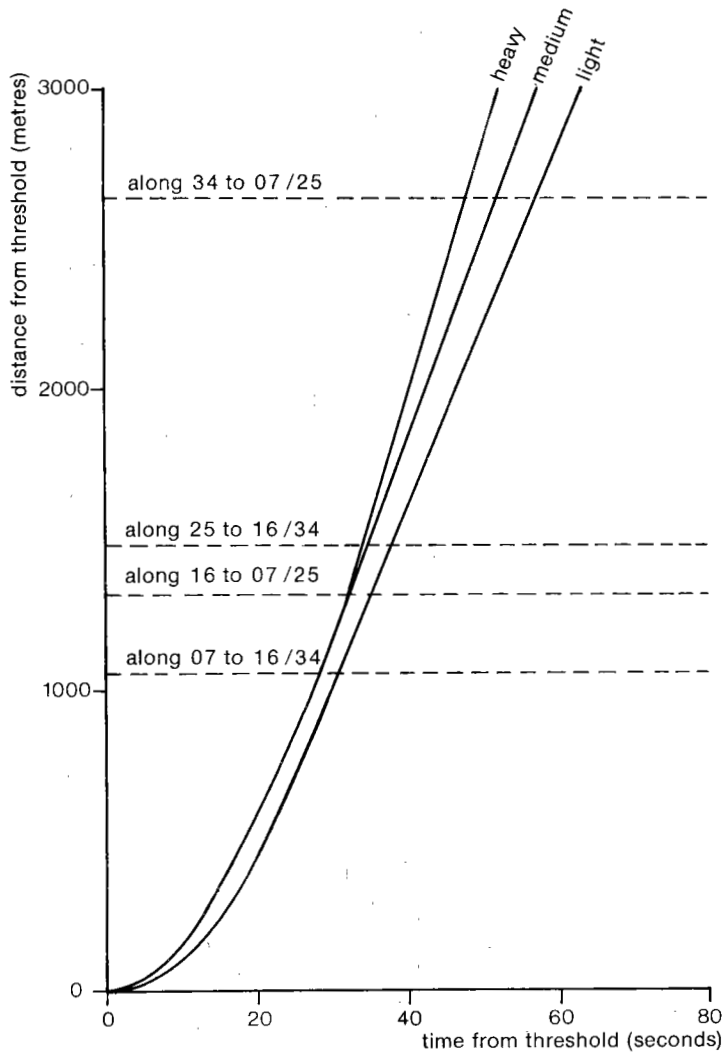

Figure I.2
Aircraft take-off performance;time-distance after brakes off at runway threshold

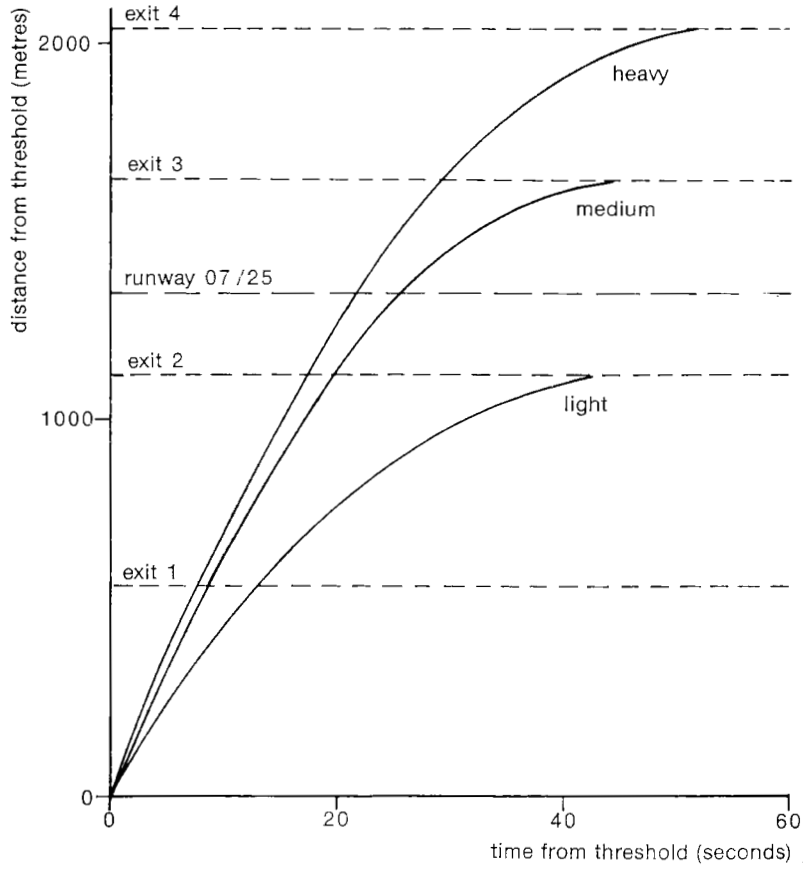

Figure I.3
Aircraft landing performance on runway 16; time-distance after crossing threshold

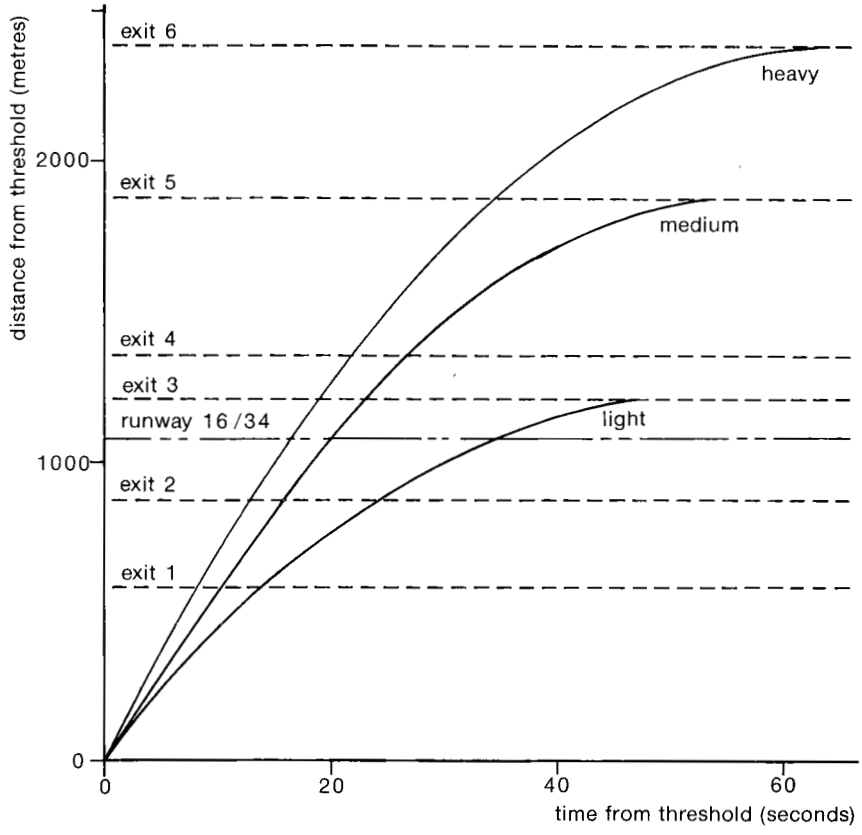

Figure I.4
Aircraft landing performance on runway 07; Time-distance after crossing threshold

		SECOND		
	(sec)	H	M	L
	H	77	170	269
FIRST	M	77	83	237
	L	77	83	120

Arrival-departure, same runway

Take-off clearance is not given until after the landing aircraft has vacated the runway. The IMT is therefore the runway occupancy time of the landing aircraft and is obtained from Figure I.3 or Figure I.4. No WTS is required.

Runway 16

(sec)	H	M	L
H	50	50	50
M	44	44	44
L	42	42	42

Runway 07

(sec)	H	M	L
H	62	62	62
M	52	52	52
L	46	46	46

Departure-arrival, same runway

Take-off clearance can be given if the following arrival is not closer than 3 nm to the threshold. The IMT is therefore the time for the arrival to fly this distance. No WTS is required. The submatrix is the same for both runways:

(sec)	H	M	L
H	77	83	120
M	77	83	120
L	77	83	120

Departure-departure, same runway

These IMT are difficult to specify precisely as they are partly dependent on the details of the departure routes. The basic requirement is that successive departures should be released so that when established on their departure routes the en route separations are not violated. Aircraft on diverging routes can therefore be released more rapidly than aircraft on the same route. The WTS (time) is applied when required. The following IMT submatrix has been assumed for both runways:

(sec)	H	M	L
H	90	120	180
M	90	90	150
L	90	90	90

When lighter (and slower) aircraft follow heavier (and faster) ones the initial WTS time coupled with the increasing separation is sufficient to prevent conflict even on the same departure route. The IMT of 90 sec between aircraft of the same type is a compromise between longer times required for aircraft on the same route and shorter times which would be possible on diverging routes. When heavier (and faster) aircraft follow lighter (and slower) ones on the same route longer IMT are required. However, the different aircraft types are less likely to be on the same route. An IMT of 90 sec has therefore been assumed for these too.

Arrival-arrival, different runways

The following arrival must be no closer than 3 nm to its threshold at the time when the leading arrival, on the other runway, crosses the intersection or stops short of it. As

neither aircraft is airborne at the intersection no WTS is required. The IMT is the time the leading arrival takes between threshold and intersection (Figure I.3 and Figure I.4) plus the time for the following arrival to fly 3 nm.

Runway 07

(sec)	H	M	L
RUN H	98	104	141
WAY M	103	109	146
16 L	119	125	162

Runway 16

(sec)	H	M	L
RUN H	93	99	136
WAY M	95	101	138
07 L	110	116	153

Arrival-departure, different runways

Take-off clearance is not given until the arrival on the crossing runway has passed the intersection or stopped short of it. The IMT is then the time the arrival takes between threshold and intersection (Figure I.3 and Figure I.4).

Runway 07

(sec)	H	M	L
RUN H	21	21	21
WAY M	26	26	26
16 L	42	42	42

Runway 16

(sec)	H	M	L
RUN H	16	16	16
WAY H	18	18	18
07 L	33	33	33

Departure-arrival, different runways

The arrival must be no closer than 3 nm to its threshold when the departure on the other runway crosses the intersection. A WTS is only required when a light arrival follows a heavy departure. The IMT is the time the departure takes from threshold to intersection plus the time for the arrival to fly 3 nm or, in the case of a light arrival following a heavy departure, the WTS time plus the time the departure takes from threshold to intersection minus the time the arrival takes from threshold to intersection.

Runway 07

(sec)	H	M	L
RUN H	109	115	179
WAY M	109	115	152
16 L	113	119	155

Runway 16

(sec)	H	M	L
RUN H	105	110	165
WAY M	105	110	147
07 L	108	114	151

Departure-departure, different runways

The following departure cannot be cleared for take-off until the leading departure on the other runway has crossed the intersection. Since both aircraft are airborne at the intersection a WTS must be applied if required. The IMT is the time for the leading departure to reach the intersection or, if a WTS is required, the WTS time plus the time for the leading departure to reach the intersection minus time for the following departure to reach it.

Runway 07

(sec)	H	M	L
RUN H	95	125	181
WAY M	32	32	152
16 L	35	35	35

Runway 16

(sec)	H	M	L
RUN H	86	115	172
WAY M	27	27	142
07 L	31	31	31

Light departures, runway 07

In the above derivation of IMT it has been assumed that departures always start their take-off run from the runway threshold. For heavy aircraft and medium jets this will generally be true. Nevertheless, some light and medium aircraft departing on runway 16 do start from the 550 metre taxiway (exit 1, Figure I.1). This has a relatively small effect on capacities and has been ignored in the analysis.

Significant advantages can be achieved, however, if light departures on runway 07 start their take-off run after the intersection with runway 16 (the remaining runway length of over 1300 metres is quite adequate). These flights originate either at the domestic terminals or at the general aviation area. both of which are in the north-east quadrant with respect to the runway intersection. By starting after the intersection light aircraft reduce their taxiing distance by at least 1200 metres and avoid crossing runway 16 on their way to the threshold. The advantage from the point of view of capacity is that departures starting after the intersection do not interfere with movements on the other runway. The times between light departures on runway 07 and movements on runway 16 can therefore be set to zero in the intermovement time matrix.

THE INTERMOVEMENT TIME MATRIX—INSTRUMENT METEOROLOGICAL CONDITIONS

The complete matrix for instrument meteorological conditions (IMC) is given below. The IMT derived above have been rounded to the nearest 5 seconds in order not to give an unwarranted impression of great accuracy.

IMC INTER-MOVEMENT TIMES (SECONDS) — *SECOND*

FIRST				RUNWAY 16						RUNWAY 07					
				ARRIVAL			DEPARTURE			ARRIVAL			DEPARTURE		
				H	M	L	H	M	L	H	M	L	H	M	L
RUNWAY 16	ARR	H		75	170	270	50	50	50	100	105	140	20	20	0
		M		75	85	235	45	45	45	105	110	145	25	25	0
		L		75	85	120	40	40	40	120	125	160	40	40	0
	DEP	H		75	85	120	90	120	180	110	115	180	95	125	0
		M		75	85	120	90	90	150	110	115	150	30	30	0
		L		75	85	120	90	90	90	115	120	155	35	35	0
RUNWAY 07	ARR	H		95	100	135	15	15	15	75	170	270	60	60	60
		M		95	100	140	20	20	20	75	85	235	50	50	50
		L		110	115	155	35	35	35	75	85	120	45	45	45
	DEP	H		105	110	165	85	115	170	75	85	120	90	120	180
		M		105	110	145	25	25	140	75	85	120	90	90	150
		L		0	0	0	0	0	0	75	85	120	90	90	90

This matrix can be assessed by calculating the runway capacity to which it gives rise under a movement mix representative of IMC. The demand profiles given in Figure VII.2 are derived from noise monitoring records and, being overall averages, are more representative of visual meteorological conditions. An approximation to an IMC mix can be obtained by assuming that in poor visibility the demand for light aircraft movements would be reduced but that other movements would be unaffected. An IMC mix is therefore obtained from these profiles by halving the light aircraft demand.

With these mixes the IMC capacity is 39 + 1 movements/hour. This figure may be compared with the value of 45 movements/hour quoted in the MANS (Major Airport Needs of Sydney) Study for these conditions but with an allowance for 'presently foreseeable improvements in equipment' (MANS Study, 1978).

As mentioned in Chapter 3 the calculated capacity is expected to be pessimistic since no account is taken of sequencing procedures. Inspection of the matrix reveals a small number of very long IMT associated with light (and slow) arrivals following heavy and medium arrivals on the same runway. Even though systematic sequencing may not be possible, the avoidance of a small number of particularly long IMT would be feasible and would increase capacity by one or two movements per hour.

The remaining error is attributable partly to the neglect of the subdivision of medium aircraft with weights between 7000kg and 25000kg. As pointed out in Appendix II this imposes an unnecessary wake turbulence separation whenever one of these is followed by a light aircraft.

Depending on the purpose of the analysis it may or may not be necessary to refine the values of the IMT in the matrix. In the context of the delay analysis, the average capacity can be assumed to be known (45 movements/hour), and the matrix used to describe the variation of capacity with changes to the movement mix around this average. It is then sufficient to scale the matrix (multiply each element by 39/45) so that the capacities do in fact match the known average. The scaled matrix, with elements rounded to the nearest 5 seconds, is given below.

IMC INTER-MOVEMENT TIMES (SECONDS)

FIRST

RUNWAY 16 ARR

SECOND	RUNWAY 16 ARRIVAL			RUNWAY 16 DEPARTURE			RUNWAY 07 ARRIVAL			RUNWAY 07 DEPARTURE		
	H	M	L	H	M	L	H	M	L	H	M	L
H	65	145	235	45	45	45	85	90	120	15	15	0
M	65	75	205	40	40	40	90	95	125	20	20	0
L	65	75	105	35	35	35	105	110	140	35	35	0

RUNWAY 16 DEP

SECOND	RUNWAY 16 ARRIVAL			RUNWAY 16 DEPARTURE			RUNWAY 07 ARRIVAL			RUNWAY 07 DEPARTURE		
	H	M	L	H	M	L	H	M	L	H	M	L
H	65	75	105	80	105	155	95	100	155	80	110	0
M	65	75	105	80	80	130	95	100	130	25	25	0
L	65	75	105	80	80	80	100	105	135	30	30	0

RUNWAY 07 ARR

SECOND	RUNWAY 16 ARRIVAL			RUNWAY 16 DEPARTURE			RUNWAY 07 ARRIVAL			RUNWAY 07 DEPARTURE		
	H	M	L	H	M	L	H	M	L	H	M	L
H	80	85	115	15	15	15	65	145	235	50	50	50
M	80	85	120	15	15	15	65	75	205	45	45	45
L	95	100	135	30	30	30	65	75	105	40	40	40

RUNWAY 07 DEP

SECOND	RUNWAY 16 ARRIVAL			RUNWAY 16 DEPARTURE			RUNWAY 07 ARRIVAL			RUNWAY 07 DEPARTURE		
	H	M	L	H	M	L	H	M	L	H	M	L
H	90	95	145	75	100	145	65	75	105	80	105	155
M	90	95	125	20	20	120	65	75	105	80	80	130
L	0	0	0	0	0	0	65	75	105	80	80	80

THE INTERMOVEMENT TIME MATRIX — VISUAL METEOROLOGICAL CONDITIONS

The basic principle is that the runway ahead of an aircraft manoeuvring during take-of or landing must be free of other aircraft. Under IFR, the main function of the standard 3 nm separation imposed ahead of arrivals is to provide time for evasive action in the event of an accident to the previous movement. Under VFR the maintenance of safe separation in the air becomes the responsibility of the pilot and, in good visibility, if wake turbulence is not a problem, separations less than the standard 3 nm can safely be used and much more flexible routing to the threshold can be allowed. The effect is to reduce many of the IMT in which the second movement is an arrival as it is these, which under IFR, are subject to the 3 nm separation. Wake turbulence separations must still be applied when necessary.

The actual separations which occur depend on circumstances and pilot judgement and can vary widely. A precise description of the situation then becomes almost impossible. The general effect on IMT can be simulated in an ad hoc way by introducing an 'effective' separation as a variable parameter in place of the standard 3 nm separation. The best value for the 'effective' separation is then found by varying it until the matrix gives the known visial meteorological condition (VMC) capacity of about 51 movements/hour for a representative VMC movement mix. As the function of these separations is to provide time for evasive action a safe separation should depend on aircraft speed. The 'effective' separations were therefore assumed to be proportional to the speed of the following arrival up to a maximum of 3 nm, the IFR standard.

The effective separation takes the place of 3 nm separation in the rules for calculating the IMT defined above. Other variations to the rules are noted below.

Arrival-arrival, same runway

Since aircraft are not obliged to follow the whole length of the common approach path the effective separation applies at the threshold, regardless of aircraft speed. For the same reason the time standard is used when WTS are required. The IMT is the WTS time (if required) or the time for the following aircraft to fly the effective separation. A lower limit on the IMT is set by the runway occupancy time of the leading aircraft which must vacate the runway before the following aircraft crosses the threshold.

Departure-arrival, same runway

The departure is considered to start its take-off run if the arrival is not less than the effective separation from the threshold. The IMT is the time for the arrival to fly the effective separation. A lower limit to the IMT is set by the requirement that the departure be airborne and at least 1800 metres from the threshold (or simply airborne if it is a light aircraft) before the arrival crosses the threshold.

Arrival-arrival, different runways

The effective separation takes the place of 3 nm in the IFR rule.

Departure-arrival, different runways

The effective separation takes the place of 3 nm in the IFR rule.

Rules with following departures

These are the same as the corresponding IFR rules.

With these modified rules for determining the IMT, the following matrix with elements rounded to the nearest 5 seconds, was obtained with effective separations of:

2.2 nm preceding heavy aircraft
2.0 nm preceding medium aircraft
and 1.4 nm preceding light aircraft

VMC INTER-MOVEMENT TIMES (SECONDS)			SECOND											
			RUNWAY 16						RUNWAY 07					
			ARRIVAL			DEPARTURE			ARRIVAL			DEPARTURE		
			H	M	L	H	M	L	H	M	L	H	M	L
FIRST	RUNWAY 16	ARR H	75	120	180	50	50	50	75	75	75	20	20	0
		ARR M	55	55	150	45	45	45	80	80	80	25	25	0
		ARR L	55	55	55	40	40	40	100	100	100	40	40	0
		DEP H	55	55	55	90	120	180	90	90	180	95	125	0
		DEP M	55	55	55	90	90	150	90	90	90	30	30	0
		DEP L	55	55	55	90	90	90	90	90	90	35	35	0
	RUNWAY 07	ARR H	70	70	70	15	15	15	75	120	180	60	60	60
		ARR M	75	75	75	20	20	20	55	55	150	50	50	50
		ARR L	90	90	90	35	35	35	55	55	55	45	45	45
		DEP H	80	80	165	85	115	170	55	55	55	90	120	180
		DEP M	85	85	85	25	25	140	55	55	55	90	90	150
		DEP L	0	0	0	0	0	0	55	55	55	90	90	90

With representative VMC movement mixes, as given in Figure VII.2, this matrix leads to runway capacities of 51 + 1 movements/hour which is the known capacity under these conditions.

APPENDIX II—WAKE TURBULENCE SEPARATION

The existence of wake turbulence behind airborne aircraft has been known since the early days of aviation, but only came to be seen as a serious problem in the mid 1960s mainly as a result of the introduction of wide bodied, heavy aircraft. The turbulence behind these heavy aircraft can be a serious hazard to following (mainly lighter) aircraft and the standard separations have had to be increased in these cases.

The wake turbulence separation (WTS) standards are set out in the Airways Operation Instructions (AOI) Volume 2 (Department of Transport, latest issue).

For the purposes of WTS, aircraft are divided into three categories, heavy (H), medium (M) and light (L). However, in the specification of the separation, two special cases are identified which have the effect of subdividing the heavy and medium categories. The five categories recognised are:

CATEGORY		*MAXIMUM TAKE-OFF WEIGHT RANGE*	*EXAMPLES*
Heavy	H1	over 200 000 kg	B747, L1011, DC10
	H2	136 000-200 000 kg	B707, A300B
Medium	M1	25 000-136 000 kg	B727, DC9
	M2	7 000-25 000 kg	F27, DC3, HS748
Light	L	under 7 000 kg	C550, B200, PA31

The WTS to be observed between a leading and a following aircraft are given in a WTS matrix in both distance (nautical miles) and time (minutes) for all possible pairs of aircraft.

WTS		*SECOND*				
(nm/min)		*H1*	*H2*	*M1*	*M2*	*L*
	H1	3/1.5	4/2	6/2	6/2	6/3
F	H2	3/1.5	4/2	6/2	6/2	6/3
I						
R	M1	*	*	*	*	5/2.5
S	M2	*	*	*	*	*
T						
	L	*	*	*	•	•

NOTE: * normal separation standards apply.

The distance and time standards are used in different situations. On the approach path the separations between arriving aircraft are monitored by radar and compared with the distance standard. The time standard is used between aircraft taking off and, when crossing runways are in use, at the intersections.

The wake turbulence behind an aircraft is associated with aerodynamic lift so that, for movements on the runways and especially at the intersections, it is necessary to know whether the leading aircraft is airborne. In general only airborne aircraft are endangered by wake turbulence. In the analysis of the runway configuration at KSA landings are assumed to touch down shortly after the threshold and not to be airborne at the intersection. Take-offs are assumed to rotate (become airborne) before the

intersection. These conditions imply that no WTS is required between a landing aircraft and a following movement on the crossing runway. On the other hand a WTS is required between take-offs on intersecting runways, and in addition a WTS is required when a heavy take-off precedes a light landing even though the landing will not be airborne at the intersection.

The five aircraft weight categories, when combined with arrivals and departures on each runway, lead to an excessive number of movement types. For the purposes of the capacity analysis therefore, the subdivisions of the heavy and medium categories are ignored and a 3 x 3 WTS matrix for heavy, medium, and light aircraft is adopted.

WTS		*SECOND*		
(nm/min)		*H*	*M*	*L*
F	H	3/1.5	6/2	6/3
I				
R	M	*	*	5/2.5
S				
T	L	*	*	*

NOTE: * normal separation standards apply.

The errors which arise from ignoring the lower subdivision of heavy aircraft are small as heavy aircraft constitute only about 10 per cent of movements and of these most are in the upper subdivision. Ignoring the medium subdivision is more serious as about 40 per cent of medium aircraft (mostly F-27s) fall within the neglected category. An unnecessary WTS is imposed whenever a light aircraft follows one of these.

APPENDIX III—THE TRIPLET CORRECTION

The runway capacity is given by:

$$C = 1/T$$

where T is the mean of the intermovement times, t_{ij}.

$$T = \Sigma_{ij}\, p_{ij}\, t_{ij}$$

Under first come, first served queue discipline

$$p_{ij} = p_i p_j$$

where p_i are the elements of the movement mix vector.

In Chapter 3 it was assumed that, during a continuously busy period, the time of occurrence of a movement is constrained only by the time of the immediately preceding movement. It can happen, however, that for a sequence of movements (i, j, k)

$$t_{ij} + t_{jk} < t_{ik}$$

In other words movement k is constrained by the time t_{ik} since the last but one movement i, rather than by the time t_{jk} since the last movement j. The probability of occurrence of the triplet (i, j, k) is p_{ijk}, given under first come, first served by:

$$p_{ijk} = p_i p_j p_k$$

This effect can be corrected for by adding to the mean intermovement time T a term

$$p_{ijk}\,(t_{ik} - t_{ij} - t_{jk})$$

for all values of the triplet (i, j, k) for which this term is positive. The capacity is then given as the reciprocal of the corrected mean intermovement time as before.

The correction for the triplet effect is typically about 5-15 per cent of the capacity. It is possible to consider effects of order higher than triplet, but the analysis is complicated and the size of the corrections typically less than 1 per cent.

APPENDIX IV—STATISTICAL MEASUREMENT OF DELAY

Arrivals

The delay model for arrivals in Chapter 4 was described formally by the relation:

$$F_{ijk} = T_i + D_j + R_{ijk} \quad (1)$$

T_i is the average undelayed time for the i-th route/aircraft combination, D_j is the average delay experienced by arrivals during the j-th period during the day and R_{ijk} represents all other influences on the actual flight time of the k-th instance of the i-th route/aircraft combination arrived during the j-th period. F_{ijk} is then the actual duration of this flight.

This can be rewritten in a form suitable for regression analysis:

$$F_{ijk} = \sum_i T_i x_i + \sum_j D_j y_j + R_{ijk} \quad (2)$$

Here, the x_i are a set of dummy variables representing the route/aircraft combination. One, and only one, member of the set takes the value one, thereby specifying the combination, while all the others take the value zero. In other words the x_i are subject to:

$$x_i = 0, 1 \quad (3)$$

$$\text{and } \sum_i x_i = 1$$

Similarly, the y_j are a set of dummy variables representing the time period of an arrival at the airport. They are subject to a similar restriction:

$$y_j = 0, 1 \quad (4)$$

$$\text{and } \sum_j y_j = 1$$

The analysis consists of regression of the actual flight times F_{ijk} against the x_i and y_j whose values are determined by a knowledge of the route, aircraft and arrival time for the trip in question. The regression yields estimates of the T_i and the D_j as coefficients of the x_i and y_j respectively.

An obvious difficulty with this procedure is that the T_i and the D_j are not uniquely determined by the regression. In expression (2) a constant could be added to all the T_i and subtracted from all the D_j without changing the F_{ijk}. This is a consequence of the restrictions (3) and (4). In order to resolve the problem one degree of freedom is removed by arbitrarily specifying one of the T_i or D_j. A natural way to do this follows from the definition of the D_j as delays due to congestion. They are therefore necessarily positive and it is reasonable to set the minimum delay during slack periods to zero. Strictly speaking, therefore, the D_j obtained in this manner *are not absolute delays but delays relative to those in the reference period.*

Departures

As explained in Chapter 4 the analysis of the departures is formally the same as that for arrivals except that the coefficients and dummy variables for the route/aircraft combinations can be dropped. The regression equation is:

$$F_{jk} = \sum_j D_j y_j + R_{jk} \quad (5)$$

where F_{jk} now represents the taxi-out time for the k-th instance of a departure during the j-th period during the day. As there is only one set of dummy variables the ambiguity which arose in the arrival analysis does not occur, but the coefficients D_j must be taken to represent the average taxi-out times during period j rather than the delay. The minimum time is taken as the reference and assumed to contain zero delay. The average departure delay during the other periods is then taken as the excess of their taxi-out times over that during the reference period.

Delay profiles by day of week

The delay coefficients D_j have been described as depending only on time of day. In fact, of course, delay is expected to depend on other factors such as weather, season and day of week. The day of week is particularly important because of the weekly cycle of demand profiles. In principle this could be taken into account by allowing the index j to represent not just time of day, but time of day on a particular day of week. This would result in a 7-fold increase in the number of variables in the set y, taking the problem beyond the bounds of the available regression program. This difficulty is circumvented by sorting the data by day of week and treating each day separately. Subsequently the results for each day are linked by choosing data for a particular time period and allowing D_j to represent day of week only in the regression.

The data

Ansett Airlines of Australia made available copies of the trip records covering all movements by their aircraft to and from KSA during 1979, a total of 28931 records. Each trip record contains the following information relevant to the delay analysis:

date
aircraft type
origin
destination
engine start time (at origin)
engine stop time (at destination)
taxi-out duration (in minutes from engine start to wheels off)
taxi-in duration (in minutes from wheels on to engine stop)
flight duration (in minutes from wheels off to wheels on).

In addition each record contains data on the scheduled trip times, the crew's perception of the causes of delays and such items as trip and flight numbers which have not been used. In the course of the analysis a day of week indicator was added to each record.

As mentioned in Chapter 4 a total of 37 days were excluded from the analysis on the basis that their demand profiles were probably atypical for their day of week. These were mainly public holidays and the days before and after them. The excluded dates were:

New Year:	1-2/1, 28-31/12
Australia Day:	26/1, 28-30/1
Easter:	12-17/4
Anzac Day:	25/4
Queen's Birthday:	15/6, 17-19/6
School Holidays:	24-26/8, 8-9/9
8 Hour Day:	28/9, 30/9, 1-2/10
Christmas:	21-26/12

By far the majority of the records referred to trips made between Sydney and one of six other cities by one of three aircraft types. In order, therefore, to reduce the number of variables required in the regression program only these cities and aircraft types were considered. The six cities were:

Adelaide (ADL)	Brisbane (BNE)	Canberra (CBR)
Melbourne (MEL)	Coolangatta (OOL)	Perth (PER)

The three aircraft types were the jets used on interstate routes:

B727-100 DC-9 B727-200

There were therefore 18 route/aircraft combinations and 18 members in the first set of dummy variables, x_i, $i = 1,2, \ldots 18$.

An aircaft's time of arrival was taken as the wheels on time given by the engine-stop time minus the taxi-in duration. This is not strictly the time of the demand on the runway system, but while delays are short compared with the time periods into which the day is divided the error is not great. The day is divided into 18 time periods, the first from midnight to 0700 and thereafter hourly periods from 0700 to midnight. The second set of dummy variables therefore has 18 members, y_j, $j = 1,2, \ldots 18$.

APPENDIX V—TIME DEPENDENT QUEUEING EQUATIONS

A qualitative description of the delay models has been given in Chapter 5. In this appendix the basic queueing equations and the formulae for the measures of performance are presented and discussed in slightly more detail. However, it is not intended to provide a comprehensive treatment of queueing theory such as can be found in the standard texts (Wagner 1969).

QUEUEING EQUATIONS—RANDOM SERVICE TIMES

The differential equations which describe the behaviour of a single channel, single server queue with Poisson requests and a negative exponential distribution of service times are:

$$\frac{dP_o}{dt} = -\lambda P_o + \mu P_1$$

$$\frac{dP_n}{dt} = \lambda P_{n-1} - (\lambda + \mu)P_n + \mu P_{n+1} \qquad 0 < n < N \tag{1}$$

$$\frac{dP_n}{dt} = \lambda P_{N-1} - \mu P_N$$

P_o is the probability of there being no 'customers' in the system, P_n is the probability that there are n 'customers' in the system (including any being served) and N is the maximum allowed value of n.

In these equations λ is the mean rate at which requests for service are made under the Poisson process in which the probability density function for the inter request times is:

$$\lambda e^{-\lambda t} \tag{2}$$

Similarly the probability density function for the service time is:

$$\mu e^{-\mu t} \tag{3}$$

with mean service rate μ (mean service time $1/\mu$).

In general λ, μ and the P_n are functions of time:

$$\lambda = \lambda(t); \quad \mu = \mu(t); \quad P_n = P_n(t)$$

The derivation of equations (1) makes use of the convenient mathematical properties of the distribution functions (2) and (3). A very short time interval δt is considered during which the probability that a request for service is made is $\lambda\delta t$ and during which the probability that more than one request is made can be ignored. Similarly the probability that a service will be completed during δt is $\mu\delta t$ (given that the system is not empty). It is then easy to obtain the change δP_n in each quantity P_n during δt and hence, by a limiting process, the rate equations (1).

Until recently most queueing analyses assumed constant request and service rate parameters and dealt only with the steady state to which the system tended in the long run. The time derivatives on the left hand side of equations (1) can then be set to zero,

and the resulting system of linear equations solved for the P_n yielding the well known simple formulae:

$$P_o = \frac{1-\rho}{1-\rho^{N+1}}$$
$$P_n = P_o\rho^n \quad (4)$$
$$\text{where } \rho = \lambda/\mu$$

This work is concerned with situations in which the rate parameters are not constant, and in which the system does not generally reach a steady state. The equations (1) must therefore be solved numerically, the results being the values of the P_n as functions of time after the specified initial conditions.

Being a complete set of probabilities, the quantities P_n must at all times satisfy the relations:

$$0 \leqslant P_n \leqslant 1 \text{ for all } n$$

and (5)

$$\sum_{n=0}^{N} P_n = 1$$

The initial conditions can be set in any way satisfying (5), but it is most convenient to make use of the fact that successive daily cycles are independent of each other owing to the disappearance of the queue each night when the demand becomes very small. In this case:

$$P_o(0) = 1.0$$

and (6)

$$P_n(0) = 0.0 \quad n > 0$$

Once the values of the P_n (at any time) are available a variety of measures of performance of the system can be calculated.

QUEUEING EQUATIONS—FIXED SERVICE TIMES

The analysis of the queue with Poisson requests and fixed service time is more complicated because use cannot be made of the convenient properties of the service time distribution, (3).

The system is considered only at regular intervals corresponding to the (fixed) service time $1/\mu$, and a set of equations is derived which relates the probabilities P_n to their values one service time earlier.

If $P_n(s)$ is the probability that there are n customers in the system after s service times then the difference equations describing the evolution of the system are:

$$P_n(s+1) = P_o(s)a(n) + P_1(s)a(n) + P_2(s)a(n-1) + \ldots$$
$$\ldots + P_n(s)a(1) + P_{n+1}(s)a(0)$$
$$\text{for } 0 \leqslant n < N \text{ and} \quad (7)$$

$$P_N(s+1) = P_o(s)u(N) + P_1(s)u(N) + P_2(s)u(N-1) + \ldots$$
$$\ldots + P_{N-1}(s)u(2) + P_N(s)u(1)$$

where:

$a(i) = \Pr$ (i aircraft request service during $1/\mu$)

$$= e^{-\rho}\rho^i/i!$$

u(i) = Pr (i or more aircraft request service during $1/\mu$)

$$= \sum_{j=i}^{\infty} a(j)$$

$$= 1 - \sum_{j=0}^{i-1} a(j)$$

As before λ and μ and hence ρ, a(i) and u(i) are time dependent. The equations (7) permit the probabilities, P_n to be followed in time (in steps of the fixed service time) from a specified set of initial conditions, again most conveniently given by equations (6).

MEASURES OF PERFORMANCE

The solutions of the sets of equations (1) and (7) which describe the queueing system are the probabilities P_n. One of them, P_o, the probability that the system is empty, is itself a useful performance indicator, but their principal value is that they allow the calculation of a wide variety of other measures of performance to suit particular requirements. The most generally useful measures have been described in Chapter 5; their formulae are given below:

Average number in the system:

(8)

Standard deviation of the number in the system:

$$\sigma_s = \left(\sum_{n=o}^{N} n^2 P_n - N_S^2 \right)^{1/2} \tag{9}$$

Average number waiting in the queue:

$$N_Q = N_S - 1 + P_o \tag{10}$$

Cumulative number turned away (requesting service when the system is full) during the period T_1 to T_2

$$N_{TWY} = \int_{T_1}^{T_2} \lambda P_N dt \tag{11}$$

Average delay to aircraft requesting service during the period T_1 to T_2

$$D_{AV} = \int_{T_1}^{T_2} \frac{1}{\mu} \left(\frac{N_S - NP_N}{1 - P_N} \right) dt \tag{12}$$

This formula takes account only of aircraft which are served. Those turned away when the system is full are not included.

Total delay to all aircraft requesting service during the period T_1 to T_2

$$D_{TOT} = \int_{T_1}^{T_2} \frac{\lambda}{\mu} \left(N_S - NP_N \right) dt \tag{13}$$

If the demand mix during the period T_1 to T_2 contains a proportion, p_i of aircraft movements of type i, then the total delay to aircraft movements of this type, $D_{TOT,i}$ will be given by

$$D_{TOT,i} = p_i D_{TOT} \quad (14)$$

In this way, by choosing the period T_1 to T_2 to represent successive hours during which the demand is specified, the total delay to each movement type throughout the day can be determined.

The above formulae for average and total delays are approximate as they do not take account of variations of mix or service rate outside the period T_1 to T_2 (they are assumed to be fixed between T_1 and T_2). So long as the average delay is small compared with T_2-T_1 the errors are not great.

Marginal delay

The marginal delay at a particular time is defined as the increase in total delay due to one extra request for service at that time. In fact attention is focussed on the hourly periods for which the demand is specified. Having performed the calculation for one daily cycle, and having calculated the total delay to all aircraft for that day, the model then calculates the total delay to all aircraft (and then the increase in total delay) which results from a unit increase in demand during the first hour of the cycle. This increase in total delay is the marginal delay during the first hour. Similar calculations are made for unit demand increments during each hour of the day.

APPENDIX VI—DESCRIPTION OF THE MODELS—INPUT AND OUTPUT

Three computer models have been developed in the course of this work and are available for use by planners:

a) AIRQR treats the system as a single queue with Poisson demand and a negative exponential (random) distribution of service times.

b) AIRQF treats the system as a single queue with Poisson demand and a deterministic (fixed) distribution of service times.

c) MARQ calculates the marginal delays - essentially by repeated application of AIRQR with a unit demand increment in each hour.

INPUT

The three models require the same input which consists of the following items:

a) The number of input periods during each of which the demand is specified.

b) The length of the input period in minutes.

c) The number of times for which output of the system characteristics is required during each input period.

d) The number of integration steps per output period. Together with items (b) and (c) this defines the length of the integration step. It is ignored by AIRQF where the integration step length is calculated internally.

e) The maximum number of aircraft permitted in the system.

f) The number of movement types in the mix.

g) The average demand rate in movements per hour for each movement type in each successive input period.

h) The matrix of the intermovement times in seconds.

COMPUTATION

AIRQR

Figure VI.1 gives a schematic flow chart of the program AIRQR.

The input is read and a record of the control parameters and the intermovement time matrix is printed. (The record of the input demand profiles is printed in parallel with the output for the appropriate time).

The calculation covers a period which will normally be one day starting with a slack period when the airport can be assumed to be empty. The probabilities are therefore set to:

$$P_0(0) = 1;\ P_n(0) = 0\ 1 \leqslant n \leqslant N$$

At the start of each demand period the total demand rate is calculated as the sum of the rates for each movement type. The capacity is calculated from the intermovement time matrix and the movement mix given by the demand.

The differential equations are then integrated numerically during the period while the demand remains constant, the cumulative totals being incremented in parallel. During this period one or more outputs of the current state of the system may be made and these are accompanied by the record of the (input) demand.

The calculation is continued for each period until the end when the cumulative totals are output.

AIRQF

The basic structure of the computation is the same as for AIRQR although the details differ because the equations describing the evaluation of the system are not differential equations (see Appendix V).

As far as the results are concerned, the only difference is that the number of aircraft turned away is not readily calculable and is omitted.

MARQ

The marginal delay at a particular time is defined as the increase in total delay due to one extra request for service at that time. MARQ determines the marginal delay during successive hours by repeated application of AIRQR. Having performed the calculation for one daily cycle, and having calculated the total delay to all aircraft for that day, the model then calculates the total delay (and then the increase in total delay) which results from a unit increase in demand during the first hour of the cycle. This increase in total delay is the marginal delay during the first hour. Similar calculations are made for unit demand increments during each hour of the day.

OUTPUT

Figure VI.2 shows an example of the output of AIRQR from which the upper delay profile in Figure 5.2 was plotted. The 12 movement types are those given in Appendix I. Following a record of the input control parameters and the intermovement time matrix the items listed below are tabulated at each output time. Finally a number of cumulative totals for the whole day are given.

At each output the following items are printed in the indicated columns:

- The time (col 1);
- The probability that the system is empty (col 2) and full (col 3);
- The mean (col 4) and standard deviation (col 5) of the number of aircraft in the system;
- The mean delay suffered by aircraft requesting service during the previous interval (col 6). This is the quantity plotted against time in Figure 5.2;
- The runway capacity during the previous interval (col 7);
- The total demand rate for all movement types during the previous interval (col 8);
- The (input) demand rate for each movement type during the previous interval. Their sum over all movement types is given as item (f) (cols 9 . . .).

At the end of the day the following cumulative totals are printed:

- The expected total delay to all aircraft.
- The expected total delay for each movement type.
- The expected number of aircraft diverted (ie which requested service when the system was full).

The output from AIRQF is similar to the above except that the integration step length and cumulative number of aircraft diverted are omitted.

The output from MARQ omits the items in columns 5, 6 and 9 onwards and substitutes the marginal delay due to a unit increase in demand during the previous interval. The cumulative totals are also omitted. A sample output from which the marginal delay profile in Figure 5.3 was plotted is shown in Figure VI.3.

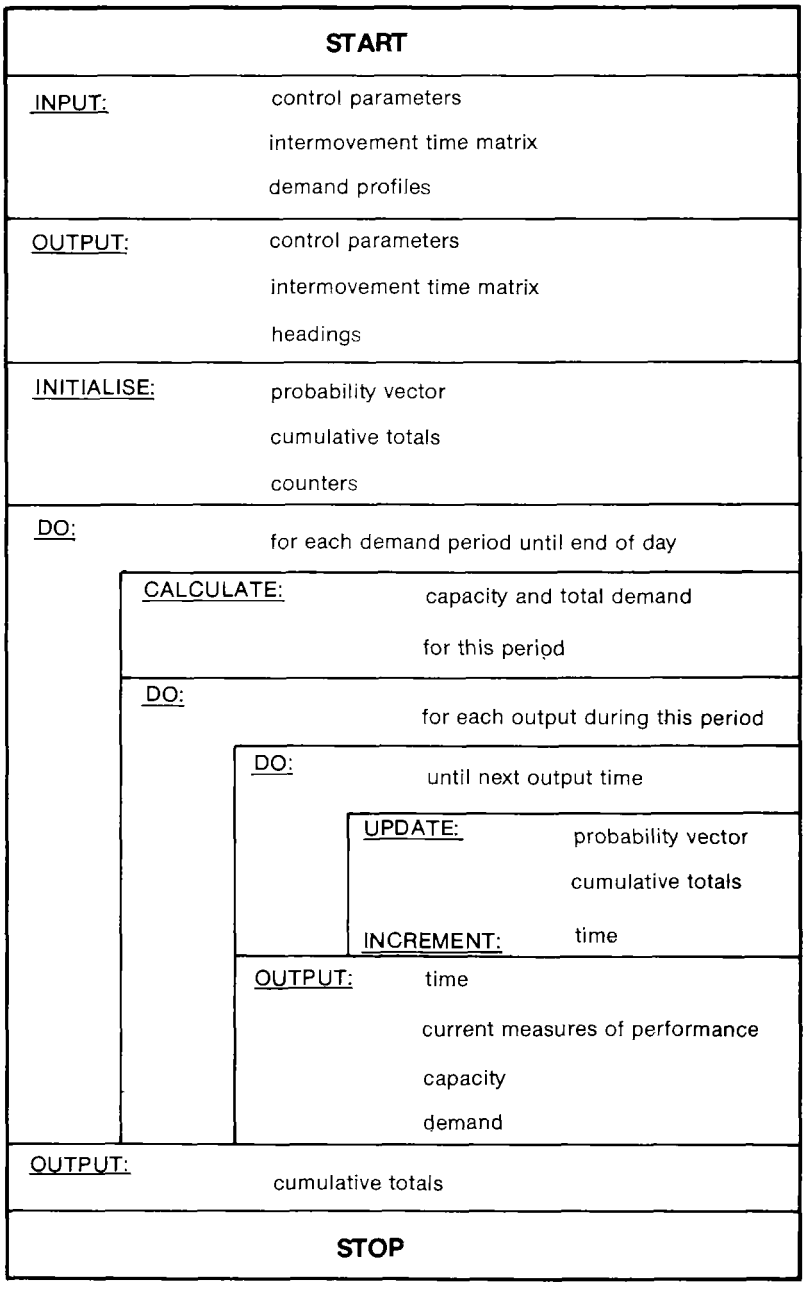

Figure VI.1
Schematic flow chart for AIRQR

Figure VI.2
Sample output from AIRQR

RUNWAY CAPACITY AND DELAY ANALYSIS
(NEG EXP DISTRIBUTION OF SERVICE TIME)
****

INPUT PERIOD (MIN): 60.0
NUMBER OF OUTPUTS/INPUT: 1
INTEGRATION STEP LENGTH (MIN): 0.40
MAXIMUM NUMBER IN SYSTEM: 50
NUMBER OF MOVEMENT TYPES: 12

INTERMOVEMENT TIME MATRIX:

75.	120.	180.	50.	50.	50.	75.	75.	75.	20.	20.	0.
55.	55.	150.	45.	45.	45.	80.	80.	80.	25.	25.	0.
55.	55.	55.	40.	40.	40.	100.	100.	100.	40.	40.	0.
55.	55.	55.	90.	120.	180.	90.	90.	180.	95.	125.	0.
55.	55.	55.	90.	90.	150.	90.	90.	90.	30.	30.	0.
55.	55.	55.	90.	90.	90.	90.	90.	90.	35.	35.	0.
70.	70.	70.	15.	15.	15.	75.	120.	180.	60.	60.	60.
75.	75.	75.	20.	20.	20.	55.	55.	150.	50.	50.	50.
90.	90.	90.	35.	35.	35.	55.	55.	55.	45.	45.	45.
80.	80.	165.	85.	115.	170.	55.	55.	55.	90.	120.	180.
85.	85.	85.	25.	25.	140.	55.	55.	55.	90.	90.	150.
0.	0.	0.	0.	0.	0.	55.	55.	55.	90.	90.	90.

TIME	PROBABILITY		SYSTEM NUMBER		MEAN DELAY	CAPACITY	--- DEMAND (MVT/HR) ---											
(HR)	EMPTY	FULL	MEAN	STDV	(MIN)	(MVT/HR)	TOTAL	BY MOVEMENT TYPE --										
1.00	0.903	0.	0.108	0.346	0.13	50.27	4.9	0.	0.4	0.6	0.	2.3	0.5	0.	0.5	0.3	0.	0.	0.3
2.00	0.937	0.	0.068	0.268	0.08	52.17	3.3	0.	0.6	0.5	0.	0.8	0.4	0.	0.8	0.1	0.	0.	0.1
3.00	0.919	0.	0.088	0.309	0.10	50.80	4.1	0.	0.3	1.0	0.	1.4	0.4	0.	0.4	0.5	0.	0.	0.1
4.00	0.935	0.	0.070	0.273	0.08	53.71	3.5	0.	0.3	1.1	0.	0.5	0.5	0.	0.3	0.5	0.	0.	0.3
5.00	0.933	0.	0.072	0.278	0.07	58.08	3.9	0.	0.1	1.3	0.	0.1	1.1	0.	0.3	0.5	0.	0.	0.5
6.00	0.937	0.	0.067	0.267	0.08	50.96	3.2	0.1	0.	0.4	0.	0.6	1.3	0.	0.1	0.1	0.	0.	0.6
7.00	0.572	0.	0.748	1.143	0.85	50.26	21.5	3.3	0.9	1.4	0.1	6.3	4.8	0.9	1.1	0.6	0.	0.	2.1
8.00	0.078	0.000	7.590	6.089	6.12	49.43	49.5	2.8	4.4	4.2	1.1	17.8	8.1	0.6	5.3	1.8	0.	0.	3.4
9.00	0.076	0.000	9.295	7.951	10.15	50.09	47.8	3.5	5.7	4.5	3.3	13.9	4.9	0.9	7.1	1.9	0.	0.	2.1
10.00	0.066	0.001	10.875	9.277	12.22	49.68	47.8	2.1	5.8	4.7	4.2	14.4	4.9	0.5	7.1	2.0	0.	0.	2.1
11.00	0.104	0.000	9.109	9.131	11.56	51.03	44.6	1.6	7.5	3.4	4.2	10.9	4.3	0.4	9.1	1.4	0.	0.	1.8
12.00	0.169	0.000	6.213	7.542	8.90	49.93	39.7	2.3	5.2	3.0	3.2	14.2	2.7	0.5	6.2	1.3	0.	0.	1.1
13.00	0.250	0.000	3.734	5.234	5.70	49.39	35.8	1.8	3.9	4.8	3.8	9.6	3.2	0.5	4.8	2.0	0.	0.	1.4
14.00	0.255	0.000	3.155	4.057	4.11	49.73	36.7	1.5	4.2	4.5	2.9	11.1	3.5	0.4	5.2	1.9	0.	0.	1.5
15.00	0.234	0.000	3.304	3.873	3.94	49.91	38.2	1.9	4.9	3.7	3.7	11.6	3.0	0.5	5.9	1.6	0.	0.	1.4
16.00	0.162	0.000	4.764	4.898	5.04	50.41	42.9	1.8	6.2	5.2	3.0	11.2	3.7	0.4	7.6	2.3	0.	0.	1.5
17.00	0.067	0.000	9.231	7.513	8.71	50.26	50.2	0.8	5.8	6.3	2.3	16.0	6.1	0.3	7.1	2.8	0.	0.	2.7
18.00	0.074	0.000	10.139	8.857	11.32	51.26	48.4	0.6	7.5	6.6	1.9	13.0	4.8	0.1	9.0	2.8	0.	0.	2.1
19.00	0.054	0.001	12.426	10.208	13.34	50.95	50.2	0.8	6.8	5.3	1.0	17.3	5.7	0.3	8.3	2.3	0.	0.	2.4
20.00	0.089	0.001	10.589	10.163	13.12	51.82	46.0	2.0	7.6	4.8	0.5	15.0	2.8	0.5	9.4	2.1	0.	0.	1.3
21.00	0.110	0.001	9.046	9.470	11.23	51.78	45.0	1.4	7.2	3.9	0.6	17.8	2.3	0.4	8.8	1.6	0.	0.	1.0
22.00	0.389	0.000	2.366	4.348	5.29	52.73	30.0	1.3	6.6	2.1	0.9	7.8	1.5	0.3	8.0	0.9	0.	0.	0.6
23.00	0.581	0.000	0.745	1.251	1.27	51.86	21.6	0.6	4.4	1.9	1.3	5.7	1.0	0.1	5.3	0.9	0.	0.	0.4
24.00	0.878	0.	0.140	0.399	0.19	52.25	6.4	0.1	1.1	0.8	0.3	1.5	0.5	0.	1.5	0.3	0.	0.	0.3

CUMULATIVE DELAY: TOTAL: 5933.6
BY MOVEMENT TYPE: 227.1 827.0 603.8 306.6 1849.4 555.7 57.7 1008.9 258.5 0. 0. 238.8

AIRCRAFT DIVERTED: 0.17

RUNWAY CAPACITY AND DELAY ANALYSIS
(NEG EXP DISTRIBUTION OF SERVICE TIME)
MARGINAL DELAYS
****

INPUT PERIOD (MIN): 60.0
INTEGRATION STEP LENGTH (MIN): 0.40
MAXIMUM NUMBER IN SYSTEM: 50
NUMBER OF MOVEMENT TYPES: 12

INTERMOVEMENT TIME MATRIX:

75.	120.	180.	50.	50.	50.	75.	75.	75.	20.	20.	0.
55.	55.	150.	45.	45.	45.	80.	80.	80.	25.	25.	0.
55.	55.	55.	40.	40.	40.	100.	100.	100.	40.	40.	0.
55.	55.	55.	90.	120.	180.	90.	90.	180.	95.	125.	0.
55.	55.	55.	90.	90.	150.	90.	90.	90.	30.	30.	0.
55.	55.	55.	90.	90.	90.	90.	90.	90.	35.	35.	0.
70.	70.	70.	15.	15.	15.	75.	120.	180.	60.	60.	60.
75.	75.	75.	20.	20.	20.	55.	55.	150.	50.	50.	50.
90.	90.	90.	35.	35.	35.	55.	55.	55.	45.	45.	45.
80.	80.	165.	85.	115.	170.	55.	55.	55.	90.	120.	180.
85.	85.	85.	25.	25.	140.	55.	55.	55.	90.	90.	150.
0.	0.	0.	0.	0.	0.	55.	55.	55.	90.	90.	90.

TIME (HR)	PROBABILITY EMPTY	PROBABILITY FULL	AVERAGE NO IN SYSTEM	CAPACITY (MVT/HR)	DEMAND (MVT/HR)	MARGINAL DELAY (MIN)
1.00	0.903	0.	0.108	50.27	4.90	0.3
2.00	0.937	0.	0.068	52.17	3.30	0.2
3.00	0.919	0.	0.088	50.80	4.10	0.2
4.00	0.935	0.	0.070	53.71	3.50	0.2
5.00	0.933	0.	0.072	58.08	3.90	0.2
6.00	0.937	0.	0.067	50.96	3.20	0.2
7.00	0.572	0.	0.748	50.26	21.50	5.1
8.00	0.078	0.000	7.590	49.43	49.50	88.3
9.00	0.076	0.000	9.295	50.09	47.80	102.4
10.00	0.066	0.001	10.874	49.68	47.80	90.4
11.00	0.104	0.000	9.108	51.03	44.60	66.2
12.00	0.169	0.000	6.213	49.93	39.70	42.9
13.00	0.250	0.000	3.733	49.39	35.80	27.4
14.00	0.255	0.000	3.155	49.73	36.70	24.1
15.00	0.234	0.000	3.304	49.91	38.20	33.0
16.00	0.162	0.000	4.764	50.41	42.90	65.5
17.00	0.067	0.000	9.230	50.26	50.20	108.0
18.00	0.074	0.000	10.139	51.26	48.40	107.1
19.00	0.054	0.001	12.425	50.95	50.20	94.5
20.00	0.089	0.001	10.589	51.82	46.00	67.7
21.00	0.110	0.001	9.045	51.78	45.00	40.7
22.00	0.389	0.000	2.366	52.73	30.00	13.1
23.00	0.581	0.000	0.745	51.86	21.60	3.0
24.00	0.878	0.	0.140	52.25	6.40	0.2

Figure VI.3
Sample output from MARQ

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ABBREVIATIONS

A	Arrival
AIL	Airborne Instruments Laboratory
ARR	Arrival
ATC	Air Traffic Control
D	Departure
DEP	Departure
FAA	Federal Aviation Administration (United States)
IFR	Instrument Flight Rules
ILS	Instrument Landing System
IMC	Instrument Meteorological Conditions
IMT	Intermovement Time
KSA	Sydney (Kingsford Smith) Airport
MANS	Major Airport Needs of Sydney
RPT	Regular Public Transport
VFR	Visual Flight Rules
VMC	Visual Meteorological Conditions
WTS	Wake Turbulence Separation