Demand for International Air Travel: a Conceptual and Operational Framework

Occasional Paper

The emphasis in this paper is on the development of a suitable conceptual and operational framework within which a policy-sensitive empirical travel choice model could be estimated. Functional Measurement and Discrete Choice Theory provide the theoretical base from which models of individuals' choices can be derived. The aim of this paper is to outline a method which is capable of identifying international air fare structures to various destinations that accord with the likely choices of individuals (both current and potential travellers). The information obtained will permit the determination of a reduced-set of feasible air fare structures which represent a compromise with the full range of possible alternatives. Particular emphasis is given to identifying the variation in frequency of choice of international air travel to various destinations as a result of changes in air fare structures.

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Demand for International Air Travel:

A Conceptual and Operational Framework

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FOREWORD

The review of Australia's International Civil Aviation Policy (ICAP) highlighted the need for the Government to have information on the implications of alternative international air fare structures. This paper represents a continuation of the work carried out as BTE's input to the ICAP Review (published as BTE Occasional Paper No 11 in 1978).

In recent years, sophisticated statistical techniques, based on choice theory and trade-off analysis, have been developed to measure perceptions and reactions of people to the attributes of products and services available for purchase. The Bureau is planning a major study of demand for international air travel using these techniques. In order to advance this work, the Bureau commissioned Dr J.J. Louviere of the University of Iowa and Dr D.A. Hensher of Macquarie University to prepare an overall study design plan. This paper reports on the first stage of this study dealing with the development of the required conceptual and operational framework and draws very heavily on the original work submitted to the Bureau by the two consultants. The overall study is being undertaken in the Economic Assessment Branch with the main researchers being Dr M. Saad, Mr C. Piccinin, Ms S. Watt and Ms Y. Dunlop (prior to her departure).

The Bureau believes that this paper contributes to our knowledge and understanding of these techniques and highlights their applicability to a wide range of demand studies and transport planning problems. An application of the choice theory approach to passenger travel between Tasmania and the Australian mainland is currently being processed for publication by the Bureau.

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June 1981

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CHAPTER 1 - INTRODUCTION

AN OVERVIEW

The market for international air travel is dynamic. In recent years there has been a marked growth in the range of destinations and ticket types available for international air travel. Ticket types are characterised by the level of the fare and the ticket conditions (for example, the cancellation penalty, number of permitted stopovers, season, advance purchase requirements, and so on) associated with the air fare ticket in question. This growth has meant that there are now a large number of alternative ticket packages from which a prospective air traveller may choose.

In considering demand for international air travel one would like to be able to analyse the effect of these (and other) changes in ticket conditions on the level of demand. Usually estimation of demand is based on either historical time series or cross section data or both. Analysis of demand frequently involves extrapolation from the range of values of certain variables in the estimated demand relationship. The above methods have limitations. The coarseness of the data base often limits modelling to a high level of aggregation because the data cannot distinguish between the behavioural variability across the population as a whole and within selected disaggregate market segments of policy interest. Also it limits the capability of assessing the impact of an extended set of future ticket types. Furthermore, when using time series data, one may need to simultaneously model supply or run into identification problems(1). This complicates an already complex task. Finally, broad aggregate measures such as price elasticities and/or income elasticities of demand for all leisure travel are useful, but are of limited value in the formulation of an overall strategy to determine a series of air fare structures. Such strategy must be based on information on individuals' responses to the availability of ticket types to various destinations and to the interaction of the different ticket conditions.

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(1) For an exposition of the identification problem see Chapter 13 of Kmenta (1971).

The approach proposed in this paper is based upon theory developed in economics and psychology which justifies the use of multilinear utility functions in choice and preference models. These models are estimated to represent the likelihood of an individual selecting a particular set of ticket conditions for travel to a given destination or selecting a destination and set of ticket conditions simultaneously. A study of this kind may be of considerable value in the development of policy on future international air fare structures, and may provide more robust predictions of the likely impact of structural changes than aggregate modelling procedures.

It should be noted that the proposed approach overcomes some important limitations of the traditional method. The data base is totally disaggregate in that a separate model for each individual can be obtained if required. The range of observations can be broadened to cover the span of policy options one wishes to analyse. There is no need to estimate a supply function. Moreover, policy-sensitive demand models can be developed across the sample of travellers and non-travellers which maintain the individual as the unit of analysis (in contrast to many transport demand studies where the data are specified at a zone-pair level). The philosophy behind the approach is that it is preferable to develop a totally disaggregate data base and modelling capability; thus retaining the maximum freedom to select the desired level of data aggregation for whatever modelling analysis needs to be undertaken.

OBJECTIVES OF THE PAPER

The emphasis in this paper is on the development of a suitable conceptual and operational framework within which a policy-sensitive empirical travel choice model could be estimated. Functional Measurement and Discrete Choice Theory provide the theoretical base from which models of individuals' choices can be derived. This approach incorporates the technique of factorial experimental design to systematically vary travel options available to travellers. Such experimentally induced responses allow one to develop estimates of actual and potential choices in view of current and possible future actions and events. The models derived from such experiments provide guidance on the likely behavioural responses of individual travellers to changes in the fare structures (including ticket conditions) between destinations. Such impacts can also be identified across a well-defined set of homogeneous traveller markets, segmented according to criteria of policy interest.

Therefore, the aim of this paper is to outline a method which is capable of identifying international air fare structures to various destinations that accord with the likely choices of individuals (both current and potential travellers). The information obtained will permit the determination of a reduced-set of feasible air fare structures which represent a compromise with the full range of possible alternatives. Given knowledge on the relationship between the sample of potential and actual travellers and the population, a scenario of predictors of market share proportions (say journey purpose by destination) can be identified. Particular emphasis is given to identifying the variation in frequency of choice of international air travel to various destinations as a result of changes in air fare structures.

STRUCTURE OF THE PAPER

Chapters 2 to 5 detail the theory underlying the disaggregated modelling approach to demand analysis. Chapter 2 develops the Functional Measurement approach to demand modelling and compares it to the Revealed Behaviour approach. Chapter 3 presents the analytical framework from which a travel choice model can be derived. Chapter 4 deals with sample design, outlining the use of factorial experimental design and fractional factorial design, and considers the restrictions imposed on the sample design by the use of a fractional factorial design. Chapter 5 includes a proposed sample design which is constructed as an international air travel demand experiment. Chapter 6 consists of concluding remarks. Finally, technical material is contained in two appendices: the first dealing with specification of individuals' preference functions and the second with orthogonality in sample design.

CHAPTER 2 - FUNCTIONAL MEASUREMENT

There are two general frameworks within which model development, estimation and verification can be accommodated in demand studies. These are the conventional Revealed Behaviour Approach (RBA) and the more recently developed approach of Functional Measurement (FM).

The RBA observes current and/or recent past choice behaviour amongst travel alternatives and attempts to relate actual/revealed choices to the attributes of both the alternatives and of the individuals by means of various statistical choice models. By comparison the FM approach (discussed in this paper) develops experimentally designed choice contexts with desirable statistical properties and assesses the experimentally induced responses of individuals to such situations. Hence, models with known statistical power and properties of the estimates can be developed.

BASIC CONCEPTS

FM refers to a variety of experimental design-based methods for assessing the responses of individuals or groups to current and potential demand (behavioural) situations. Mathematically, it is an approach which leads to development of quantitative expressions (or models) which describe the process by which the attributes of alternatives are combined by an individual or a group of individuals to choose among various bundles or collections of attributes (that is, alternatives). The actual choice or decision of the individual or group of individuals is inferred by examining their choices over different sets of offered alternatives.

In the present context of assessing the future demand for international air travel, FM defines any procedure which seeks to measure or observe the potential travel choices of individuals under various destinations and ticket type scenarios. This is achieved by expressing the offered travel alternatives (such as different air fare packages) as combinations (or bundles) of the levels of the different attributes which comprise the alternatives. To predict traveller choices and to assess the sensitivity of these choices to policy alternatives, a model is specified by relating these choices to both the attributes of the alternatives and the characteristics of individuals.

AVAILABILITY SCENARIOS

In order to observe travel choices of individuals, it is necessary to create a number of hypothetical 'availability' scenarios in which various combinations of destinations and fare packages (that is, alternatives) are offered (or not offered) to travellers and potential travellers. These experimentally designed scenarios are required because it is necessary to generate sets of alternatives that span as much of the entire range of possible future conditions as possible in ensuring that <u>any</u> future alternative could be represented by interpolation. Moreover, to ensure realism of results, the hypothetical choice alternatives should be related to current and recent past alternatives which are a subset of all the travel choices to which the derived choice models should apply. In this way the experimentally induced responses can be related to actual choices. This is a major advantage of the FM approach particularly in the case of new products/services. That is, it enables one to make maximum use of a limited data base of actual alternatives supplemented by a range of possible future alternatives.

LEVEL OF ANALYSIS

With the FM approach, it is possible to develop choice models at an aggregate or an individual level of analysis. It is proposed that the individual be employed as the basic unit of data collection and analysis for two reasons. First, such a level is behaviourally more meaningful and valid in that impacts accrue to single individuals or similar traveller 'groups' whose composition cannot be fully known a priori. Second, the validity of the assumptions inherent in various methods of aggregation are questionable (see Hensher and Johnson, 1981).

METHODOLOGY

The FM approach outlined in this paper is based upon work in psychology known as Behavioural Decision Theory (Slovic, Fischhoff and Lichtenstein 1977) and related methods in discrete choice theory in econometrics (McFadden 1974; Hensher and Johnson 1981). Moreover, methods developed in classical approaches to the design and analysis of experiments to formulate mathematical models of choices or decisions are employed (Anderson 1970; Winer 1971;

Louviere 1980). The FM approach consists of four discrete but necessarily interrelated steps. In the first step, the basic causal variables which influence the choice or decision of an individual are identified. In the second step, the variables identified in step one are experimentally manipulated and choices or decisions of interest (hereafter termed 'responses') are observed. The object of this second step is to express the responses as an algebraic function (model) of the manipulated experimental variables. A third step is required for validation and involves the models being applied to predict the recent past choices of a subsample of the respondents who have been interviewed. This permits assessment of models performance by application to actual choices and also indicates whether any further specification and estimation is necessary. A final step involves the use of the models to make forecasts of choices of destination/ticket type/trip frequency for some period of interest (say, the next two years) for various policy alternatives.

COMPARISON WITH REVEALED BEHAVIOUR APPROACH

The RBA has the advantage of being related directly to actual behaviour, with the limitation that the behaviour was only observed within a limited context. In relation to international air travel, the context is limited to whatever variability currently exists between various destination and fare package alternatives. Hence, the RBA relates actual choices to alternatives which <u>cannot</u> be assessed a priori regarding statistical power and properties of estimates. Thus, the covariance structure of real alternatives is biased toward current and recent past situations and, therefore, any policy decisions which change the covariance structure (as many do) limit the parameter estimates for forecasting. On the other hand, the FM approach is very strong in that it can ensure that the model parameters are relatively independent of existing covariance structures, but weak in direct validity to actual behaviour. This is attributable to the unknown link between choices made in an experimental environment and those made in a real-life situation.

In the present area of interest the RBA would be particularly weak in that it appears future conditions are likely to be significantly different from those at present or in the past. Hence the FM approach is developed in this paper as an appropriate approach to formulate a model to assess the future demand for international air travel. By using estimated FM models in a simulation of actual choices made by the sample respondents, it is possible to verify the models, thus minimising the disadvantages of this approach.

CHAPTER 3 - ANALYTICAL FRAMEWORK

As indicated in Chapter 2, FM is a statistical approach to the design and analysis of choice studies. This approach is based on research in mathematical psychology and is characterised by two aspects. Firstly, FM provides the analytical framework from which models of individuals' choices can be derived. Secondly, the approach incorporates the use of experimental methods to estimate choice models rather than observations on individuals' revealed choices. This chapter together with Appendix 1 develops the theory which provides the framework for analysis(1).

BASIC ASSUMPTIONS

The FM approach involves four assumptions. The first is that individuals have perceptions or beliefs about the levels of attributes possessed by alternatives. These perceptions or beliefs are a function of physically measurable qualities or properties of the alternatives. That is:

$$x_i = f_i\ (X_i) \qquad (3.1)$$

where x_i is the perceived or subjective level of attribute i,
X_i is the actual or measured level of attribute i,
and f defines the mapping unique to each measured levels of attribute (X_i), onto the perceived levels (x_i) of attribute i.

It is on the perceived levels (x_i) of attributes, that individuals base their value judgments rather than on the measured value (X_i) of the attributes. Equation (3.1), therefore, is often termed the 'psychophysical function' in that it describes how individuals' beliefs about the levels of attributes vary with the physical properties of the observed variable.

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(1) Chapter 4 discusses the experimental measurement techniques required for the estimation of these models.

The second assumption is that individuals place different values on the levels of the various attributes. Hence, the 'worth' of a perceived level of an attribute must be a function of the observed level of that attribute. That is(1)

$$v\ (x_i) = g_i\ (X_i) \tag{3.2}$$

where $v(x_i)$ is the worth of the perceived level of attribute i, and g_i defines the mapping, unique to each attribute i, from the observed value of attribute i, (X_i), onto the worth of the perceived level of attribute i, $(v(x_i))$.

The third assumption is that individuals determine their overall preference or value judgment for each alternative (which is described as a combination of attributes) by combining the perceived values of the respective attributes represented by equation (3.2). That is:

$$V_j = d_j\ (v(x_{1j}), v(x_{2j}), \ldots, v(x_{ij}), \ldots v(x_{Ij})),\ j = 1,\ldots,J \tag{3.3}$$

where V_j is the overall value or worth of the jth alternative,
$v(x_{ij})$ is the marginal value of attribute i associated with alternative j,
d_j is a mapping defined over the I attributes of alternative j,
and J is the number of alternatives.

Equation (3.3) is known as the 'joint value function' because it represents the manner in which the separate marginal values are combined by individuals to produce a single value for each alternative.

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(1) The worths assigned to levels of attributes are frequently referred to as 'marginal' values. This term is used because a full analysis of all possible alternatives is technically a factorial experimental design or a cross-classification table. The sums, averages etc, recorded on the margins of this table are technically estimates of the partial worths of each level of each attribute holding all other attributes constant. Hence, the term 'marginal' value. This term is adhered to throughout the remainder of this discussion.

The final assumption is that the observed response or choice is a function of the joint value given to each of the J alternatives. In the most general case, individuals might be permitted to choose frequently among the J alternatives. Thus, it would be possible to observe repetitions of choices by a single individual. This assumption can be mathematically expressed as

$$P_j = h(V_j), \tag{3.4}$$

where P_j is the probability (relative frequency) of choices allocated to alternative j over a particular observation period,
V_j is defined above,
and h is a mapping defined over all the values of the alternatives.

The above four assumptions, mathematically described by equations (3.1), (3.2), (3.3) and (3.4) can be incorporated into one composite choice function. That is:

$$P_j = h(d_j(v(x_{1j}), v(x_{2j}), \ldots, v(x_{Ij})))$$

$$= h(d_j(g_1(X_{ij}), g_2(X_{2j}), \ldots, g_I(X_{Ij}))) \tag{3.4'}$$

Defining the vector function $\underset{\sim}{\phi}_j(\underset{\sim}{X}_j)$ - where $\underset{\sim}{X}_j = (X_{1j}, X_{2j}, \ldots, X_{Ij})$ - as

$$\underset{\sim}{\phi}_j\ (\underset{\sim}{X}_j) = (g_1(X_1), g_2(X_2), \ldots, g_I(X_1)), \tag{3.5}$$

renders it feasible to represent P_j as

either $$P_j = h(d_j(\underset{\sim}{\phi}_j(\underset{\sim}{X}_j))) \tag{3.6}$$

or $$P_j = F_j(\underset{\sim}{X}_j), \tag{3.6'}$$

where F_j is the composite function determining the probability of selecting alternative j given the measured level of its attributes.

IDENTIFICATION GUIDELINES

To operationalise this theoretical framework for modelling purposes, it is necessary either to make explicit assumptions about the functional form in equations (3.1) to (3.5) or to identify the functional forms empirically. Both alternatives require an error theory for specification/testing. In the absence of *a priori* expectations concerning the functional form the latter alternative is usually selected. The most commonly used specifications of the models represented by equations (3.1) to (3.5) are reviewed below.

Marginal value functions

Previous research has indicated that the functional forms of the marginal values are usually non-linear (Louviere 1978, 1979; Louviere and Henley 1977; Louviere and Levin 1979, Louviere and Meyer 1976, Meyer 1977, Norman 1977). The most common specifications of equation (3.2) employed in empirical work are the following

$$v(x_i) = a_i + b_i X_i^{c_i} \tag{3.7a}$$

$$v(x_i) = a_i + b_i X_i + c_i X_i^2 \tag{3.7b}$$

$$v(x_i) = a_i + b_i (\exp (c_i + d_i X_i)) \tag{3.7c}$$

It should be noted that each individual can have a unique shape for his or her marginal value functions. The actual shapes of the curves are determined by the coefficients of the above three equations. It is hypothesised that differences between individuals' coefficients are related to the personal characteristics of the individuals concerned. Thus, if the coefficients can be estimated for each individual, it would be possible to test for differences in these coefficients among individuals as a function of differences in their interpersonal characteristics(1).

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(1) This is discussed in Chapter 5.

General multilinear form of the joint value function

A common specification of the joint value equation (3.3), is that of the general multilinear form. In the case of three attributes, such specification is:

$$
\begin{aligned}
V_j = k_0 &+ k_1\ v(x_{1j}) + k_2\ v(x_{2j}) + k_3\ v(x_{3j}) \\
&+ k_4\ v(x_{1j})v(x_{2j}) + k_5\ v(x_{1j})v(x_{3j}) \\
&+ k_6\ v(x_{2j})v(x_{3j}) + k_7\ v(x_{1j})v(x_{2j})v(x_{3j}) + \varepsilon_j
\end{aligned}
\tag{3.8}
$$

where V_j, $v(x_{ij})$ are defined above (there are a total of 3 attributes defining alternative j in this case),

k_i are scaling coefficients,

and ε_j represents an error term.

The single terms in equation (3.8) are commonly referred to as 'main effects' because they represent the 'main' contribution of each attribute (independent of other attributes) to the overall value or response. The product terms are 'interactions' representing any combined effects that two or more attributes have, above and beyond the sum of their main effects.

The additive and multiplicative specifications of the general multilinear form

The most commonly employed restricted specifications of equation (3.8) are simple additive and multiplicative forms. These are:

$$V_j = k_0 + k_1 v(x_{1j}) + k_2 v(x_{2j}) + k_3(x_{3j}) + \varepsilon_j \tag{3.9a}$$

and

$$V_j = l_0 + l_1 v(x_{1j})v(x_{2j})v(x_{3j}) + \varepsilon_j \tag{3.9b}$$

where all the variables are as defined above while the k's and l's are scaling coefficients.

Previous studies of individuals' choices (Louviere and Levin 1979; Louviere and Meyer 1976; Meyer 1977; Norman 1977; Louviere 1980) have repeatedly suggested that multiplicative forms are better descriptors of the value judgments of individuals than are strictly additive forms. Similarly, it has been found that when two or more attributes are considered simultaneously (jointly), if one attribute is at a very undesirable level, it makes little difference what the values of the remaining attributes might be. Moreover, multiplicative model forms are extremely useful for evaluation and planning studies because the theory provides a means by which threshold levels for attributes can be estimated and their potential joint effects determined prior to initiating say a transport service(1).

Probability choice function

An accepted form of the probability choice function (equation 3.4)) is that of the multinomial logit model. This is a disaggregate choice model in which it is generally assumed that the joint value functions alternatives, (V_j), are linear functions of the attributes, that is

$$V_j = d_j(v(x_{1j}), v(x_{2j}), \ldots v(x_{Ij})). \qquad (3.3)$$

Assuming $v(x_{ij}) = X_{ij}$, then

$$V_j = \beta_{1j} X_{1j} + \beta_{2j} X_{2j} + \ldots + \beta_{Ij} X_{Ij} + \varepsilon_j \qquad (3.10)$$

where ε_j is an error term,

and β_{ij} are coefficients to be estimated.

Defining:

$$V'_j = \sum_{i=1}^{I} \beta_{ij} X_{ij}, \qquad (3.11)$$

results in the probability choice function being specified in the multinomial logit model as:

(1) See Appendix 1.

$$P_j = h(V_j), \qquad (3.4)$$

$$= \exp V'_j / \sum_{j=1}^{J} \exp V'_j, \qquad (3.12)$$

$$= \exp\left(\sum_{i=1}^{I} \beta_{ij} X_{ij}\right) / \sum_{j=1}^{J} \exp\left(\sum_{i=1}^{I} \beta_{ij} X_{ij}\right) \qquad (3.12')$$

The theoretical base of the logit model and appropriate estimation procedures are well documented in the literature (Hensher and Johnson 1981).

CHAPTER 4 - SURVEY METHODOLOGY

This chapter deals with a technique for constructing questionnaire and survey forms. These questionnaires are the basis of a demand experiment which permits the specification of individual and group response functions as they apply to the range of alternatives offered. These designs are constructed to induce responses that span over a broad range of policy circumstances because they cover a wider range of options than those presented by existing choices.

FACTORIAL DESIGN

Modelling individuals' responses to multi-attribute alternatives involves selecting combinations of levels of attributes for individuals to evaluate. This is similar to more traditional travel analyses in which the observations of choice employed in transportation data sets consist of observations on combinations of attributes at different levels. Because of the nature of the real world, however, the vectors of real attributes are almost always highly correlated. For example, values for stopovers, air fares and travel times are almost certainly related to distance, and will be strongly correlated across most trips. Moreover, in many cases only a subset of the possible alternatives is available to individuals, thus limiting the inferences that can be drawn. In other words, the data collection methods discussed in this section are extensions of traditional data collection efforts which improve the range of alternatives and the descriptions of their attributes over those available in actual choice data. Furthermore, they are an improvement over traditional data collection techniques in that they permit one to designate *a priori* which statistical effects can be examined and with what power.

It is theoretically possible (although not practical to implement in most instances) for these sampling plans to guarantee independence of all attributes so as to permit estimation of a large number of joint effects of these attributes. One method of collecting data, termed a factorial design, is a procedure for constructing a set of combinations of levels of attributes by which it is possible to guarantee the independence of both single and joint effects of the attributes (this statistical independence allows the estimation of all the coefficients in equation (3.8)).

For example, various 'tickets' to a particular destination could be developed as combinations of levels of fare, advance purchase requirement, and cancellation penalty. If, say, two levels are allowed for each attribute (eg, fare - $800, $1200; advance purchase requirement - 30 days, 45 days; cancellation penalty - 25 per cent, 50 per cent), there are eight possible 'tickets' (alternatives) given by these combinations as shown in Table 4.1. Note that each combination provides information regarding all of the attributes. This sampling plan is very efficient because it provides information about all single (marginal) effects and all joint (interaction or cross-product) effects. The approach to using this kind of plan (from which the present study is derived) is to present individuals with sets of alternatives developed according to a factorial design plan similar to those alternatives in Table 4.1. The individuals are requested to provide some response of interest, such as degree of 'preference', 'likelihood of use', or number of trips for each alternative. A statistical model fitted to such data permits one to simulate the effects of changing these ticket conditions as if they had been changed in the real world. This kind of data collection scheme is called a factorial design because all combinations of all attributes are enumerated and responses to all are observed. Technically, the sampling design in Table 4.1 is a 2^3 factorial. If each attribute is assigned a third level, (say $500 for fare; 60 days for advance purchase requirement; and 75 per cent cancellation penalty), the design would become a 3^3 factorial, comprising 27 alternatives.

TABLE 4.1 - FACTORIAL COMBINATIONS IN TICKET EXAMPLE

Ticket alternatives	Fare ($)	Advance purchase requirement (days)	Cancellation penalty (per cent)
1	800	30	25
2	800	30	50
3	800	45	25
4	800	45	50
5	1 200	30	25
6	1 200	30	50
7	1 200	45	25
8	1 200	45	50

With increases in the number of attributes or the number of levels, or both, the total number of possible combinations increases rapidly. For example, in a design of 4 levels over 4 attributes, there would be 4^4 (or 256) sets of attribute bundles. As the total number of combinations increase, the requirements placed upon respondents become prohibitive.

As a result, a number of ways to reduce the total number of judgments required in a factorial design experiment have been developed. These modified data collection plans are called Fractional Factorial Designs.

FRACTIONAL FACTORIAL DESIGNS

The following are important questions which should be answered before selecting a fractional design:

a) What type of information does one require for modelling purposes?

- main effects only(1); main effects plus selected interaction effects(2); or all main and interaction effects.

b) What is the nature of the levels of each attribute?

- do different attributes have the same or different numbers of levels.

c) How many attributes does the researcher want to vary in any single set of combinations?

- all attributes; or some subset of them.

Question (a) essentially determines the complexity of the information one needs to obtain from the respondent: the collection of information on all interaction effects is rarely practical and hence the selection of certain interaction effects is usually required.

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(1) Main effects are those effects which are due to an attribute <u>alone</u> .
(2) Interaction effects are those effects which are due to the presence of a number of attributes.

Question (b) concerns whether one can employ a symmetric or asymmetric fractional design(1). Although it is easier to obtain symmetric designs from available sources, a catalogue produced by Hahn and Shapiro (1966) covers a very wide range of both types of designs and should ordinarily suffice.

Question (c) concerns the possible use of designs that present less than all attributes at a time. In particular, one type of design in which attributes are varied two-at-a-time (called trade-off analysis) has been frequently employed (for example, see Eberts and Koeppel 1977).

Only Questions (a) and (b) are considered truly relevant to the design of a Functional Measurement study. While it is impossible to provide a general rule for the selection of fractional designs a guide to selection of plans for a specific problem is provided by Hahn and Shapiro (1966).

Admittedly, some information is lost by employing a fractional factorial design. Such information loss means that some interaction effects in the general multilinear form (equation (3.8)) cannot be revealed (or estimated) by the data(2). The number of interaction effects that cannot be estimated is dependent upon the size of the design. Therefore it can be decided *a priori* by the analyst which effects can be assumed to be negligible.

Nevertheless, the information loss by employing a fractional factorial design is not a serious restriction. For example, if the 'true' specification of the general multilinear form is multiplicative (equation 3.9b), using a fractional factorial design that eliminates the joint effects will mean that the actual estimated form will be additive (equation 3.9a). This need not be a problem, because the linear function provides an estimate of the true marginal value function, provided lower-order interactions are negligible (Lerman and Louviere 1978). The linear additive specification will almost always reproduce the correct rank order of observed data, even if the true specification is multiplicative.

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(1) Symmetric designs are those in which all attributes have the same number of levels, asymmetric designs are those in which at least one attribute has a different number of levels compared with the others.

(2) It is usually the higher order interaction effects that cannot be estimated.

Prediction of rank order is stressed because this is all that is necessary to predict order of choice. Provided an individual will most frequently select the alternative with the highest predicted value, then this prediction can come from knowledge of rank order alone.

Thus, if one can estimate a linear value equation for each individual in a sample as an approximation to predicting how the values of their responses will vary over bundles of attributes, one can forecast choice.

ORTHOGONALITY

It is frequently desirable to employ designs which provide that all main effects and two-way interactions can be estimated independently of one another and all other interaction effects. To illustrate this idea, consider a 3 x 3 x 3 full factorial sampling plan. The levels are labelled 1, 2 and 3.

The full design is given in Table 4.2 for hypothetical attributes A, B and C. To fractionate this design so that one can infer the main effects of A, B and C, one needs to know which terms or effects are correlated with which others. For example, if one wants to estimate the main effects independently of one another, it is obviously desirable that all main effects be uncorrelated with each other. So, one would want to choose a fractional sampling plan that guaranteed this statistical independence. Likewise, if one suspected that there would be significant interaction effects, one would want to try to minimise correlations between all relevant variables, both linear and joint terms.

As a rule of thumb, less and less variation is accounted for by interactions after main effects have been accounted for, even if the interactions are statistically significant. It is usually the case, in fact, that two-way (eg, A x B) interactions account for less variance than main effects, but more than three-way interactions (eg, A x B x C). Hence, one usually wants to collapse the factorial design across as many interaction effects that are three-way or higher as possible. In fact, one tries to minimise correlations with two-way effects because these correlations could be large and hence affect both the estimation and interpretation of results.

To illustrate how one selects such a fraction, it is instructive to turn to a multiple linear regression format. For the design in Table 4.2 the following regression equation may be specified.

$$
\begin{aligned}
V_j = {} & \beta_0+\beta_1A_i+\beta_2B_i+\beta_3C_i+\beta_4A_i^2+\beta_5B_i^2+\beta_6C_i^2 \\
& +\beta_7A_iB_i+\beta_8A_iC_i+\beta_9B_iC_i \\
& +\beta_{10}A_i^2B_i+\beta_{11}A_i^2C_i+\beta_{12}A_iB_i^2+\beta_{13}B_i^2C_i \\
& +\beta_{14}B_iC_i^2+\beta_{15}A_iC_i^2 \\
& +\beta_{16}A_i^2B_i^2+\beta_{17}A_i^2C_i^2+\beta_{18}B_i^2C_i^2 \\
& +\beta_{19}A_iB_iC_i+\beta_{20}A_i^2B_iC_i+\beta_{21}A_iB_i^2C_i+\beta_{22}A_iB_iC_i^2 \\
& +\beta_{23}A_i^2B_i^2C_i+\beta_{24}A_i^2B_iC_i^2+\beta_{25}A_iB_i^2C_i^2 \\
& +\beta_{26}A_i^2B_i^2C_i^2+\varepsilon_i,
\end{aligned}
\tag{4.1}
$$

where β_0 is a constant,
β_1 to β_6 are the main effects of the relevant attributes,
β_7 to β_{18} are the two-way interaction effects,
β_{19} to β_{26} represent the three-way interaction effects,
ε_i is an error term,
and A, B and C are the three attributes in Table 4.2.

It is desirable that the fractional factorial design chosen minimises the correlation amongst the terms associated with the coefficients β_1 through to β_{18}. This will permit the estimation of those coefficients. The remaining coefficients however can not be estimated with the same fractional factorial design.

In order to obtain independent estimates of the main effects and two-way interaction effects, one needs to specify A and A^2, B and B^2, C and C^2 in such a way that they are uncorrelated or at least minimally correlated. In this case a factorial or fractional factorial data collection plan is a necessary but not sufficient condition to guarantee this independence. This data collection plan is termed 'design orthogonal' to distinguish it from the corresponding estimation problem, although the latter always follows directly from the former. Unless one can transform A, B and C into separate uncorrelated linear and quadratic terms, A and A^2 will be correlated, as will B and B^2 and C and C^2.

The creation of separate orthogonal terms of interest will be termed 'estimation orthogonal' as it refers to estimation properties. The creation of 'estimation orthogonality' is accomplished by means of a transformation procedure termed the 'method of orthogonal polynomial transformations'. This method is used in research tasks to ensure independence of main linear and quadratic effects and corresponding interaction effects in analysis. An outline of the method of orthogonal polynomials is given in Appendix 2.

TABLE 4.2 - FULL FACTORIAL CODING EXAMPLE

A	B	C	A	B	C	A	B	C
1	1	1	2	1	1	3	1	1
1	1	2	2	1	2	3	1	2
1	1	3	2	1	3	3	1	3
1	2	1	2	2	1	3	2	1
1	2	2	2	2	2	3	2	2
1	2	3	2	2	3	3	2	3
1	3	1	2	3	1	3	3	1
1	3	2	2	3	2	3	3	2
1	3	3	2	3	3	3	3	3

CHAPTER 5 - A PROPOSED INTERNATIONAL AIR TRAVEL DEMAND STUDY DESIGN

INTRODUCTION

In the field of international air travel, there are a very large number of possible influences on travellers' choices that are of potential interest. It is therefore necessary to carefully specify the variables of interest. For example:

- there are destinations, which have unique effects,

- there are ticket types, which have unique and joint effects, and these effects may be different for different destinations, and

- there are travel and tour package options which may have unique effects as well as effects that differ by destination.

It is recognised that the 'effect' of these variables is related to the availability of various combinations of destinations and ticket types. Thus it is important to vary the availability of various fare packages to various destinations. For example, it is realistic to assume that current first class and economy class fares will continue to be available to all destinations and that only some combinations of discount fare packages and tour options will be available to only some destinations. However, it is the joint effect of availability of packages to destinations that enters into the decision making process as well as the components of the packages.

In order to obtain as complete an information base as possible regarding each of these influences a study design should consist of four sections.

(1) A series of situations in which destination is held constant and ticket types are varied (eg, by asking travellers what changes they would make in their journey in response to the availability of other types of tickets). That is, different sets of conditions and discounts are made available in different combinations and choice behaviour is assessed.

(2) A series of situations in which the availability of destinations is varied subject to current first class and economy class fares or these fares plus an 'acceptable' discount fare. The respondent is asked to choose among available destinations. This provides an assessment of the joint effects of destinations, holding conditions of tickets constant.

(3) A series of situations in which destination availability and ticket type availability are simultaneously varied and respondents are asked to choose among available tickets and destinations. This final design permits one to estimate marginal, joint and conditional effects and combinations of these. However, in practice not all combinations of effects can be estimated(1).

(4) A series of personal, demographic and historical background questions in order to assist in segmenting traveller responses and choices into groups who potentially will respond differently to alternative travel options.

An extensive series of pilot interviews will usually be required to determine how much of these four sections respondents could be expected to complete. Previous research experience suggests that the following are reasonable procedures.

(A) Arrange the contents of the sections so that each individual answers a subset of the total target situations of interest in each section, thereby reducing the overall data requirements but still obtaining sufficient information on each section from *all* individuals.

(B) Divide individuals into three groups and assign them on an equal probability basis to the following sections:
- sections (1), (4);
- sections (2), (4);
- sections (3), (4).

(1) As alluded to in Chapter 4, the use of fractional factorial designs result in a loss of information required to estimate some of the interaction effects.

(C) Divide individuals into two groups and assign them on an equal probability basis to the following sections:
   - sections (1), (3), (4);
   - sections (2), (3), (4).

The logic underlying the above procedures is that section (4) (socio-demographic data) is required for market segmentation while section (3) provides the most information and permits estimation of marginal, joint and conditional effects of the various combinations of destinations and ticket conditions. However, in view of the possible complexity of section (3), it is advisable to augment it with either section (1) or section (2). The two procedures (B and C) greatly reduce the requirements imposed on any one individual; of procedures B and C, C is more desirable if A is not feasible. Thus, procedure B should be employed only if all else proves infeasible because it supplies the least information.

POTENTIAL STUDY DESIGNS

Three potential study designs appear feasible at this point in time.

Design I: Hold destination constant and vary ticket conditions

The design logic for this plan is exhibited in Table 5.1 containing the codes for the levels (values) of each of the attributes. The levels are arbitrarily assigned code values 0 and 1 to identify them in a sampling plan drawn from a full factorial design. The full factorial consists of 2^7 or 128 total combinations. One quarter of the total or 32 combinations have been selected which have the following properties:

- every column is orthogonal to every other column;
- every main effect is independent of second order (or two-way) interaction terms,
- there are 15 interactions of linear x linear terms (out of a possible 21) which can be estimated in addition to all main effects.

- it is assumed that all other interactions are zero or small relative to these interactions and the latter interactions are larger than the measurement errors introduced in the assignment plan.

In order to examine as wide as possible a range of the independent variables and to gain as much advantage as possible from the properties of the design in Table 5.1, the procedure is as follows.

- Arbitrarily divide the range of each experimental variable into 'high' and 'low'. In most instances these are fairly obvious divisions. However, some variables such as 'departure certainty' have no such division and must be retained as dummy variables.

- Sample from the 'highs' and 'lows' in a random manner so that a range of values on each variable is present under 'high' and a second range under 'low'. Create several different questionnaire sets with different random samplings, if possible, to finely sample the ranges of interest.

- Create at least one condition, if possible, in which only two levels at the extremes are employed to permit a good test for interactions.

The design plan presented in Table 5.1 has the properties outlined above and can be used to estimate non-linear main effects as well. The minimum number of degrees of freedom required to estimate the effects of all the linear and quadratic terms in this design is 14, so the 32 treatment combinations should be more than adequate to estimate these plus the 15 remaining linear x linear interaction terms. It is doubtful that quadratic interaction effects could be detected or estimated with this design plan. Hence, this represents a fairly powerful approach to depicting statistically most of the major effects.

Design II: Hold ticket types constant, vary destination availability

The design logic for this aspect of the study is given in Table 5.2. The basic intent is to assess substitutability in the choice of alternative destinations in order to assist in interpreting the final results.

TABLE 5.1 - VARIATION IN TICKET CONDITIONS, HOLDING DESTINATION CONSTANT

Ticket Package Number	Discount off Full Economy Fare	Cancellation penalty (% of your fare you lose if you cancel)	How many days ahead you must purchase the ticket	No of stopovers permitted en route	Min no of days you must stay at your destination	Can you select the exact day you depart or the exact week?	Is a tour package available and if so how much is it in $ per person per day(a)
1	0	0	0	0	0	0	0
2	0	0	0	0	1	1	1L
3	0	0	0	1	0	1	1H
4	0	0	0	1	1	0	0
5	0	0	1	0	0	1	1H
6	0	0	1	0	1	0	0
7	0	0	1	1	0	0	0
8	0	0	1	1	1	1	1L
9	0	1	0	0	0	1	0
10	0	1	0	0	1	0	1H
11	0	1	0	1	0	0	1L
12	0	1	0	1	1	1	0
13	0	1	1	0	0	0	1L
14	0	1	1	0	1	1	0
15	0	1	1	1	0	1	0
16	0	1	1	1	1	0	1H
17	1	0	0	0	0	1	0
18	1	0	0	0	1	0	1H
19	1	0	0	1	0	0	1L
20	1	0	0	1	1	1	0
21	1	0	1	0	0	0	1L
22	1	0	1	0	1	1	0
23	1	0	1	1	0	1	0
24	1	0	1	1	1	0	1H
25	1	1	0	0	0	0	0

TABLE 5.1 - VARIATION IN TICKET CONDITIONS, HOLDING DESTINATION CONSTANT (Cont'd)

26	1	1	0	0	1	1	1L
27	1	1	0	1	0	1	1H
28	1	1	0	1	1	0	0
29	1	1	1	0	0	1	1H
30	1	1	1	0	1	0	0
31	1	1	1	1	0	0	0
32	1	1	1	1	1	1	1L

(a) The H and L values refer to 'High' and 'Low' and are nested under code 1 = 'Yes', a tour package is available, H and L refer to costs.

NOTES: 0 = Low values (or levels) or one of 2 levels.
1 = High values (or levels) or one of 2 levels.

Levels of each variable are arbitrarily divided into 'high' and 'low' and are randomly assigned to the 32 rows depending upon the code sequence in each particular column. For example, discount may be above 50% (High) or below 50% (Low); levels for '0' are selected at random from values below 50%; levels for '1' are likewise selected for values above 50%.

Levels suggested for use

Discount off full economy fare:

'0' = 20-49%; '1' = 51-80%.

Cancellation penalty:

'0' = 0-25%; '1' = 26-50%.

Advance purchase requirement:

'0' = 0, 7, 14, 21, 28, 35 or 42 days; '1' = 45, 60, 75, 90, 120, 150 or 180 days.

Stopovers:

'0' = none; '1' = some.

Minimum stay:

'0' = none, 3, 5, 7 or 10 days; '1' = 14, 18, 21, 28 or 30 days.

Departure certainty:

'0' = you decide the exact day; '1' = you decide the week.

Tour package:

'0' = none; '1L' = \$20-\$50/person/day; '1H' = \$51-\$100/person/day.

TABLE 5.2 - VARIATION IN DESTINATION AVAILABILITY HOLDING TICKET RESTRICTIONS CONSTANT

Choice set no	New Zealand	Fiji	Singapore	Japan	The Netherlands	Thailand	UK	Germany	Italy	US	Canada	Hong Kong	I would choose not to travel overseas
1	0	0	0	0	0	0	0	0	0	0	0	0	
2	0	0	0	1	0	1	1	0	0	1	1	1	
3	0	0	1	0	1	1	1	0	1	0	1	0	
4	0	0	1	1	1	0	0	0	1	1	0	1	
5	0	1	0	0	0	1	0	1	1	1	1	0	
6	0	1	0	1	0	0	1	1	1	0	0	1	
7	0	1	1	0	1	0	1	1	0	1	0	0	
8	0	1	1	1	1	1	0	1	0	0	1	1	
9	1	0	0	0	1	1	1	1	1	1	0	1	
10	1	0	0	1	1	0	0	1	1	0	1	0	
11	1	0	1	0	0	0	0	1	0	1	1	1	
12	1	0	1	1	0	1	1	1	0	0	0	0	
13	1	1	0	0	1	0	1	0	0	0	1	1	
14	1	1	0	1	1	1	0	0	0	1	0	0	
15	1	1	1	0	0	1	0	0	1	0	0	1	
16	1	1	1	1	0	0	1	0	1	1	1	0	

NOTES: 0 means 'Low' % discount ('Low' = 20% or 30%).
1 indicates 'High' % discount ('High' = 50% or 60%).

Respondents are requested to indicate which destinations they would select given various levels of discounts from economy class fares and assuming that restrictions on these discounted fares were 'acceptable'. Thus, the only condition being imposed is a personal budget constraint.

Design III: Simultaneous variation in both tickets and destinations

The layout given in Table 5.3 represents one possible ticket type/destination choice design. Such a design permits an analysis of the joint ticket type and destination choice by individuals. This is achieved by varying the availability of certain ticket types to different destinations between different choice sets.

It is of interest to estimate how many trips would be made in the next, say, x years to each destination given a particular situation, or to estimate the likelihood of any person making a single choice. The layout in Table 5.3 permits the estimation of changes in travel plans as a function of alternatives available to the traveller. Thus, it should be possible to predict the travel choices of an individual or the sample given any set of alternatives. An alternative is defined as a particular set of destinations and associated fare packages which are available for travel. It is proposed that first class and economy class fares will always be available to all destinations and the aim is to estimate substitutions among destinations and ticket types away from these always available two alternatives.

MODELLING CONSIDERATIONS

The most general model specification for the data may be written as follows:

P_{qdt} = Mean Effect + Marginal Effects of Destinations +
Marginal Effects of Ticket Types +
Marginal Effects of Conditions within Tickets +
Marginal Effects of Socio-demographic Characteristics (Covariates) +
Joint Effects of Destinations and Tickets +
Joint Effects of Destinations and Conditions Within Tickets +
Joint Effects of Covariates and Destinations +
Joint Effects of Covariates and Tickets +

TABLE 5.3 - CHOICE SETS FOR EXAMPLE DESTINATION/TICKET CHOICES

Choice Set No.	Destinations for Leisure/Discretionary Travel D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	I would choose not to travel overseas
1	E,F	1,3,4,5,9 E,F	2,3,4,8 E,F	1,2,3,7,9 E,F	1,2,6,8,9 E,F	1,5,7,8 E,F	4,6,7,9 E,F	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	
2	1,2,4,5,6 E,F	E,F	1,3,4,5,9 E,F	2,3,4,8 E,F	1,2,3,7,9 E,F	1,2,6,8,9 E,F	1,5,7,8 E,F	4,6,7,9 E,F	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	
3	2,3,5,6,7 E,F	1,2,4,5,6 E,F	E,F	1,3,4,5,9 E,F	2,3,4,8 E,F	1,2,3,7,9 E,F	1,2,6,8,9 E,F	1,5,7,8 E,F	4,6,7,9 E,F	3,5,6,8,9 E,F	
4	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	1,2,4,5,6 E,F	E,F	1,3,4,5,9 E,F	2,3,4,8 E,F	1,2,3,7,9 E,F	1,2,6,8,9 E,F	1,5,7,8 E,F	4,6,7,9 E,F	
5	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	1,2,4,5,6 E,F	E,F	1,3,4,5,9 E,F	2,3,4,8 E,F	1,2,3,7,9 E,F	1,2,6,8,9 E,F	1,5,7,8 E,F	
6	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	1,2,4,5,6 E,F	E,F	1,3,4,5,9 E,F	2,3,4,8 E,F	1,2,3,7,9 E,F	1,2,6,8,9 E,F	
7	4,6,7,9 E,F	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	1,2,4,5,6 E,F	E,F	1,3,4,5,9 E,F	2,3,4,8 E,F	1,2,3,7,9 E,F	
8	1,5,7,8 E,F	4,6,7,9 E,F	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	1,2,4,5,6 E,F	E,F	1,3,4,5,9 E,F	2,3,4,8 E,F	
9	1,2,6,8,9 E,F	1,5,7,8 E,F	4,6,7,9 E,F	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	1,2,4,5,6 E,F	E,F	1,3,4,5,9 E,F	
10	1,2,3,7,9 E,F	1,2,6,8,9 E,F	1,5,7,8 E,F	4,6,7,9 E,F	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	1,2,4,5,6 E,F	E,F	
11	2,3,4,8 E,F	1,2,3,7,9 E,F	1,2,6,8,9 E,F	1,5,7,8 E,F	4,6,7,9 E,F	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	1,2,4,5,6 E,F	
12	1,3,4,5,9 E,F	2,3,4,8 E,F	1,2,3,7,9 E,F	1,2,6,8,9 E,F	1,5,7,8 E,F	4,6,7,9 E,F	3,5,6,8,9 E,F	2,4,5,7,8,9 E,F	1,3,4,6,7,8 E,F	2,3,5,6,7 E,F	

Explanation of Table

(a) D1-D10 are ten (10) travel destinations of interest (eg, Fiji, New Zealand,....,France).
(b) Numbers 1-9 refer to nine (9) tickets of interest listed below. E and F respectively refer to Economy and First Class tickets.
(c) A choice set is represented by a row of the table, eg, Row one (Choice Set 1) consists of only E and F tickets to D1, tickets 1,3,4,5,9,E and F to D2,....,and tickets 1,3,4,6,7,8,E and F to D10, as well as the 'not travel' option.
(d) An individual's task is to choose one and only one destination/ticket combination from each row, eg, in Row one there are 65 such combinations. Only one from the 65 can be picked by each individual. Each individual makes 12 choices - one from each row.

TABLE 5.3 - CHOICE SETS FOR EXAMPLE DESTINATION/TICKET CHOICES (Cont'd)

	Ticket Types								
	1	2	3	4	5	6	7	8	9
Advance Purchase (days ahead)	30	30	30	30	90	90	90	90	60
Departure Certainty (WK = within 1 week; YD = you decide)	WK	WK	YD	YD	WK	WK	YD	YD	YD
Stopovers	no	no	yes	yes	yes	yes	no	no	no
Minimum Stay (days)	21	7	21	7	21	7	21	7	14
Cancellation Penalty (%)	25	75	25	75	75	25	75	25	50
Discount (% off full economy fare)	30	70	70	30	70	30	30	70	50

Joint Effects of Covariates and Conditions Within Tickets +

Joint Effects of Covariates, Destinations and Tickets +

Joint Effects of Covariates, Destinations and Conditions Within Tickets +

Joint Effects of Covariates. (5.1)

where P_{qdt} is the probability of an individual q choosing a particular destination d, and ticket type t.

Because the dependent variable is discrete in all of the above study designs the data are amenable to analysis by means of available multinomial logit (MNL) procedures which provide discrete analogs to generalised regression/analysis of variance procedures (covariance analysis). A novel use of a covariance type of analysis is also proposed to make use of the data on personal traveller background and current characteristics (age, income, occupation, previous destination choice(s), etc). The traveller characteristics data can be used to determine market segments in a very general manner and these characteristics can be inserted in the model as explanatory variables. This is a preferable procedure to the usual practice of running ad hoc tests on various 'groupings' or segment configurations.

ASSESSING VALIDITY

As mentioned previously, a major weakness of the Functional Measurement approach is that one does not observe actual behaviour. The direct link between choices in an experimental environment and in a real life situation is not known *a priori*.

In order to test the validity of the estimated models, it is proposed that data be collected in the traveller characteristics section of the survey which will provide information on past travel experience, say over the last five years. Such information may be used to effect comparisons with simulated choice behaviour.

CHAPTER 6 - CONCLUDING REMARKS

In 1978 the Bureau of Transport Economics released Occasional Paper 11 on 'Factors Affecting Demand for International Travel to and from Australia'. This study was the first attempt by the Bureau to make use of attitudinal data on air travel choices for policy analysis. Since then the BTE has attempted to develop a more systematic framework for the development and analysis of such studies. This paper outlines such a framework and draws heavily from the consultants report submitted to the BTE by Dr J.J. Louviere and Dr D.A. Hensher.

This paper provides a framework for an empirical travel choice model which is sensitive to relevant policy options. The aim was to outline a method which is capable of identifying international air fare structures to various destinations that accord with present and likely future choices of individuals. The Functional Measurement approach adopted develops experimentally designed choice situations with desirable statistical properties which assist in evaluating the induced responses of individuals. Such experimentally induced responses allow one to develop estimates of actual and potential choices in response to a range of options which includes those currently available but extends to options which may become available in the future. Chapter 4 outlines a technique for constructing questionnaires based on choice contexts in general, whereas Chapter 5 specifically deals with an application of this approach to an International Air Travel Demand Study.

The emphasis in this paper has been on the development of a suitable conceptual and operational framework within which a travel choice model could be estimated. No attempt has been made to present the method of analysis for the model estimation.

The Bureau is at present conducting an empirical study in the area of international air travel based on the survey designs presented in this paper. The Bureau plans to report the method of analysis and results of this study in a separate future publication.

APPENDIX 1 - IDENTIFICATION OF THE FUNCTIONAL FORM OF THE JOINT VALUE EQUATION

This appendix is concerned with the specification and estimation of the joint value equation (3.3) viz

$$V_j = d\ (v(x_{1j}),\ v(x_{2j}),\ldots,v(x_{Ij})) \qquad \text{(A1.1)}$$

where all terms are as previously defined in the text.

This equation provides information on the value individuals assign to various alternatives. If function (A1.1) can be specified and estimated then the estimated values of V_j's can be used to predict order of choice (assuming that individuals will choose the alternative which has the highest value/worth).

The most generally accepted specification of the joint value function (A1.1) are subsets of the general multilinear form given below for the case of 3 attributes:

$$V'_j = k_0 + k_1v(x_{1j}) + k_2v(x_{2j}) + k_3v(x_{3j}) + k_4v(x_{1j})v(x_{2j})$$
$$+ k_5v(x_{1j})v(x_{3j}) + k_6v(x_{2j})v(x_{3j}) + k_7v(x_{1j})v(x_{2j})v(x_{3j}) \qquad \text{(A1.2)}$$

where V'_j and $v(x_{ij})$ are as previously defined in the text,
k_1 to k_3 are the main effects of the attributes 1, 2 and 3 respectively
and k_4 to k_7 are the effects of the interaction of the attributes 1, 2 and 3.

Two important subsets of the general linear form (A1.2) are particularly considered here.

First, the additive form (A1.3) implies that all the interaction effects are zero. That is

$$V'_j = k_0 + k_1v(x_{1j}) + k_2v(x_{2j}) + k_3v(x_{3j}). \qquad \text{(A1.3)}$$

Second, the multiplicative form is given as

$$V'_j = k_0 + k_7v(x_{1j})v(x_{2j})v(x_{3j}). \qquad \text{(A1.4)}$$

In order to identify which of these two forms or any other subset of equation (A1.2) is the appropriate specification, it is necessary to estimate the parameters $k1$ to $k7$ and test for their statistical significance. This can be achieved by employing a linear estimation technique such as analysis of variance or ordinary least squares regression method. This involves the introduction of an error term into the equation (A1.2).

However, prior to estimating equation (A1.2), it is necessary to observe or measure the dependent variable V_j and the explanatory variables $v(x_{ij})$s. This is achieved through the use of (fractional) factorial designs which are discussed in Chapter 4 of this paper. Such designs allow data to be collected on the values/worths that individuals assign to combinations of various levels of each of the attributes under consideration. Individuals are asked to rank these combinations on some numerical scale (say for example 0 to 100). The rank given to each combination which defines a particular alternative becomes a proxy for the value/worth, V, of that alternative. Furthermore, the marginal values $v(x_{ij})$s can be approximated by the marginal means(1) of the (fractional) factorial designs. This is proved below.

For ease of exposition consider the general multilinear joint value equation for the case of two attributes(2):

$$V_{i\ell} = k0 + k1v(x1_i) + k2v(x2_\ell) + k3v(x1_i)v(x2_\ell) + \varepsilon_{i\ell} \qquad (A1.5)$$

where

$V_{i\ell}$ represents the value or rank assigned by an individual to the particular alternative where attribute 1 is at level i and attribute 2 is measured at level ℓ

$v(x1_i)$, $v(x2_\ell)$ are the marginal values of attributes 1 and 2 measured at levels i and ℓ respectively,

and $\varepsilon_{i\ell}$ is an error term for a particular alternative.

---

(1) The marginal mean is the average of the assigned ranks where one specific attribute is held constant.
(2) The subscript j representing the alternative under consideration has been replaced for ease of exposition by the levels i and ℓ of the attributes 1 and 2.

Averaging over the subscript ℓ yields the following

$$V_{i.} = k_0 + k_1 v(x1_i) + k_2 \left[\sum_{\ell}^{L} v(x2_\ell)/L \right] +$$

$$k_3 v(x1_i) \sum_{\ell}^{L} \left[v(x2_\ell)/L \right] + \sum_{\ell}^{L} \varepsilon_{i\ell}/L \qquad \text{(A1.6)}$$

where $v(x_{mn})$ are as described above,

$V_{i.}$ is the average or marginal mean of the joint value function over all levels of attribute 2,

L represents the total number of levels of attribute 2.

Equation (A1.6) can be expressed as

$$V_{i.} = K_0 + K_1 v(x1_i) + \varepsilon_{i.} \qquad \text{(A1.7)}$$

where $K_0 = k_0 + k_2 \left[\sum_{\ell}^{L} v(x2_\ell)/L \right]$ and

$K_1 = k_1 + k_3 \left[\sum_{\ell}^{L} v(x2_\ell)/L \right]$ are the collected terms and $\varepsilon_{i.}$ is the average error over the L levels of attribute 2.

Similarly,

$$V_{.\ell} = K_2 + K_3 v(x2_\ell) + \varepsilon_{.\ell} \qquad \text{(A1.8)}$$

where the terms K_2, K_3 are analogous to K_0 and K_1,

$V_{.\ell}$ is the marginal mean for level ℓ of attribute 2 averaged over all levels of attribute 1 and similarly

$\varepsilon_{.\ell}$ is the average error term.

This demonstrates that the marginal means $V_{i.}$ and $V_{.\ell}$ are approximately equal to the marginal values $v(x1_i)$ and $v(x2_\ell)$ up to a linear transformation. An important underlying assumption of this conclusion is however, that the data was generated by the general multilinear joint value equation or any subset of that form.

Thus the ranks assigned by individuals and the marginal means can be employed as proxies for the joint value and marginal values in the general multilinear value function (A1.2). Furthermore, the marginal means must be transformed to orthogonal form to take into account the dependencies between the explanatory variables as explained in Appendix 2.

To sum up, available general linear modelling procedures such as ordinary least squares or analysis of variance can be used to estimate the parameters of the joint value equation. Statistical tests which indicate whether the parameters are significantly different from zero, can then be applied. In this way, the functional form of the joint value equation may be identified and information can be obtained on the way attribute values contribute to the joint value function.

Alternatively, a more simplistic graphical approach may be undertaken to gain insight into the functional form of the joint value function. This approach involves an examination of a graphical representation of the interactions between attributes. As an example of this approach assume that individuals combine the attribute values linearly, (ie, the linear model (A1.3) is the correct specification of the joint value equation).

When the assigned value or rank $V_{i\ell}$ is plotted against the levels of either of the attributes 1 or 2 the resultant plot should be a series of parallel lines.

This can be proved as follows: assuming the additive form is correct, consider the effect of subtracting the value judgments ascribed by individuals to level 1 of attribute 1 with a specific level of attribute 2, from the value judgments for level 2 of attribute 1 with the same level of attribute 2. This yields

$$\begin{aligned} V_{2\ell} - V_{1\ell} &= (k_0 + k_1 v(x_{1_2}) + k_2 v(x_{2_\ell}) + \varepsilon_{2\ell}) - \\ &\quad (k_0 + k_1 v(x_{1_1}) + k_2 v(x_{2_\ell}) + \varepsilon_{1\ell}) \\ &= k_1(v(x_{1_2}) - v(x_{1_1})) + (\varepsilon_{2\ell} - \varepsilon_{1\ell}) \end{aligned} \tag{A1.9}$$

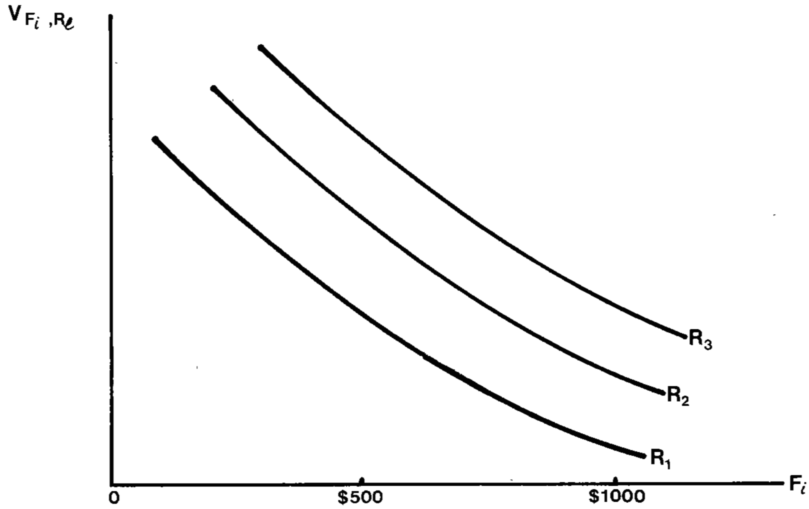

Figure A1.1 : Additive joint value function

Note
F_i = ith level of fare
R_ℓ = ℓth level of ticket restrictions

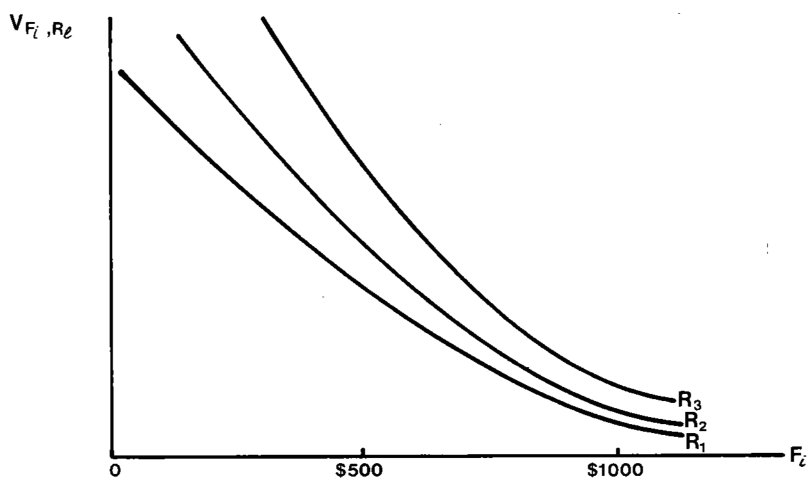

Figure A1.2 : Multiplicative joint value function

Equation (A1.9) indicates that the difference between the joint values when attribute 2 takes on any value is always the same (that is, $k_1(v(x1_2) - v(x1_1))$ ie, a constant, except for randomness. Hence, a graph should yield a series of parallel lines.

As a simple illustration (in reference to international air travel), consider the joint values assigned to combinations of fare levels and levels of ticket restrictions. If the additive specification is correct (ie, all interaction effects are not significant), then, as is demonstrated in Figure A1.1, the difference between levels of ticket restrictions (ie, R_1, R_2 and R_3 where R_3 is the least restricted) is constant over all levels of fare.

Figure A1.1 illustrates the concept of 'independence' in that the response to one (or more) attribute(s) in combination with one (or more) other attribute(s) is entirely independent of the levels of such attributes.

In contrast, if the multiplicative specification is appropriate ($V_{i\ell} = k_0 + k_3 v(x1_i)v(x2_\ell)$), then a graphical plot would indicate a convergence of the levels of the ticket restriction attribute over the levels of the fare attribute. In Figure A1.2 the difference between each R_ℓ curve is inversely proportional to the levels of fare. That is, as fare increases, the distance between the curves decreases. There is an intuitive way of explaining this preference structure. If the fare is very expensive (very low preference), the levels of restrictions make little, if any, difference, however, as levels of fare decrease, differences in restrictions begin to matter.

So, to reiterate, Figure A1.1 would indicate that the level of fare has no bearing on the change in the joint value (V_{F_i}, R_ℓ) when restrictions are eased, that is, the change is constant. On the other hand, Figure A1.2 would indicate that as the level of fare increases, the change in the joint value associated with the easing of restrictions diminishes.

The convergence as indicated in Figure A1.2, when the multiplicative model is assumed to be correct can be mathematically demonstrated as follows:

$$
\begin{aligned}
V_{2\ell} - V_{1\ell} &= (k_0 + k_3 v(x_{12}) v(x_{2\ell}) + \varepsilon_{2\ell}) - \\
&\quad (k_0 + k_3 v(x_{11}) v(x_{2\ell}) + \varepsilon_{1\ell}) \\
&= k_3 v(x_{2\ell}) \left[v(x_{12}) - v(x_{11}) \right] + (\varepsilon_{2\ell} - \varepsilon_{1\ell})
\end{aligned}
\tag{A1.10}
$$

Equation (A1.10) indicates that the difference between the value judgments at levels 2 and 1 of attribute 1 over any level of attribute 2 are proportional to the marginal value of the level of attribute 2. As the marginal value of attribute 2 decreases, the distance between the joint values decreases. This corresponds to the illustration in Figure A1.2. (Intuitively, as fare increases the marginal value of fare would decrease.)

The two approaches discussed above provide methods of identifying the joint value equation (A1.1). They both rely on the following two important assumptions.

First, the joint value equation can be specified as some subset of the general multilinear model expressed for 3 attributes in equation (A1.2).

Second, the marginal means of a factorial design can be used as proxies for the marginal values of the attributes. This is based upon the proof given above which implies that the marginal values are a linear transformation of the marginal means (equation (A1.7)). This is true for both the additive and multiplicative models.

To reiterate, the data required for this procedure must be collected on the values individuals assign to various combinations of attributes (ie, alternatives) by asking respondents to rank each alternative on some numerical scale. These lead to the identification of the joint value function which can then be used to provide information on the relative importance of each attribute and the way they combine to the joint value of each alternative, ie, additivity or in a multiplicative way.

APPENDIX 2 - PURPOSE AND METHOD OF CONSTRUCTION OF ORTHOGONAL POLYNOMIALS

The purpose of 'estimation orthogonality' is to permit inferences to be made regarding higher order terms in an expanded polynomial equation as well as to make inferences regarding the significance of cross-product (interaction) terms. That is, it is not generally possible to test $x1$, $x2$, $x1^2$, $x2^2$, $x1.x2$, $x1^2.x2$, $x1.x2^2$, $x1^2.x2^2$, each as untransformed separate terms in a model which includes all of these effects because of the collinearity problems which result. Although it is frequently alleged that collinearity is a 'sample size' problem, this is at best misleading. Rather, collinearity should properly be viewed as a problem confounding of effects, such that one cannot really say whether one is estimating the separate effect of $x1$ or some other term that might be highly correlated with $x1$. Hence, for various reasons, not the least of which is interpretability, collinearity should be reduced to as minimal a level as possible. The method of orthogonal polynomials accomplishes this in a straightforward manner, but at the sacrifice of 'direct' interpretation in the original units of measurement. However, as we shall note, it is always possible to return to the original units and to interpret some of the effects directly, as well.

In the situation in which one were to have a balanced factorial sampling plan for obtaining observations, the method of orthogonal polynomials guarantees that <u>all</u> possible testable effects will be orthogonal to one another and therefore their effects will be independent and amenable to tests of significance. The problem arises, therefore, in non-balanced data collection schemes such as the usual revealed behaviour data obtained in detailed travel or trip surveys. That is, it is usually the case that travel times and costs are correlated in real data and there is not a balanced sampling of high costs and high times, high costs and low times, low costs and high times, and low costs and low times, with equal numbers of observations in each of these combinations. It would be desirable to be able to estimate a non-linear marginal effect for both time and cost, independently of the linear effect of time and cost, as well as any interactions between the two factors, as they may affect the dependent variable. While it cannot remove the collinearity between the linear components of time and cost, the method of orthogonal polynomials can remove <u>all</u> correlation between time and time squared and between cost and cost squared and reduce the correlations between the various cross-products and those other effects, as well. In particular, the method of orthogonal polynomials has the property that it provides an exact test for the highest-order effect. Thus, if a squared term is appropriate in the model, this method will find it with a high degree of confidence.

The particular approach that is adopted is patterned after an article by Robson (1959). As Robson demonstrates, a least-squares regression equation of the form:

$$Y_t = b^*_0 + b^*_1 x_t + b^*_2 x^2_t + \ldots + b^*_r x^r_t \qquad t = 1,\ldots,n > r \qquad \text{(A2.1)}$$

where

Y is the function of a single x,

t is the observation number,

r is the degree of polynomial, and

n is the number of observations

may be expressed in the following form:

$$Y_t = b_0 f_0(x_t) + b_1 f_1(x_t) + \ldots + b_r f_r(x_t), \qquad t = 1,\ldots,n > r \qquad \text{(A2.2)}$$

where $f_i(x_t)$ is a polynomial of degree i in x_t and where $f_0, f_1 \ldots f_r$ are normal orthogonal functions, ie, where

$$\sum_{t=1}^{n} f_i(x_t) f_{i'}(x_t) = \begin{cases} 0 & \text{if } i \neq i' \\ 1 & \text{if } i = i' \end{cases} \qquad \text{(A2.3)}$$

The construction of the functions $f_i(x_t)$ is accomplished by application of the following formulae (for the linear and quadratic terms):

$$\underline{\text{Linear Term}} \qquad f_L(x_t) = (x_t - \bar{x}) \qquad \text{(A2.4)}$$

$$\underline{\text{Quadratic Term}} \qquad f_Q(x_t) = x_t^2 - 1/n(\Sigma C) - (x_t - \bar{x})(\Sigma A/\Sigma B) \qquad \text{(A2.5)}$$

where

$$\Sigma A = \sum_t x_t^2 (x_t - \bar{x})$$

$$\Sigma B = \sum_t (x_t - \bar{x})^2$$

$$\Sigma C = \sum_t x_t^2$$

Note that these formulae may also be applied to dummy variates, yielding a transformed dummy variate that has the values of $\pm$ the mean. When <u>all</u> variates are transformed to orthogonal form in this manner by centering about their means, the intercept estimates the zero order effect or the mean of Y. Approximately orthogonal estimators may be obtained by squaring the linear terms and centering about their mean. Indeed, if the values are evenly spaced, this method is exact. In a similar way, approximately orthogonal higher-order estimators may be formed by finding the appropriate products of linear and squared terms and centering about their mean.

As mentioned earlier, the Robson method assures that the quadratic terms are exact, but the linear terms must be adjusted to take account of equation (A2.5). This can be easily seen in the approximation suggested above in which the linear term is squared and centered, subtracting the mean of the quadratically transformed observations from each observation one obtains:

$$(x_t - \bar{x})^2 - \left[(x_t - \bar{x})/n \right] \qquad \text{(A2.6)}$$

There is clearly an unbiased x_t^2 term, but x_t also contains $-2\bar{x}(x_t)$ which is involved in the quadratic term. This algebra clearly shows that one gets a 'real' test on the highest order terms, but the lower order terms must be reconstructed.

For example, in Table 4.2, one can use the method of orthogonal polynomials to create all the terms of interest by replacing the codes in Table 4.2 in the text with the following:

a) For a linear effect, wherever the value '1' appears, replace it with '-1'; replace '2' with '0'; replace '3' with '1'.

b) For a quadratic effect, wherever the value '1' appears, create a value of a new vector equal to '1'; for the level '2', create a value of '-2'; for the level '3', create a '1'(1).

These new codes are given in Table A2.1. In Table A2.1, L stands for a linear effect, Q for a quadratic effect. Note that in each column the sum of the elements of the vector equals zero. The correlation between each pair of vectors is zero. The cross-product, A(L).B(L) is formed simply by multiplying each pair of elements within a row (observation); thus, the first observation under A(L) is -1, and under B(L) is -1; their cross-product is A(L) x B(L), or (-1) x (-1) = +1. All cross-products are formed in the same way.

Table A2.2 shows that there are six independent main effects which can be estimated; similarly, by expansion, one could find 20 interaction terms. Now consider the effect of taking a 1/3 fraction or 9 combinations from the total 27; an example of a 1/3 fraction is given in Table A2.2.

All of the terms shown in Table A2.2 are independent of each other; however, they are not independent of other terms not shown. For example, $AC = AB^2C$ and $AB = ABC^2$. Thus, one could estimate an effect or coefficient for AC or AB, but one would not be able to ascribe the coefficient to either effect. Note that in traditional market demand data there will rarely be observations over all possible combinations, nor will there be balance in the number of observations of each level of each factor. Thus, traditional market demand data constitutes a non-orthogonal (non-independent) sampling plan with some effects confounded in a manner which cannot be determined prior to data collection. On the other hand, factorial sampling plans permit one to know *a priori* what the confoundment structure is and what effects can be reliably estimated. This is important because in contrast to traditional analyses

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(1) This result is taken from standard orthogonal polynomial tables available in standard texts on experimental design. These values are not immediately obvious based on the previous algebra. Let us reconstruct these values: first calculate the mean of the levels (1,2,3) = 2; now subtract the mean from each level = (-1,0, and 1). Now square these new levels = (1,0,1) and calculate the mean of these squares = 2/3. Subtract this mean from the square levels (1/3, -2/3, 1/3); divide by the common denominator = (1,-2,1), and obtain the necessary result.

of market choice data, the exact analytical structure is known in advance. Furthermore, because all respondents face the same sampling structure, they all can be compared. Thus, the differences between respondents in the judgments can be due to real differences in characteristics of the respondents such as income or sex.

TABLE A2.1 - ORTHOGONAL CODING OF THE DESIGN OF TABLE 4.2

Main Effects						Some Example Interactions		
A(L)	A(Q)	B(L)	B(Q)	C(L)	C(Q)	A(L)B(L)	B(Q)C(L)	A(Q)B(Q)C(L)
-1	+1	-1	+1	-1	+1	+1	-1	-1
-1	+1	-1	+1	0	-2	+1	0	0
-1	+1	-1	+1	+1	+1	+1	+1	+1
-1	+1	0	-2	-1	+1	0	+2	+2
-1	+1	0	-2	0	-2	0	0	0
-1	+1	0	-2	+1	+1	0	-2	-2
-1	+1	+1	+1	-1	+1	-1	-1	-1
-1	+1	+1	+1	0	-2	-1	0	0
-1	+1	+1	+1	+1	+1	-1	+1	+1
0	-2	-1	+1	-1	+1	0	-1	-2
0	-2	-1	+1	0	-2	0	0	0
0	-2	-1	+1	+1	+1	0	+1	-2
0	-2	0	-2	-1	+1	0	+2	-4
0	-2	0	-2	0	-2	0	0	0
0	-2	0	-2	+1	+1	0	-2	+4
0	-2	+1	+1	-1	+1	0	-1	+2
0	-2	+1	+1	0	-2	0	0	0
0	-2	+1	+1	+1	+1	0	+1	-2
+1	+1	-1	+1	-1	+1	-1	-1	-1
+1	+1	-1	+1	0	-2	-1	0	0
+1	+1	-1	+1	+1	+1	-1	+1	+1
+1	+1	0	-2	-1	+1	0	+2	+2
+1	+1	0	-2	0	-2	0	0	0
+1	+1	0	-2	+1	+1	0	-2	-2
+1	+1	+1	+1	-1	+1	+1	-1	-1
+1	+1	+1	+1	0	-2	+1	0	0
+1	+1	+1	+1	+1	+1	+1	+1	+1

TABLE A2.2 - 1/3 FRACTIONAL FACTORIAL PLAN, POLYNOMIALLY CODED

Main Effects						Some Two-Way Interactions						
A(L)	A(Q)	B(L)	B(Q)	C(L)	C(Q)	AB	AB^2	A^2B	A^2B^2	AC	BC	ABC
-1	+1	-1	+1	-1	+1	+1	-1	-1	+1	+1	+1	-1
-1	+1	0	-2	+1	+1	0	+2	0	-2	-1	0	0
-1	+1	+1	+1	0	-2	-1	-1	+1	+1	0	0	0
0	-2	-1	+1	+1	+1	0	0	+2	-2	0	-1	0
0	-2	0	-2	0	-2	0	0	0	+4	0	0	0
0	-2	+1	+1	-1	+1	0	0	-2	-2	0	-1	0
+1	+1	-1	+1	0	-2	-1	+1	-1	+1	0	0	0
+1	+1	0	-2	-1	+1	0	-2	0	-2	-1	0	0
+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1

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