# **BTE Publication Summary**

## Railway Track Design: A Review of Current Practice

## **Occasional Paper**

This Paper reviews the current practice for the design of conventional railway track. Current track design practice has historically developed slowly and is predominantly based upon years of operating experience. Consequently many of the developed design expressions are at best semi empirical in nature. The scope of this report is confined to the design of the track structure above the level of the formation, i.e., rails, sleepers and ballast.





#### BUREAU OF TRANSPORT ECONOMICS

#### RAILWAY TRACK DESIGN A REVIEW OF CURRENT PRACTICE

N.F. Doyle BHP Melbourne Research Laboratories

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#### FOREWORD

In the design of conventional railway track the designer must select from various available procedures and criteria which influence the resultant final design. This report is a review of design methodology and examines in detail current track design practices. The report contains detailed information relating to the engineering design of rails, sleepers and ballast, and where available presents alternate design formulae and assumptions.

This report was prepared as a source of basic design information included in an interactive track design model now being developed by the author. This model, which is the subject of present research, has the initial objective of producing designs for track having minimum capital costs.

While not presenting any new design procedures, this report presents a comprehensive review of current knowledge and provides new insights into current practices. As such it should be a useful document for all those involved in railway track design and research.

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#### SUMMARY

This report reviews the current practice for the design of conventional railway track. Current track design practice has historically developed slowly and is predominantly based upon years of operating experience. Consequently many of the developed design expressions are at best semi-empirical in nature.

The scope of this report is confined to the design of the track structure above the level of the formation, i.e., rails, sleepers and ballast.

Conventional railway track can to some extent be regarded as a panel consisting of rails and sleepers floating on supporting ballast and subgrade layers. For any given traffic environment this panel is subjected to both dynamic and static loading conditions which are largely dependent upon the strength of track structure and the degree and quality of track maintenance.

The current practice for the design of railway track is based upon satisfying several criteria for the strength of the individual track components. These criteria, which include limits on rail stresses, sleeper stresses, the pressure between the sleeper and the ballast and the pressure on the subgrade, are extensively reviewed in the report.

Design concepts essential to the analysis of the track components (also reviewed in detail) include impact factor determination, beam on an elastic foundation analysis, the track modulus and the rail seat load analysis. The track component that most seriously limits the progression to higher design axle loads is the rail. Therefore, the report also includes a detailed review of rail head contact analysis and methods currently used to estimate the life of the rail.

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At present a standard of track maintenance is implied by the current track design procedure through the use of various factors of safety. These factors appear to be based upon the considered judgement of earlier railway researchers, and whilst the resultant track design is obviously safe no attempt is made to relate the design procedure to the expected track maintenance requirements. Consequently, for tracks that are to be maintained to a high standard, the current track design procedures do not allow design benefits via the reduction of the calculated stress levels in any of the individual track components. It is also apparent that the ballast and subgrade properties need to be examined and included in the current design procedure. Allowable limits of deformation can then be based upon the requirements of track geometry retention and therefore the standard of maintenance can be included in the design procedure.

#### CHAPTER 1 - INTRODUCTION

Despite over 100 years of operating experience, the design of railroad track usually depends to a large extent upon the engineering experience of the designer. Due to the problems of relating an applied track loading to a large number of observed track responses, the developed design expressions which are available are at best semi-empirical. Therefore an understanding of the assumptions and limitations that are implicit in the use of such expressions is required. The dynamic response characteristics of the track are not sufficiently well understood to form the basis of a rational design method. At present the design of railroad track relies on relating the observed dynamic response to an equivalent static response, by making use of various load factors.

It is intended that this report will be a detailed review of the design of conventional railway track.

The objectives of this study are:

- to collate the available track design expressions together with the relevant engineering rationale, and from this literature review produce a report explaining the design of conventional railway track which incorporates a listing of the relevant track design expressions
- to use this report as the source document for an Interactive Track Design Model which will optimise the track design parameters on a basis of cost such that sensitivity analyses can be conducted to identify limitations in the current design practice
- identify areas for future design development to improve the cost efficiency of track design practice.

Although the capability of the formation to resist the bearing pressures imposed by the loaded track should be analysed the scope of this study is confined to the design of the track structure above the level of the formation, i.e., rails, sleepers, and ballast.

The design methods currently used are generally based on estimating the stresses at various critical locations of the track structure, and comparing these stresses with suitable design criteria. It is important that these calculations be based upon sound data derived from experience with commonly used track designs, current axle loads and operating speeds and therefore representing the actual track conditions as closely as possible.

As this report is limited to the current practice used in the design of conventional railway tracks, the following aspects of the permanent way assembly, which have not been quantitatively related to service performance, are outside the scope of this report:

- track fasteners, i.e., fish plates, rail welds, rail fasteners, sleeper bearing plates etc
- . turnouts and crossings
- . ballast material considerations, i.e. grading, breakdown, deformation.

However, it is recommended that available data on these aspects should be assembled for future use and completeness, particularly the ballast deformation properties which have a significant bearing on track surfacing maintenance costs.

The structure of this report generally follows the flow chart, Figure 1.1, developed at Battelle Columbus Laboratories (Prause et al. 1974). The report expands on this general design flow chart in order to develop a more comprehensive description of the interrelationships between design parameters.

#### CHAPTER 2 - THE CURRENT PRACTICE FOR DESIGNING CONVENTIONAL RAILWAY TRACKS

The conventional rail track structure is a load distributing system in which the cyclic loads associated with the passage of vehicle wheels are transmitted from the rails to the sleepers and then to the formation through a protective ballast layer. The magnitude of the stresses imposed on the formation is dependent upon the depth of the ballast layer.

The current practice for designing the railway track is based upon satisfying several criteria for the strength of individual components (Prause, Meacham 1974).Some of the important criteria are:

- allowable rail bending stress
- . allowable sleeper bending stress
- . allowable ballast pressure
- . allowable subgrade pressure.

These criteria are generally evaluated in sequence as indicated in Figure 1.1. The main purpose of establishing these strength criteria is to reduce to a tolerable level the amount of track damage caused by particular track responses. The main track responses and the associated track damage caused by excessive loading are presented in Table 2.1.

## TABLE 2.1 - RELATIONSHIP BETWEEN TRACK RESPONSE AND TRACK DAMAGE (a)

Track Response	Track Damage	
Rail head contact stresses	Rail batter and shelling	
Rail shear forces and web shear stresses	Web and bolt hole failures	
Rail bending moments	Rail fracture and fatigue	
Sleeper loads	Ballast and Sleepers	
Track displacement	Ballast	

(a) Jenkins, Stephenson, Clayton, Morland and Lyon, 1974.



#### CHAPTER 3 - RAIL ANALYSIS

The usual starting point for determining the suitability of a particular rail to carry out its function of withstanding the applied vehicular loading is to calculate the design wheel load. Having calculated the design wheel load, the beam on elastic foundation model is then used to calculate the rail bending stresses caused by this loading. Winkler (1867) and Zimmerman (1888) developed this model over 100 years ago for use in the design of rails. Others who have used rail design approaches based on this model include Talbot (1918-1934), Timoshenko and Langer (1932) and Clarke (1957).

The rail bending stress is usually calculated at the centre of the rail base, but the stress at the lower edge of the rail head may be critical if the vehicles impose high guiding forces during curving between the wheel flange and rail head. Temperature stresses induced in the rail are then calculated, and the total combined rail stress is compared with the allowable rail stress, which is based upon fatigue considerations. Having satisfied the bending stress criteria the rail is evaluated for excessive vertical deflection under the imposed design loading. The rail is then analysed to establish its capacity to withstand the contact stresses at the point of wheel/rail interaction.

The track component that most seriously limits the progression to higher axle loads in railway operations is the rail itself (Tante, Botha 1971). Due to the current trend towards higher axle loads, the selection of the ultimate strength of the rail steel has a significant influence upon the operating life of the rail in service.

The track panel is further analysed for buckling and lateral track stability under the expected in-service conditions. Finally an estimate of the rate of rail head wear can be empirically calculated to determine the expected life of the rail under particular operating conditions.

#### THE DESIGN VERTICAL WHEEL LOAD

The nominal vehicle axle load is usually measured for the static condition, but in the design of railway track the actual stres æs in the various components of the track structure and in the rolling stock must be determined from the dynamic vertical and lateral forces imposed by the design vehicle moving at speed. The dynamic wheel loads cause increases in the rail stress values above those of the static condition due to the following factors:

- . lateral bending of the rail
- eccentric vertical loading
- . transfer of the wheel loads due to the rolling action of the vehicles
- . vertical impact of wheel on rail due to speed
- . irregularities and non-uniformities in the track and the wheel and rail profiles.

The general method used in the determination of the design vertical wheel load is to empirically express it as a function of the static wheel load, i.e.,

$$P = \phi P_{c'}$$
(3.1)

where P = design wheel load (kN),  $P_s$  = static wheel load (kN), and  $\phi$  = dimensionless impact factor (always >1).

The expression used for the calculation of the impact factor is determined empirically and is always expressed in terms of train speed. When developing expressions of the impact factor, the number of above mentioned factors considered depend upon the amount and quality of the track instrumentation used, and the assumptions used in relating the parameters.

Earlier empirical expressions of the impact factor are expressed in terms of the square of the vehicle speed; thereby implying a

solution related to kinematic theory. Some European formulae developed for the impact factor are of this form. The main criticism of this type of empirical formulae is that they neglect any vertical track elasticity which absorbs some of the impact blow on the rail. Clarke (1957) has stated that for a given axle load, experimental results have shown that the rail flexural stresses vary with the vehicle speed V, to the power of 1 to  $1.2^{(1)}$ . In the following sections various types of expressions that have been developed for determining the impact factor are reviewed.

## Types of Impact Factor Formulae currently used to calculate the Design Vertical Wheel Load

The main types of formulae currently used to determine the value of the impact factor to enable the calculation of the design wheel load are discussed below.

AREA Formula: The American Railroad Engineering Association (AREA) formula<sup>(2)</sup> quoted by Prause et al. (1974) recommended for determining the impact factor used in the calculation of the design wheel load is a function of the vehicle speed and the wheel diameter, i.e.,

$$\phi = 1 + 5.21 \frac{V}{D} , \qquad (3.2)$$

where V = vehicle speed (km/h), and D = wheel diameter (mm).

Eisenmann's Formula: This method adopts a statistical approach to determine the magnitude of the impact factor. Eisenmann (1972) suggests that the rail bending stress and deflection are normally distributed and that the mean values can be calculated

<sup>(1)</sup> Clarke does not reference these in-track experiments in his paper.

<sup>(2)</sup> This formula was prior developed by the Association of American Railroads (AAR).

from the beam on elastic foundation model which is discussed in detail later. This normal distribution is illustrated in Figure 3.1 for both rail stress and rail deflection values, (which are obviously inter-related).

Denoting the mean rail stress (or rail deflection) by  $\bar{x}$  the corresponding standard deviation of this mean value  $\bar{s}$ , can be expressed by

 $\overline{s} = \overline{x} \delta \eta$  (3.3)

where  $\delta = a$  factor dependent upon the track condition, and n = a speed factor.

The value of  $\delta$  is determined by the quality of the track, and the following values have been suggested for use:

 $\delta$  = 0.1, for track in very good condition  $\delta$  = 0.2, for track in good condition  $\delta$  = 0.3, for track in poor condition.

The value of  $\eta$  is determined by the speed of the vehicle, V (km/h), and the following values have been suggested for use:

 $\eta = 1, \text{ for vehicle speeds up to 60 km/h}$   $\eta = 1 + \frac{V-60}{140}, \text{ for vehicle speeds in the range of 60 to}$ 200 km/h.

The product  $\delta.\eta$  is referred to as the coefficient of variation. The corresponding maximum rail stress (or rail deflection) is given by

$$X = \bar{X} + \bar{S} t, \qquad (3.4)$$

where t depends upon the chosen upper confidence limits defining the probability that the maximum rail stress (or rail deflection)



Speed

Note: Maximum Value = Mean Value  $\times$  (1 + t $\overline{S}$ ) where t = Student 't' statistic  $\overline{S}$  = standard deviation of the mean

## Figure 3.1

Statistical distribution of measured rail stress and deflection values, showing the effect of increased speed upon the range of the standard deviation, (Eisenmann 1972)

Rail Stress or Deflection

will not be exceeded. The maximum rail stress (or rail deflection), X can be defined by the simple relationship

$$X = \phi \overline{X}$$
(3.5)

where x = mean rail stress (or rail deflection) from beam on elastic foundation model, and

 $\phi$  = impact factor relating to the track condition and train speed.

Combining Equations 3.3 and 3.4 and equating to Equation 3.5 the expression for the impact factor reduces to

 $\phi = 1 + \delta n t,$  (3.6)

where values of t for the chosen upper confidence limits (UCL) can be obtained from the following:

t = 0, UCL = 50 per cent t = 1, UCL = 84.1 per cent t = 2, UCL = 97.7 per cent t = 3, UCL = 99.9 per cent.

ORE Formula: By far the most comprehensive method to determine the impact factor is that developed by the Office of Research and Experiments of the International Union of Railways (ORE) and is entirely based upon measured track results (ORE 1965).

The impact factor is defined in terms of three, dimensionless speed coefficients  $\alpha'$ ,  $\beta'$  and  $\gamma'$ , i.e.,

$$\phi' = 1 + \alpha' + \beta' + \gamma', \qquad (3.7)$$

where  $\alpha'$  and  $\beta'$  relate to the mean value of the impact factor and  $\gamma'$  to the standard deviation of the impact factor.

The coefficient a' is dependent upon:

- the level of the track
- . the suspension of the vehicle
- . the vehicle speed.

The correlation between the value of  $\alpha$ ' and the influence of the level of the track is difficult to estimate due to errors in measurement. In a perfectly levelled track  $\alpha$ ' is virtually zero. In tangent track with levelling defects and very fast traffic  $\alpha$ ' was found to approach 35 per cent. Whereas in curved track values of  $\alpha$ ' did not exceed 18 per cent.

For the most unfavourable case  $\alpha$  ' increases with the cube of the speed and for the locomotives examined it was empirically expressed as

$$\alpha' = 0.04 \left(\frac{V}{100}\right)^3, \qquad (3.8)$$

where V = vehicle speed (km/h).

The numerical coefficient (in this case 0.04) is dependent mainly on the resilience of the vehicle suspension.

The coefficient  $\beta'$  is dependent upon:

- . the vehicle speed
- . superelevation deficiency of the track
- . the centre of gravity of the vehicle.

The coefficient  $\beta'$  is the contribution resulting from the wheel load shift in curves, and may be defined by either:

. the French (SNCF) formula

$$\beta' = \frac{2d.h}{g^2}, \qquad (3.9)$$

the German (DB) formula

$$\beta' = \frac{V^2(2h+c)}{127 \text{ Rg}} - \frac{2c.h}{g^2} , \qquad (3.10)$$

where g = gauge width (m),

- h = height of the centre of gravity of the vehicle (m),
- d = superelevation deficiency (m),
- c = superelevation (m),
- R = radius of curve (m), and
- V = vehicle speed (km/h).

The two formulae are approximately equivalent. The DB-formula is strictly accurate, whereas the SNCF-formula is approximate to within about 1 per cent.

Birmann (1965-1966) states that for the SNCF operating conditions with a superelevation error of 150 mm, the value of  $\beta$ ' from Equation 3.10 ranges from 0.13 to 0.17.

It was observed, under otherwise equal conditions, that the measured coefficient  $\alpha$ ' in tangent track is in almost all cases larger than the value  $(\alpha'+\beta')$  for curved track  $(\alpha'$  measured and  $\beta'$  calculated). It would therefore seem more appropriate to only consider values of  $\alpha'$  as the mean value of the impacts factor for use in design.

The coefficient  $\gamma'$  is dependent upon:

- . the vehicle speed
- . the age of the track
- . the possibility of hanging sleepers
- . the vehicle design
- . the maintenance conditions of the locomotive power units.

The measured coefficient  $\gamma'$  increases with speed, and, as a first approximation the following formula can be used if experimental data are not available.

$$\gamma' \simeq \gamma_0 = 0.10 + 0.017 \left(\frac{v}{100}\right)^3$$
, (3.11)

where V = vehicle speed (km/h).

If the effects of other variables are to be incorporated the above formula can be generalised as

$$\gamma' = \gamma_0 \cdot a_0 \cdot b_0 \quad , \tag{3.12}$$

where  $\gamma_{0}$  = value determined by Equation 3.11,

a a<sub>o</sub> = a locomotive factor relating to the maintenance condition (including the effects of locomotive age), and

b<sub>o</sub> = a track maintenance factor relating to the standard of the track.

The ORE (1965) have obtained typical values of the coefficients  $\gamma_{O}$ ,  $a_{O}$  and  $b_{O}$  for various track and locomotive conditions. Implied in these values are the standards of track and locomotive maintenance that allow particular operating speeds to be safely maintained. The ORE emphasise the values of these coefficients were determined entirely from the average observed relative increase in the standard deviation of the mean rail force level. The following values of  $\gamma_{O}$ ,  $a_{O}$ , and  $b_{O}$  were recommended:

- for normal track with a maximum permissible speed of up to 140 (km/h):
  - $\gamma_{o} = 0.11$ , (from observations<sup>(1)</sup>) locomotive maintenance factor,  $a_{o} = 2.0$ track maintenance factors,  $b_{o} = 1.3$
- for special track with an authorised speed of 200 km/h, assuming new vehicles:

(1) This compares with 0.15 obtained from Equation 3.11.
$\gamma_0 = 0.24$ , (from Equation 3.11) locomotive maintenance factor,  $a_0 = 1.5$ track maintenance factor,  $b_0 = 1.2$ .

The first of the above mentioned values are those most relevant to Australian conditions. A maximum value of the track maintenance factor,  $b_0$  was also determined for a track with relatively poor ballast compaction beneath the sleepers. Under load this track was observed to have 3 mm voids between the sleeper and the ballast, and this condition coupled with high speed traffic gave a maximum value of  $b_0$  equal to 1.7.

In summary, the ORE have observed that the maximum value of the impact factor occurs in tangent track. Consequently, the formula for the impact factor  $\phi$ , Equation 3.7, reduces to

$$\phi = 1 + \alpha' + \gamma'.$$
 (3.13)

Using Equations 3.8 and 3.12 and the ORE recommended values of  $a_0$ ,  $b_0$  and  $\gamma_0$  from test data for normal tracks with permissible speeds of up to 140 km/h, the maximum value of the impact factor can be estimated by <sup>(1)</sup>

$$\phi = 1.29 + 0.04 \left(\frac{V}{100}\right)^3, \qquad (3.14)$$

where V = train speed (km/h).

The relationship between the maximum value of the impact factor and the train speed has been plotted for various standards of track and is presented in Figure 3.2.

(1) This compares with  $2 \\ \phi = 1.26 + 0.08 \ (\frac{V}{100}) \\ \text{using Equations 3.8, 3.11 and 3.12 in the absence of test data for <math>a_0$ ,  $b_0$  and  $\gamma_0$ .



The relationship between the ORE impact factor and the train speed for various track classes (ORE 1965) (Birmann 1965–66)

All these formulae suffer from the basic limitation in that they have no adequate theoretical backing, and therefore extrapolation is not reliable while little published data exist on the statistical variation of loading under Australian operating conditions.

British Railways Formula: A simple model for a discrete irregularity such as dipped rail joint has been developed which illustrates the combined effect of vehicle speed, unsprung mass and track irregularities (Jenkins et al. 1974, Railway Gazette 1970, Koffmann 1972).

The model was developed for current BR main line track consisting of 54 kg/m continuously welded rails and with concrete sleepers spaced at 760 mm.

The resultant dynamic wheel load due to a wheel striking a dipped rail joint was determined from

$$P = P_{s} + 8.784 (\alpha_{1} + \alpha_{2}) V \begin{bmatrix} D_{j}P_{u} \\ g \end{bmatrix}^{0.5}, \qquad (3.15)$$

where  $P_s$  = static wheel load (kN),  $P_u$  = unsprung weight at one wheel (kN),  $D_j$  = track stiffness at the joints (kN/mm), g = gravitational constant (m/S<sup>2</sup>),  $(\alpha_1 + \alpha_2)$  = total rail joint dip angle (radians), and V = vehicle speed (km/h).

Therefore the dynamic factor can be defined as

$$\phi = 1 + \frac{8.784 (\alpha_1 + \alpha_2) V}{P_s} \left[ \frac{D_j^P u}{g} \right]^{0.5}$$
(3.16)

For BR conditions and with a track consisting of continuously welded 54 kg/m rails on concrete sleepers spaced at 760 mm, a value of D<sub>j</sub> equal to 88 kN/mm is considered sufficiently accurate for use in Equation 3.15 (Koffman, 1972). Also for these conditions a rail joint dip of 10 mm and a corresponding value of  $\alpha_1 + \alpha_2 = 0.015$  radians can be used as representative values of the welded joint.

The resultant dynamic wheel load due to locomotives striking a dipped rail joint at speed, and estimated by Equation 3.15, has been evaluated for a range of currently used British Rail locomotives, and is presented in Figure 3.3. It can be seen that the dynamic wheel load caused by poorly maintained joints can be as high as two to three times the static wheel load depending upon the speed of the locomotive.

The four main types of impact factor formulae, so far discussed, are compared in Figure 3.4. Although the formulae are not specifically interrelated a few general observations about the predicted magnitude of the impact factor can be made. The envelope defined by Eisenmann's curves of impact factor for very good and good track conditions (at UCL of 99.9 per cent), contains both the AREA and ORE impact factor curves which have been derived for probable average track conditions. Also the impact factor curve corresponding to Eisenmann's poor track condition is approximately of the same order of magnitude as the British Rail formula for locomotives striking poorly maintained rail joints.

Other developed expressions that have been used to determine the magnitude of the impact factor are presented in Table 3.1. The Indian Formula (Agarwal 1974) attempts to relate the track condition to the impact factor by making use of the measured values of the track modulus. The German Formula (Schramm 1961) is a typical expression developed solely on kinematic considerations. The South African Formula (Lombard 1974) is of the same form as the AREA Formula (Prause et al. 1974), Equation 3.2, but calculated for narrow gauge track structure. The Clarke Formula (Clarke 1957) is simply the algebraic combination of the AREA Formula and the Indian Formula and as such is not modelled on any experimental results. This emphasises the problem of using highly empirical formulae without adequate knowledge of the track conditions and assumptions used in their derivation.



Speed (km/n)
--------------

Legend:	No.	Vehicle	Static Axle Load (tonnes)	Unsprung Mass (tonnes)
	1	KESTREL (Diesel Elec) Loco, Co-Co	23	4.25
	2	Class 86 Elec. Loco, Bo-Bo	20.45	4.22
	3	100 Tonne Tank Wagon	25	2.8
	4	45 Tonne Tank Wagon	22.5	2.7
	5	Class 55 Deltic Diesel Elec. Loco, Co-Co	17.6	3.3
	6	Freightliner Wagon	17.5	1.92
	7	Special Elec, Multiple Unit	16.5	2.0
	8	6 Axle Freightliner Wagon	13.5	1.9
	9	Advanced Passenger Train	12	1.5

Comparison of dynamic wheel loads on BR showing the relative effects of higher speeds, static axleloads and unsprung masses (Railway Gazette 1970)



6 Ore  $\beta' \text{ assumed} = 0.20 \ a_0 = 1.5, \ b_0 = 1.2$ 

7 B.R.	Class 55 Deltic Diesel Elec. F	Ps = 86.2 kN	Pu = 65 1
--------	--------------------------------	--------------	-----------

8 B.R. Kestrel Diesel Elec. Ps = 112.7 kN Pu = 2.12 t

### **Figure 3.4** Comparison of impact factor formulae

### TABLE 3.1 - SUMMARY OF EXPRESSIONS THAT HAVE BEEN USED TO DETERMINE THE VALUE OF THE IMPACT FACTOR<sup>(a)</sup>

1. Indian Formula (Agarwal 1974):  $\phi = 1 + \frac{V}{58.14(k)}^{0.5}$ 2. German Formula (Schramm 1961): (a) For speeds up to 100 km/h  $\phi = 1 + \frac{v^2}{3 \times 10^4}$ (b) For speeds above 100 km/h  $\phi = 1 + \frac{4.5v^2}{10^5} - \frac{1.5v^3}{10^7}$ South African Formula (Lombard 1974), (Narrow gauge track): 3.  $\phi = 1 + 4.92 \frac{V}{D}$ Clarke Formula (Clarke 1957): 4.  $\phi = 1 + \frac{19.65V}{D(k)} 0.5$ WMATA Formula<sup>(b)</sup> (Prause et al. 1974): 5.  $\phi = (1 + 3.86 \times 10^{-5} V^2)^{0.67}$ 

- (b) WMATA is the abbreviation of the Washington Metropolitan Transit Authority.

A comparison of the vehicle and track parameters that have been included in formulae developed for the impact factor are presented in Table 3.2. It is apparent that the early formulae developed to determine the impact factor are rather simplistic in that they relate only to vehicle parameters (e.g. train speed and wheel diameter). Although the track maintenance conditions is somewhat implicit in the early empirical formulae, it is only recently that attempts have been made explicitly to determine its significance to the value of the impact factor.

WMATA	CLARKE	SAR	GERMAN	IR	BR	ORE	EISENMANN	AREA	FORMUL/		TABLE 3.2
$(1 + 3.86 \times 10^{-5} v^2)^{2/3}$	$1 + \frac{19.65v}{v_k}$	1 + 4.92 V/b	$1 + \frac{v^2}{3 \times 10^4}$ (etc)	$1 + \frac{V}{58.14 \sqrt{k}}$	$1 + \frac{8.784(\alpha_1 + \alpha_2)V}{\Pr_s} \sqrt{\frac{D_j P_u}{g}}$	$1 + \alpha' + \beta' + \gamma'$	1 + &nt	1 + 5.21 V/D	IMPACT FACTOR	VEHICLE AND RACK PARAMETERS	
X	×	X	X	×	×	×	X	х	TRAI	N SPEED	
	×	×						×	WHEEL	L DIAMETER	
	-	:			×		İ		STAT LOAD	IC WHEEL	VEH
		-			×	1			UNSP MASS	RUNG WHEEL	ICLE
						X			CENT OF V	RE OF GRAVITY EHICLE	
		-		1 1 1 1	k	×			LOCC TENA	MOTIVE MAIN- NCE CONDITION	
	×			×					TRAC	CK MODULUS	
					X			-	TRAC AT I	CK STIFFNESS RAIL JOINT	
					×	1			TRAC	CK JOINT DIP LE	TRA
						×			CAN' IN	F DEFICIENCY CURVES	Ċ <b>K</b>
						×			CUR	VE RADIUS	
						X	×		TRA CON	CK MAINTENANCE DITION	

τz

# VEHICLE AND TRACK PARAMETERS INCLUDED IN IMPACT FACTOR FORMULAE

TABLE 3.9

### THE DESIGN LATERAL WHEEL LOAD

The magnitude of the lateral guiding force imposed on the rail head can be considered to be dependent upon the following:

- . the curve radius of the track
- . the vehicle speed
- . the length of the vehicle wheelbase and its bogie configuration
- . the tracking motion of vehicles in the train consist.

Few in-track test programmes have been carried out to determine the magnitude of the lateral forces caused by the wheel flanges of vehicles contacting the rail head when negotiating curves. Of the published literature the results of Birmann (1966), the ORE (1965, 1970), and Olson and Johnsson (1960) appear to be of the most use.

Birmann (1966) has carried out a series of experiments to determine the magnitude of the lateral guiding force caused by the wheel flanges of vehicles (in particular locomotives) contacting the rail head when negotiating curves. Results of guide force measurements for various locomotive and wagon bogie configurations shown in Figure 3.5 are plotted against curve radius in Figure It can be seen that the guide force, H(kN) is to a degree 3.6. more dependent upon the curve radius than the vehicle speed. for radii greatee than 800 m and at the same speed, the magnitude of forces exerted by a given vehicle are similar to those exerted on tangent track due to the tracking motion of the train consist. However with radii less than 800 m the lateral forces increase significantly with decreasing radii. The recorded maximum values of the guiding force were found to be of the order of 30 to 60 per cent higher than the mean of the recorded values. The relationship of the mean lateral guiding force exerted by various vehicle types to the curve radius of the track has been presented to Eisenmann (1970) and is shown in Figure 3.7.



DB locomotives and freight wagons, with axle loads (kN) and distances between axles (m) (Birmann 1966)



Lateral guide forces as a function of the track curve radius at various mean speeds for DB locomotives and freight wagons (Birmann 1966)



Relation between the guide force and the track curvature for vehicles at high speeds (mean values) (Eisenmann 1970)

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The ORE (1965, 1970) used Birmann's results and carried out an expanded test programme for speeds up to 200 km/h. The expression suggested by the ORE for determining the magnitude of the lateral force H(kN), caused by the wheel flanges of locomotives contacting the rail head when negotiating curves is dependent only upon the radius of the curve R(m). It is based upon the observed maximum envelope of experimental results for all vehicles and is expressed as

$$H = 35 + \frac{7400}{R} . \tag{3.17}$$

A series of field tests were also carried out by the Swedish Railways (Olson et al 1960) to determine the magnitude of locomotive guiding forces. Measurements of the magnitude of the lateral force caused by the wheel flanges of the locomotive contacting the rail head were obtained from a small number of tests and only for a curve radius of 600 m. Since the curve radius was kept constant the empirical expression of the magnitude of the mean lateral force  $H_{mean}(kN)$  is dependent only upon the speed of the locomotive V(km/h), i.e.,

$$H_{\text{mean}} = 17 + \frac{V}{27.6} . \tag{3.18}$$

It was specifically noted that the magnitude of the mean lateral force increases in a similar way for both the light and heavy axle load locomotives used in the tests. It is usually assumed that the magnitude of the lateral force would increase more rapidly in the case of heavy axle load locomotives.

The lateral forces caused by the wheel flanges of locomotives contacting the rail head when negotiating curves have also been investigated by British Rail (Koffmann 1972). Results of the values of the magnitude of lateral guiding forces imposed upon the rail head by Bo-Bo and Co-Co type locomotives for various locomotive speeds and wheel loads and ranges of curve radii are presented in Table  $3.3^{(1)}$ .

TABLE	3.3	-	GUIDING	FOR	ES	IMPOSED	ON	TRACK	ΒY	TWO	TYPE	S OF	
			LOCOMOTI	IVES	TR/	VELLING	AT	SPEED	ON	VAR	LOUS	CURVE	RADII
			(Koffmar	n 197	72)								

Wheel	Locomotive				G	uidin	g Ford	ce H(]	kN)	
Load P (kN)	Туре	1	Cu: 80	rve 35	Radi	us (m 65	1)	Tang Trac	gent Ck	Maximum Permissible
				S	peed	(km/	'n)			Values
		40	60	60	80	100	110	110	120	
	Во-Во	52	82	61	83	77	87	66	83	_
115	Co-Co	48	78	52	75	73	87			100
	Во-Во	64	96	74	98	90	101	<b>7</b> 7	99	
135	Co-Co	55	96	62	93	88	101			100
1 5 0	Во-Во	70	106	80	107	98	113	86	110	
150	Co-Co	65	113	69	104	98	113			100

Having determined the design vertical wheel load and the design lateral wheel load for a given vehicle, the next step in the design procedure is to determine the resultant maximum rail stresses caused by this imposed loading.

THE LOCATION OF THE MAXIMUM STRESSES IN THE RAIL SECTION

The location of the maximum rail stresses are shown in Figure 3.8 (Eisenmann 1969a). The evaluation of each must satisfy the corresponding design criteria for the allowable rail stress. The maximum shear stress occurring in the web is not usually considered

Clark (1973) has defined these locomotive axle and bogie configurations and presents a summary of Australian locomotives for comparison with Figure 3.5.



Legend: A, B, C are areas of critical stress. P and H are applied vertical and horizontal wheel loads respectively.

### **Figure 3.8** Loads on the rail and positions of high rail stresses

in the current rail design methods, since it is unlikely that the rail will be subjected to a shear failure (unless there is an occurrence of a rail fatigue defect, e.g. transverse defect).

The bending stress which occurs at the centre of the rail base (point A) is independents of the magnitude of the guide force and the eccentricity of the point of attack of the wheel. The design criteria for this bending stress is established to prevent the occurrence of cracks in the rail base.

The bending stress at the lower edge of the rail head (point B) is important in the evaluation of plastic deformation of the rail head in the horizontal direction.

High values of rail shearing stress are generated near the contact point between the rail and the wheel (point C) as a result of constant repetitive load introduction. When the fatigue strength is exceeded, fracture of the rail head occurs. This is commonly termed shelling of the rail head and problems of this nature will be discussed in detail later.

In order to facilitate the calculation of the rail bending stress at the centre of the rail base and also the amount of vertical rail deflection under load, it is useful at this stage to introduce the concept of the rail considered as a beam on a continuous linear elastic foundation.

THE RAIL CONSIDERED AS A BEAM ON A CONTINUOUS LINEAR ELASTIC FOUNDATION

The concept of a foundation modulus to represent the rail support was first introduced by Winkler (1867), when he analysed the rail as an infinite beam supported on a continuous linear elastic foundation. The differential equation for the bending theory of an elastic beam from Figure 3.9(a) is



Figure 3.9(a)

Equilibrium position of a deformed beam subjected to load q (x)



**Figure 3.9(b)** Representation of a continuously supported infinite beam on an elastic foundation subjected to load q (x)

## Deformation of an infinite beam on an elastic foundation

$$EI\frac{d^{4}y}{dx^{4}} + p(x) = q(x), \qquad (3.19)$$

where y(x) = vertical deflection at x,

q(x) = distributed vertical load, EI = flexural rigidity of the rail, p(x) = continuous contact pressure between the sleeper and ballast, p(x) = ky(x), and k = modulus of the foundation.

Hence the Winkler equation becomes

$$EI\frac{d^{4}y}{dx^{4}} + ky(x) = q(x).$$
 (3.20)

This equation may be represented as the response of an infinite beam attached to a spring base, subjected to a load q(x), Figure 3.9(b). The general solution of the Winkler equation has been developed in detail by Hetenyi (1946).

Since the rail is subjected to wheel loads, which are concentrated loads, the relevant solution to Winkler's equation must be restated in terms of the design wheel load, P, instead of load q(x). The solution of the rail deflection rail shear force and rail bending moment at any position x, (x positive), from the load point are:

. rail deflection

$$y_{x} = \frac{P_{\beta}e^{-\beta x}}{2k}(\cos\beta x + \sin\beta x) , \qquad (3.21)$$

. rail shear force

$$V_{\rm X} = \frac{{\rm Pe}^{-\beta {\rm X}}}{2} \cos\beta {\rm X}$$
(3.22)

rail bending moment

$$M_{x} = \frac{Pe^{-\beta x}}{4} (\cos\beta x - \sin\beta x)$$
(3.23)

Here  $\beta$  includes the flexural rigidity of the beam as well as the elasticity of the supporting medium, and is an important factor influencing the shape of the elastic beam. For this reason the factor  $\beta$  is called the characteristic of the system, and, since its dimension is (length)<sup>-1</sup>, the term  $1/\beta$  is frequently referred to as the characteristic length. Consequently, the product  $\beta x$  will be a dimensionless number with

$$\beta = \left(\frac{k}{4EI}\right)^{0.5},$$
 (3.24)

where k = track modulus (MPa), E = Young's modulus of the rail steel (MPa), and I = rail moment of inertia (mm<sup>4</sup>).

The Winkler equation was originally developed for longitudinallysleepered track, and has since been applied to transversely sleepered track, thereby raising questions concerning the validity of the assumption of continuous rail support. But although there have been many methods developed to analyse track on the basis of discrete elastic supports, notably by Schwedler (1882), Zimmerman (1888) and Engesser (1888), according to Ken (1976) the results obtained are not significantly different from those using the Winkler model. Hence considering the rail as a beam on a continuous linear elastic foundation is generally regarded as the most acceptable method for the analysis of rail stresses and deflections.

### Limitations of the beam on elastic foundation analysis

The following are the main limitations of the beam on an elastic foundation analysis as applied to railway track conditions:

- the model neglects any continuity or coupling of the ballast and subgrade layers that make up the track foundation. The magnitude of the coupling effect depends upon the sleeper spacing, the sleeper size, the ballast depth, and the subgrade properties (Eisenmann, 1969a)
- . there is no adequate modelling of the stress-strain behaviour of the ballast and the subgrade. Thus the model is of limited value in considering the behaviour of the sub-structure beneath the rail (Robnett, Thompson, Hay et al. 1975)
- the simple Winkler model does not include several additional factors which are known to affect the stresses and deflections in railroad track. These include longitudinal loads from thermal stresses, a restoring moment proportional to the rotation of the rail and sleeper, the eccentricity of the vertical load on the rail head, and track dynamic effects, such as inertial and damping forces (Prause et al. 1974).

But despite these deficiencies the Winkler model has proven quite useful for design purposes because of its simplicity, its ease of use and its degree of accuracy when compared with measured results.

### The base parameter of the foundation<sup>(1)</sup>

Including the foundation modulus as used in the beam on an elastic foundation analysis (Equation 3.26), there are currently three alternative ways to define the base parameter of the foundation from field measurements of railway track (Bhatia, Romualdi and Theirs 1968).

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<sup>(1)</sup> The term base parameter refers to the assumptions used in the analysis to define the elastic foundation support beneath the beam. The general solution to Equation 3.19 for beam deformation must be redefined for each set of assumptions to ensure consistent dimensions.

Spring Constant Method: This method used to define the base parameter is analagous to the spring constant used in vibration theory. (Figures 3.10(a) and (b)).

For the track structure loaded by a known static axle load, (i.e. twice the known static wheel load  $P_s$  on each rail) the total rail support deflection Y, is the sum of the rail deflections occurring at each sleeper location to the left and right of the position of the known static axle load.

The total rail support deflection can therefore be expressed as

$$Y = \sum_{m=0}^{m=\infty} Y_{n-m}^{+}, \qquad (3.25)$$

where  $Y_{n-m}^+$  = rail deflection (mm) at the m<sup>th</sup> sleeper location away from the position of the applied axle load at sleeper n.

The foundation spring constant D (kN/mm) for the rail support derived from this mechanical model of the railway track is

$$D = \frac{P_s}{Y}, \qquad (3.26)$$

where  $P_s = known$  static wheel load on each rail (kN), and Y = total measured rail support deflection (mm).

Kerr (1976) defines the spring constant in terms of the track subjected to a loading of  $2P_s$  (i.e. the axle load) as twice the spring constant determined from only one rail being subjected to a load of  $P_s$ , i.e.,

$$D_{\text{track}} = 2D_{\text{rail}} = 2D \tag{3.27}$$

Talbot (1918-1935) observed that the static track deflection caused by a single point load on the rail head spread over a total number of between seven and nine sleepers in track.



### Modelling of foundation deflection properties

Kurzwell (1972) states that for average track structures the deflection due to a point load on a track is negligible beyond about 3.000 m of the point of load application. Adopting a typical sleeper spacing, this suggests that about nine sleepers resist the applied loading. Hetenyi (1946) also reports the results of experiments conducted by Wasiutynski (1937) who noted that the value of D determined by means of Equation 3.36 for the track assembly is about half the value obtained when only a single separate sleeper is loaded and is due to foundation interaction between adjacent sleepers, thus

 $D_{\text{track}} \simeq \frac{D_{\text{sleeper}}}{2}$  (3.28)

There is a marked decrease in the track stiffness, D, in the vicinity of a fish-plated rail joint (Meacham, and Ahlbeck 1969). Values of the average track stiffness at the rail joint D<sub>j</sub> range from 0.25D, for a joint in a very bad condition, to 0.77D for a joint in excellent condition. It can clearly be seen that the rail joint is the weak link in the track structure.

<u>Track Modulus Method</u>: This method is analagous to Young's Modulus used in determining the strength of materials. The continuous support deflections, shown in Figure 3.10(c), can be approximated by a series of stepped deflections, considered to be constant over the length of the sleeper spacing. This stepped deflection curve is presented in Figure 3.10(d). Using this assumption the sleeper spacing can be introduced into the expression to determine the rail support parameter. The track modulus k (MPa) is defined as the force/ unit of delfection/unit of track length<sup>(1)</sup>. For the case of a single rail it can be expressed as

(1) The track modulus k is sometimes denoted by the symbol U in the railway literature.

$$k = \frac{P_{s}}{Y \ s \ 10^{3}} , \qquad (3.29)$$

where P<sub>s</sub> = known static wheel load on each rail (kN), Y = total measured rail support deflection (mm), (by Equation 3.25), and S = sleeper spacing (mm).

This is the method used by the AREA and Clarke to determine the rail support base parameter for use in the beam on an elastic foundation analysis.

<u>Coefficient of Subgrade Reaction Method</u>: This method is based upon the original Zimmerman theory which was developed for a longitudinally sleepered track considered to be resting on a compressible foundation. Since track is now transversely sleepered, a transformation of this track type to an equivalent longitudinally sleepered track is required in the analysis. This can be achieved if the assumption is made that the effective rail support area provided by the sleeper remains constant for both types of track. (Eisenmann, 1969b).

For transversely sleepered track the effective sleeper support area beneath one rail seat,  $A_{s}$  (mm<sup>2</sup>), can be assumed to be

$$A_{g} = B(\ell - g),$$

where B = sleeper breadth (mm),

- l = sleeper length (mm), and
- g = distance between the centre line of the rail seats
   (mm).

This is discussed in detail later.

Referring to Figure 3.11 the breadth B' (mm) of an equivalent longitudinal sleeper supporting one rail can be calculated (assuming constant rail support area) as







Note: Effective rail support area = B (  $\ell - g$ )

Transformation of transversely sleepered track to an assumed longitudinally sleepered track for use with Zimmermann theory (Eisenmann 1969 b)

$$B' = \frac{A_{S}}{S} ,$$

where S = sleeper spacing (mm).

The coefficient of subgrade reaction C  $(kN/mm^3)$  for one rail is then defined as

$$C = \frac{P_{S}}{Y S B'}, \qquad (3.30)$$

where P = known static wheel load on each rail (kN),

S = sleeper spacing (mm), and

Under German Railway (DB) track conditions the sleepers are spaced 630 mm apart and the effective rail support area per rail, is usually assumed to be 2.6 x  $10^5 \text{ mm}^2$ . Therefore the equivalent breadth of an equivalent longitudinal sleeper supporting one rail can be expressed as

$$B' = \frac{2.6 \times 10^5}{630} = 413 \text{ mm}.$$

It is important to realise that the above methods to determine the base parameter of rail support are interrelated since

 $D = Sk = SB'C \tag{3.31}$ 

and that the only field measurement carried out is the calculation of the value of Y for a known static axle load.

Values of the track modulus k have been proposed by Hay (1953) and Ahlf (1975) for various track structures (Tables 3.4 and 3.5). Westrail (1976) have published measured values of track modulus for current mainline track structures (Table 3.6). The results of German measurements of the track stiffness D have been published by Birmann (1965-1966) (Table 3.7). Birmann also reports the results of Luber (1961) for the stiffnesses of individual track components and these are reproduced in Table 3.8. Typical values of the coefficient of subgrade reactions have been proposed by Eisenmann (1969b) for German Railways (DB) track conditions (Table 3.9).

The main factors influencing the value of the track modulus k are (Lundgren, Martin and Hay, 1970):

- . the sleeper spacing, dimensions and quality
- . the quality, depth and degree of compaction of the ballast layer which defines the solidarity of the track construction
- . the subgrade quality and the degree of its compaction which determines the strength of the foundation
- . the rail size affects the load distribution of adjacent sleepers in the track panel.

As a first attempt to determine the significance of some of the above factors which influence the track spring rate (and therefore from Equation 3.31 the track modulus) a theoretically derived ballast pyramid model has been suggested by Prause et al. (1974). This theoretical method of determining the track spring rate is presented in Annex A.

The track modulus as defined is strictly a static parameter and is not intended to include any dynamic effects such as frequency dependent damping or the mass of the rail support system (Eisenmann 1969b). Having estimated a value of the track modulus the maximum rail bending stress at the rail base, Figure 3.8 point A, can be calculated by using the beam on an elastic foundation analysis.

### TABLE 3.4 - VALUES OF TRACK MODULUS (HAY 1953)

Rail (kg/m)	Sleeper Size (mm)	Track and Ballast Condition	Track Modulus (MPa)
42	178 x 229 x 2590	150 mm fine cinder ballast in poor condition on loam clay subgrade	3.7
42	178 x 229 x 2590	150 mm cinder ballast, fair condition, loam clay subgrade	5.2
42	152 x 203 x 2440	150 mm limestone on loam clay subgrade, good before tamping	6.7
42	152 x 203 x 2440	300 mm limestone on loam clay, subgrade after tamping	7.5
42	178 x 229 x 2440 .	300 mm limestone on loam clay subgrade, good before tamping	7.3
42	178 x 229 x 2440	300 mm limestone on loam clay subgrade, good before tamping	7.3
42	178 x 229 x 2590	600 mm crushed limestone on loam and clay	8.3
64 R.E. <sup>(a)</sup>	178 x 229 x 2590	600 mm gravel ballast plus 200 mm heavy limestone on well-compacted subgrade	20.0 - 20.7
55 R.E. <sup>(a)</sup>	178 x 229 x 2440 560 mm centres G.E.O. fastenings <sup>(b)</sup>	Flint gravel ballast on wide stable roadbed	17.3, 17.9, 24.8 Av 20.0
55 R.E. <sup>(a)</sup>	178 x 229 x 2440 560 mm centres G.E.O. fastenings <sup>(b)</sup>	Limestone ballast on wide stable roadbed	25.5, 38.0, 42.8 Av 35.4

- (a) R.E. indicates an AREA rail specification.
- (b) The G.E.O. fastening is a German manufactured fastener and is very similar to the k fastener used by D.B.
- Source: From the first and sixth progress reports of special committee on stresses in Railroad Track (Talbot 1918; 1934).

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	TRACK,	(AHLF 1975)		
Sleeper Condition	Ballast Depth (mm)	Ballast Condition	Subgrade Condition	k (MPa)
Poor	150	Relatively unsound material, fouled with mud	Poorly drained, soft	6.9
Fair	150	Fair soundness, reasonably free of mud	Average, some drainage	13.8
Good	150	Sound, crushed stone, free of mud	Average, some drainage	20.7
Good	300	Sound, crushed stone, free of mud	Average, some drainage	27.6
Good	450	Clean, sound, crushed stone	Good, compact, well drained	34.5

TABLE 3.5 - TRACK MODULUS VALUES FOR FIVE DIFFERENT TYPES OF

TABLE 3.6 - MEASURED VALUES OF TRACK MODULUS, STANDARD GAUGE

MAINLINE	TRACK	WESTRAIL	(at	1976)	ł
	the second se				

Date of Test	Sleeper Crs.(mm)	Ballast Depth (mm)	Season	k (MPa)
Aug 1972	610	250	Winter (moist)	13.2
30.8.72	610	250	Winter (dry)	15.2
31.8.72	610	250	Winter (dry)	13.8
4.9.72	610	230	Winter (dry)	14.8
3.10.72	627		Spring	21.3
12.9.73	617		Winter	12.7
12.9.73	640	228 300	Winter	17.9
	Date of Test Aug 1972 30.8.72 31.8.72 4.9.72 3.10.72 12.9.73 12.9.73	Date of Test       Sleeper Crs.(mm)         Aug 1972       610         30.8.72       610         31.8.72       610         4.9.72       610         3.10.72       627         12.9.73       617         12.9.73       640	Date of Test       Sleeper Crs.(mm)       Ballast Depth (mm)         Aug 1972       610       250         30.8.72       610       250         31.8.72       610       250         4.9.72       610       250         3.10.72       627       230         12.9.73       640       228 300	Date of Test         Sleeper Crs.(mm)         Ballast Depth (mm)         Season           Aug 1972         610         250         Winter (moist)           30.8.72         610         250         Winter (dry)           31.8.72         610         250         Winter (dry)           4.9.72         610         230         Winter (dry)           3.10.72         627         Spring           12.9.73         640         228 300         Winter

Component	Softwood Sleeper (kN/mm)	Hardwood Sleeper (kN/mm)	Steel Sleeper (kN/mm)	Concrete Sleeper (kN/mm)
Intermediate Wood Layer	50 <b>-</b> 500	50 <b>-</b> 500	50 - 500	50 <b>-</b> 500
Sleeper	50 <b>-</b> 150	300 <b>-</b> 500	2000 -4000	8000 - 20000
Ballast	50 - 300	50 <b>-</b> 300	50 - 300	50 - 300
Overall Spring Rate (without ballast and formation)	30 - 110	40 - 290	50 <b>-</b> 310	50 - 430
Overall Spring Rate	20 - 80	20 - 130	20 - 170	30 - 180

TABLE 3.7 - SPRING RATES OF INDIVIDUAL RAIL TRACK COMPONENTS,

LUBER (1961)

### TABLE 3.8 - RESULTS OF GERMAN RAILWAYS (DB) TRACK SPRING RATE MEASUREMENTS, BIRMANN (1965-1966)

Subgrade	Туре	Track Spring Rate (kN/mm)
Track on	marshy soil	5 - 15
Track on	clay soil	15 <b>-</b> 20
Track on	gravel	20 - 60
Track on	rock	30 - 40
Track on	frozen ballast and formation	80 - 160
Most free	quent mean value	30

# TABLE 3.9 - VALUES OF THE COEFFICIENT OF SUBGRADE REACTION FOR A TRACK WITH A SLEEPER SPACING OF 630mm, AND ASSUMED EFFECTIVE SLEEPER BEARING AREA OF 5.2 x 10<sup>5</sup> mm<sup>2</sup> (EISENMANN 1969b)

Subgrade Type	Coefficient of Subgrade Reaction 10 <sup>6</sup> (kN/mm <sup>-3</sup> )			
Very poor subsoil (marshy soil, fine grained	sand)	20		
Poor subsoil (cohesive soil)		49		
Good subsoil (gravel)		98		

CALCULATION OF THE RAIL BENDING STRESS AT THE BASE OF THE RAIL

Using the beam on an elastic foundation analysis under the action of a single design wheel load the rail bending moment M (kNm) at a distance x (m) from the load point is

$$M_{\rm x} = \frac{P \cdot e^{-\beta x}}{4 \cdot \beta} \left( \cos \beta x - \sin \beta x \right), \qquad (3.32)$$

where P = single wheel load (kN)

 $\beta = \left(\frac{k}{4EI}\right)^{0.25},$  k = track modulus (MPa), E = Youngs Modulus of the rail steel (MPa), and  $I = \text{rail moment of inertia (mm}^4).$ 

A master diagram (Hay, 1953) has been developed, for the case of a single point load, which relates the rail bending moment under the load point  $M_O(kNm)$ , to the rail bending moment at any other location (Figure 3.12). The rail bending moment under the load point is the maximum for the single load case and is expressed as

$$M_{O} = \frac{P}{4\beta} . \qquad (3.33)$$

The distance  $x_1(m)$  (Figure 3.12) is that distance to the position of zero rail bending moment from the point of load application and is given by

$$x_1 = \frac{\pi}{4} \cdot \frac{1}{\beta}$$
 (3.34)

For the actual track loading conditions the rail at any point will be subjected to a combination of bending moments caused by the interaction of adjacent wheel loads. The total length of this zone of interaction is approximately a distance  $6x_1$ , to the left and right of the load point, for the case of a single point load (Figure 3.12). Using the principle of super-position the rail bending moment M(kNm) under a particular wheel including the rail bending moments due to the interaction of adjacent wheels is

$$M = \sum_{i=0}^{L=n} P_{x_{i}} A_{x_{i}}'$$
(3.35)

where i = 0, 1, 2 ..., = number of adjacent wheels in the interaction length (Figure 3.12). i = 0 refers to the reference wheel,

x<sub>i</sub> = distance to the adjacent wheels from the reference
 wheel<sup>(1)</sup> (Figure 3.13),

$$P_{x_i}$$
 = magnitude of the impact factored wheel loads at distances  $x_i$  from the reference wheel, and

It can clearly be seen that the maximum rail bending moment depends to a large extent upon the axle spacing.

(1)	Wheels at distances $X_i > 6X_1$ are not included, for the case									
	of the reference wheel $X_0 = 0$ .									
(2)	For a general case the expression can be written as:									
	$A_{X_{i}} = \frac{e^{-\beta X_{i}}}{4\beta}  (\cos X_{i} - \sin X_{i})$									



Master diagram for moments, pressure intensity and rail depression under a single wheel load (Hay 1953)



Note: X1<X2<X3<X4<6x

### Figure 3.13

General load interaction diagram for the calculation of maximum rail bending moment and deflection

Some railway operators restrict the length of the vehicle wheel base to less than  $6x_1$ , thereby reducing the occurrence of numerous fully unloaded cycles in the rail during the passage of a train. This is an important consideration in the fatigue life of a rail.

The maximum rail stress at the centre of the rail base  $\sigma_{\rm b}$  (MPa) is readily calculated using simple applied mechanics, and can be stated as

$$\sigma_{\rm b} = \frac{M_{\rm m} \cdot 10^6}{Z_{\rm o}}, \qquad (3.37)$$

where 
$$M_m = \max \min rail$$
 bending moment (kNm) from Equation  
3.35, and  
 $Z_o = \text{section modulus of the rail relative to the rail}$   
base  $(mm^3)$ .

The section properties of current Australian rail sizes as manufactured by BHP are presented in Table 3.10.

### Allowable rail bending stress at the rail base

The following are methods currently used by railway organisations for evaluating the maximum allowable rail bending stress at the rail base.

The General Method as used by AREA: The AREA (1973) recommends that the acceptable rail stress for continuous welded rail be established at the rail base (Robnett et al. 1975). The current procedures limit the allowable rail bending stress in the rail base to implicitly avoid fatigue cracking. Clarke (1957) has suggested that the value of the allowable rail bending stress should not exceed 50 per cent of the rail yield stress,  $\sigma_{\rm Y}$ . Although the value of the design load P contains the amplification effect of the impact factor which implicitly includes the effects of locomotives, track condition etc., the allowable rail bending stress as determined by the general method makes further reductions for these and other factors.

Mass and Section of Rail	Area of Head	Area of Web	Area of Foot	Total Area	Height of Neutral Axis	Total Height	Width of Foot	Width of Head T	Vertical Moment of Inertia	Horízontal Moment of Inertía Z	Section Modulus (Head) Z	Section Modulus (Foot)
	(mm <sup>2</sup> )	(mm <sup>2</sup> )	(mm <sup>2</sup> )	(mm <sup>2</sup> )	(mm)	(mm)	(mm)	-xx (mm)	<sup>-</sup> YY 10 <sup>6</sup> (mm <sup>4</sup> )	$10^{6}  (mm^4)$	<sup>3</sup> xx 10 <sup>3</sup> (mm <sup>4</sup> )	10 <sup>3</sup> (mm <sup>4</sup> )
31 kg/m AS 1085-1977	1826	794	1394	4014	58.7	117	108	63.5	7.66	1.71	130.4	130.4
41 kg/m AS 1085-1977	2148	£110	1929	5187	64.7	137	127	63.5	13.24	2.80	184.4	204.5
47 kg/m AS 1085-1977	2548	1277	2103	5928	68.3	141	127	69.9	15.92	3.21	217.8	233.7
50 kg/m <sup>(a)</sup> AS1085-1977	2680	1640	2150	6470	74.8	154	127	70.0	20.10	3.26	254.1	269.3
53 kg/m AS 1085-1977	2710	1529	2510	6749	73.0	157	146	69.9	22.80	4.62	271.2	311.5
59 kg/m AREA Profile	2787	1961	2768	7516	79.3	173	140	67.5	29,72	4.68	317.9	375.3
60 kg/m <sup>(a)</sup> AS 1085-1977	2960	2040	2770	7770	79.1	1.70	146	70.0	29.40	5.07	323.2	371.4
66 kg/m AREA Profile	2852	2361	3142	8355	81.3	181	152	76.2	36.71	6.07	368.7	452.7
68 kg/m AREA Profile	3136	2336	3142	8614	85.0	186	152	74.6	39.50	6.48	391.7	463.8

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TABLE 3.10 - SECTION PROPERTIES OF AUSTRALIAN RAIL SIZES (BHP, 1975)

(a) New Australian rail section.
The allowable rail bending stress should be sufficiently below the elastic limit (or yield stress) of the rail steel in order to account for any variability in the rail support, the wheel loading or other existing service conditions, which may result in excessive rail bending stresses in the inelastic domain or in the worse case, actual rail fracture. The general approach for calculating the allowable bending stress in the rail,  $\sigma_{all}$  (MPa) (Hay, 1953), relies on the application of various factors of **safety** as follows, consequently

$$\sigma_{all} = \frac{\sigma_{y} - \sigma_{t}}{(1+A)(1+B)(1+C)(1+D)}, \qquad (3.38)$$

= yield stress of the rail steel (MPa), where σv = temperature induced stress in the rail (MPa), σt = stress factor to account for lateral bending of the А rail, в = stress factor to account for track conditions, С = stress factor to account for rail wear and corrosion, and = stress factor to account for unbalanced superelevation D of track.

According to Magee (1965) the recommended values of these stress reduction factors can be explained as follows:

- (a) Lateral Bending: Due to the wheel loading having a horizontal component that produces bending of the rail in a horizontal plane, a lateral bending stress is produced in the base of the rail which is additive to the vertical bending stress on one side or the other. Examination was made of Talbot's reports and a value of 20 per cent was considered adequate for lateral bending at all speeds, (Talbot 1918-1934).
- (b) <u>Track Condition</u>: Due to the occurrence of mechanically worn, deteriorated, or low sleepers, increased rail bending stresses can be induced. The standard of maintenance or the attention which is given to proper sleeper support will of course determine the extent to which rail stresses will be

increased as a result of track condition. After examination of the Special Committee's reports (Talbot 1918-1934), a factor of 25 per cent was considered adequate to provide for the effect of track condition at all speeds in mainline track and 35 per cent in branch line track.

- (c) <u>Temperature Stresses</u>: In jointed rail track, the temperature stresses of concern will be the tensile stresses in winter due to joint bar resistance to rail slippage augmented to some extent by rail anchor restraints. Available data indicate that for main line track, temperature induced rail tensile stress will not exceed 69 MPa and for branch line track, 34.5 MPa. With continuous welded rail, an allowance of 138 MPa is recommended in order to account for the rail dropping to a temperature of 38°C below the laying temperature.
- (d) <u>Rail Wear</u>: On the outer rail of curves an allowance should be made for reduction in strength due to loss of area by wheel flange wear and corrosion. A study of the section modulus about the base of typical curve worn sections indicates that an allowance of 15 per cent is adequate.
- (e) <u>Unbalanced Elevation</u>: On curves, AREA recommendations limit the speed of operation to account for a probably 75 mm of unbalanced elevation. For a height of the centre of gravity of a vehicle of 2.13 m, this would result in an increase in wheel load on the outer rail of 15 per cent.

A comparison of other recommended values of the stress reduction factors is presented in Table 3.11. This table outlines the stress factors recommended by Hay (1953), Clarke (1957) and Magee (1965-1971). Magee includes the effects of both jointed and continuous welded track, whereas Hay and Clarke list an additional stress factor to account for the effects of the locomotive driving wheels. The additional rail stresses caused by the impact of rolling wheel flats are not considered. The calculation

of the temperature stress  $\sigma_t$  used in Equation 3.38 will be investigated in detail in a later section. It would appear from inspection that the majority of the stress safety factors are related to the maintenance condition of the track, and the magnitudes are based entirely upon the researchers' own judgement.

	Stress Factor	Researcher				
Symbol	Description	Hay Clarke	Magee			
σy σt	Rail Yield Strength (MPa) Temperature Stress (MPa)	$\begin{array}{ccc} 413 & 413 \\ _{48}(f) & _{48}(f) \end{array}$	$\begin{array}{c} 483 \\ 34.5 (a) \\ 69 (b) \\ 138 (c) \end{array}$			
А	Lateral Bending	15 per 15 per cent cent	20 per cent			
B	Track Condition	25 per 25 per cent cent	25 per <sup>(d)</sup> cent 35 pèr <sup>(e)</sup> cent			
С	Rail Wear and Corrosion	l0 per 10 per cent cent	15 per cent			
D	Unbalanced Superelevation	15-20 25 per cent cent	15 per cent			
	Locomotive	5 per 5 per cent cent	-			

TABLE 3.11 - CRITERIA FOR CALCULATING THE ALLOWABLE BENDING STRESS

Note: (a) Branch Line - Jointed Track

(b) Main Line - Jointed Track

(c) Continuous Welded Rail

(d) Main Line Track

(e) Branch Line Track

(f) Jointed Track.

Wheel flats are a form of wheel defect resulting from the wheels sliding along the rails during inefficient braking. These flats apply dangerous impacts on the rails, while the wheels roll along. Further dangers for the rail are due to rail or brake block material being built up on the wheel surface during slow slippage of wheels (material welded onto the wheel tread surface). The danger to which the rails and the track stability are exposed in this case, and which also affects the life of the wheel sets and the vehicles in general, has been considered so great, that a European agreement limits the versine of depth of wheel flats to l mm, and the flat length to not more than 85 mm (ORE 1965). However it is often noted in practice that the size of a wheel flat is considerably greater than the limits given above.

The AREA (1952) in association with the Association of American Railroads (AAR) have carried out experiments to determine the magnitude of the rail bending stresses generated by wheel flats.

The following general conclusions were drawn:

- . the rail stresses caused by wheel flats increase rapidly with speed reaching a maximum value at 30 km/h decrease to a minimum at 60 km/h and then begin to rise up again to 140 km/h, although not reaching the value at 30 km/h
- . the bending stress in the rail is proportional to the length of the wheel flat
- . the then current AAR maximum wheel flat length limit of 2.5 inches, 36 mm, did not produce excessive rail stress, but stresses were however 100 to 150 per cent greater than without the flat spot
- . rail stresses increased with increasing wheel load
- . the depth rather than the length of the wheel flat had a much greater bearing on the impact on the rail.

Schramm (1961) supports these views and states that the rail bending moment which is attributable to a wheel flat is largest at speeds of approximately 30 km/h and is always dependent upon the depth of the wheel flat. The rail bending moment  $M_{f}(kNm)$  attributable to a wheel flat can be calculated from the following equation

$$M_{f} = (15.4 + 0.11 P_{s}) d_{f}^{0.5}, \qquad (3.39)$$

where  $P_s$  = static wheel load in the range of 50 to 100 kN  $d_f$  = depth of wheel flat (mm); commonly occurring depths are 2 mm.

It is not readily apparent whether the effects of wheel flats are implied in the value of the impact factor used in the calculation of the design wheel load or in the locomotive factor in Table 3.11.

<u>The German Method</u>: At a specified constant tensile stress  $\sigma_t$ due to temperature and internal stresses, the allowable tensile stress  $\sigma_{all}$  based upon fatigue considerations can be determined for a particular rail steel material. Eisenmann (1969a,b,c) uses a fatigue diagram previously developed by Smith (1942) which reduces the allowable fatigue bending stress at the centre of the rail base as the apparent constant longitudinal rail stresses varies with fluctuations in the rail temperature. This is similar to the South-African approach of analysing rail fatigue and referred to by Westrail (1976).

Values of the allowable rail tensile stress developed for Central European conditions are listed in Table 3.12. The calculation of the constant rail temperature stress  $\sigma_t$  will be investigated in detail in a later section.

Rail Strength <sup>o</sup> ult <sup>(MPa)</sup>	Allowable fatigue bending stress at the centre of the rail base $\sigma_{all}$ (MPa)					
	Jointed Rail $\sigma_t = 78(MPa)$	$CWR^{(a)}\sigma_{t} = 176(MPa)$				
686	$0.40^{\sigma}$ ult = 274	$0.33\sigma_{\rm ult} = 225$				
882	$0.36^{\sigma}$ ult = 313	$0.31\sigma_{ult} = 274$				

TABLE 3.12 - ALLOWABLE FATIGUE BENDING STRESS AT THE CENTRE OF THE RAIL BASE (EISENMANN 1969b)

(a) This constant temperature induced rail stress relates to very low temperatures experienced under Central European conditions. For calculations of  $\sigma_{\pm}$  refer to p. 68 et seq.

Birmann (1968) consideres the effects of the lateral guiding forces on the magnitude of the tensile stresses in the rail base. According to laboratory tests and measurements in the track, the highest rail flange stress  $\sigma_R$  caused by vertical and simultaneously acting horizontal forces is approximately 1.4 to 1.6 times the stress at the centre of the rail flange,  $\sigma_b$ , under vertical wheel load alone. Taking into account the longitudinal rail stress arising from temperature fluctuations the maximum combined stress in the rail is limited by

$$1.6^{\sigma}_{b} + \sigma_{t} \leq \sigma_{all} = 0.9^{\sigma}_{y} , \qquad (3.40)$$

where

 $\sigma_{all}^{=}$  allowable tensile stress in the rail (MPa),  $\sigma_{y}^{=}$  yield stress of the particular rail material (MPa),

 $\sigma_{\rm b}$  = calculated rail stress at the centre of the rail base (MPa), and

 $\sigma_t = longitudinal stress in the rail arising from temper ature changes (MPa).$ 

Having determined that the allowable bending stress at the centre of the rail base is not exceeded for a particular rail section, the rail is further analysed for its capacity to withstand the expected lateral loads in service. CALCULATION OF THE RAIL BENDING STRESS AT THE LOWER EDGE OF THE RAIL HEAD

The magnitude of the rail bending stress at the lower edge of the rail head (Figure 3.8 Point B) is dependent upon the magnitude of the lateral guiding forces imposed on the rail head.

According to Navier's hypothesis, an additional tensile bending stress has to be added to the usual tensile bending stress calculated at the lower edge of the rail head as a result of the discrete non-uniform shape of the rail profile and due to the fact that the wheel force acts to some extent eccentrically and in the horizontal direction (Eisenmann 1970a).

Combining the applied rail head load due to the eccentric and skew wheel load application, (Figures 3.14), gives an equation for determining the stress at the lower edge of the rail head σ<sub>v</sub>, i.e.,

> $\sigma_{\kappa} = \sigma_{p} + \Delta \sigma_{1} + \Delta \sigma_{2,p} + \Delta \sigma_{2,H} + \sigma_{H},$ (3.41)

- where  $\sigma_{D}$  = stress due to vertical load applied centrally, Δσ<sub>1</sub> = additional bending stress caused by a centrally applied force due to the discontinuity effects at the rail web,
  - additional bending stress caused by the torsional <sup>Δσ</sup>2,Ρ moment due to the eccentricity of the axle load P,
  - additional bending stress caused by the torsional <sup>∆ σ</sup> 2,Н moment due to the eccentricity of the guiding force H,

σ bending stress due to the horizontal guiding force H = Н applied at the fulcrum.

The magnitudes of the terms  $\sigma_{\rm P}$ ,  $\Delta\sigma_1$ ,  $\Delta\sigma_2$ ,  ${\rm P}$ ,  $\Delta\sigma_2$ ,  ${\rm H}$ , and  $\sigma_{\rm H}$  are calculated in the following manner.

The stress at the level of the lower edge of the rail head  $\sigma_{\rm p}\,(MPa)$  due to an equivalent centrally located vertical design wheel load, P is



Note:  $M_T = H \cdot h - P \cdot e$ 

## Figure 3.14

Superposition of rail stresses caused by vertical and horizontal loads to obtain resultant rail stresses caused by a skewed load (ORE 1966)

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$$\sigma_{\rm P} = M_{\rm m} \cdot \lambda_{\rm l} \cdot 10^6, \qquad (3.42)$$

where  $M_m = \max \min rail bending moment (kNm) calculated from$ Equation 3.31, $<math>\lambda_1 = rail section parameter (mm^{-3}) = \frac{C_1}{I}$ ,  $C_1 = height of the position of the lower edge of the rail$ head above the neutral axis of the rail (mm), and $<math>I = moment of inertia of the rail (mm^4)$ .

The additional bending stress  $\sigma_1$  (MPa) caused by the disturbance of the rail section where the web meets the rail head edge is calculated from the equation

$$\Delta \sigma_1 = \lambda_2 P \cdot 10^3, \qquad (3.43)$$

where P = design wheel load (kN), and  $\lambda^2$  = rail section parameter (mm<sup>-2</sup>);

 $\lambda_2$  is given by

$$\lambda_{2} = 1.5 \left[ \frac{2.3 \ln(a_{2}/a_{1})}{3 b^{3}.a_{1}^{4}.d} \right]^{0.25}, \qquad (3.44)$$

where a<sub>1</sub> = average rail head height (mm), a<sub>2</sub> = height of rail head + web (mm), b = mean rail head width (mm), and d = web thickness (mm).

The additional bending stress  $\Delta \sigma_2$  (MPa) caused by the torsional moment, at the level of the lower edge of the rail is calculated from the equation

$$\Delta \sigma_2 = \lambda_3 M_{\rm T} \cdot 10^6, \qquad (3.45)$$

where  $M_T$  = applied torsional moment to the rail (kNm) (Figure 3.14c)

= H.h - P.e,

H = lateral guiding force (kN),

P = design wheel load (kN),

- h = distance between the position of the loading point
   of the lateral force and the fulcrum (m),
- e = eccentricity of the vertical force (m), and
- $\lambda_3$  = rail section parameter (mm<sup>-3</sup>) (the theoretical solution is given by Timoshenko and Langer 1932.)

For any rail section the applied torsional moment  $M_{\rm T}$  will be transmitted partially in the form of simple twist and partially by bending of the head and base of the rail, thus

$$M_{T} = M_1 + M_2$$

where  $M_1$ 

From these considerations the value of the rail section parameter  $\lambda_3$  can be calculated neglecting the effect of the rail web, by

$$\lambda_{3} = \frac{E \cdot h}{2 \cdot GI_{p}} \cdot \frac{\alpha \left(1 - e^{\alpha S}\right)}{1 + e^{\alpha S}} \cdot \frac{b}{2} , \qquad (3.46)$$

where

е

e E = Young's Modulus of the rail steel (MPa),

- G = Shear Modulus of the rail steel (MPa),
  - natural log base,
- b = mean width of rail head (mm),
- S = distance between the rail supports (mm) (i.e. the sleeper spacing)

$$I_p = polar moment of inertia of the rail (mm4),
 $I_p = I_{p(bead)} + I_{p(web)} + I_{p(flange)}$ , and$$

$$\alpha = \begin{bmatrix} G \cdot I_{p} \\ E \cdot I_{H} (h_{H} + h_{F}) \cdot h_{H} \end{bmatrix}^{0.5}$$
(3.47)

The distances  $h_{\rm H}(\rm mm)$  and  $h_{\rm F}(\rm mm)$  refer to the distances from the neutral axis of the centroid of the rail head and rail flange respectively, and are defined by

$$h_{H} = h_{t} \frac{I_{F}}{I_{H} + I_{F}}, \text{ and} \qquad (3.48)$$

$$h_{F} = h_{t} \frac{I_{H}}{I_{H} + I_{F}}, \qquad (3.49)$$

where  $I_H = horizontal moment of inertia of the rail head (mm<sup>4</sup>)$  $<math>I_F = horizontal moment of inertia of the rail flange$ (mm<sup>4</sup>) $<math>h_t = distance between the rail head and rail flange$ centroids (mm), $<math>= h_H + h_F$ . (3.50)

The bending stress  $\sigma_{\rm H}\,(MPa)$  at the level of the lower edge of the rail head due to the lateral guiding force H(kN) applied at the fulcrum is

$$\sigma_{\rm H} = \lambda_4 \cdot {\rm H.10}^3,$$

where  $\lambda_4$  = rail section parameter (mm<sup>-2</sup>);

 $\lambda_{\mathbf{A}}$  is determined by

$$\lambda_4 = \frac{I_H}{I_H + I_F} \cdot \frac{S}{4Z_h}$$
(3.51)

where  $z_h$  = horizontal bending resistance moment of the rail head, and is defined by

$$z_h = \frac{2 \cdot I_H}{b} (mm^3)$$

and  $I_{H}$ ,  $I_{F}$  S, and b are as previously defined.

Calculated values of the rail section parameters  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ , and  $\lambda_4$  for common AREA and German rail profiles are presented in Table 3.13. The theoretically calculated value of the stress at the lower edge of the rail head is generally 10 per cent higher than the experimentally measured value.

Rail Section	AREA 115RE	AREA 132RE	AREA AREA 132RE 140RE		S54	S64	
Rail Weight	(kg/m)	57.5	66	70	49	54	64
	(lb/yd)	115	132	140	98	108	128
Moment of Ir	nertia						
I	(10 <sup>6</sup> mm <sup>4</sup> )	27.20	35.80	40.88	18.9	20.73	32.52
Section Modurail base	lus about						
<sup>z</sup> o	(10 <sup>3</sup> mm <sup>3</sup> )	369	441	474	240	262	356
λl	$(10^{-6} \text{mm}^{-3})$	2.2	1.8	1.5	2.0	1.7	1.5
<sup>λ</sup> 2	$(10^{-3} \text{mm}^{-2})$	0.655	0.612	0.519	0.540	0.490	0.480
<sup>λ</sup> 3	$(10^{-6} \text{mm}^{-3})$	19.7	17.9	14.9	16.0	14.0	13.0
<sup>λ</sup> 4	$(10^{-3} \text{mm}^{-2})$	1.30	1.06	1.00	1.60	1.40	1.00
Distance fro upper side o	om fulcrum of rail head	to d	- <u>-</u>		· ·		
h	(mm)	130	140	145	99	102	124

TABLE 3.13 - PARAMETERS TO CALCULATE THE ADDITIONAL BENDING STRESS ON THE LOWER EDGE OF THE RAIL HEAD (EISENMANN 1970)

NOTE: Calculated values are 10 per cent higher than experimental.

Eisenmann (1970a) states that the flexural stresses in the rail head reach a considerable value only for comparatively high lateral forces. The experience gained from numerous in-track measurements has shown that locomotives initiate the higher values. The wheel sets of locomotives with a wheel load of 110 kN exert lateral forces of about 44 kN to 59 kN, while freight cars with a wheel load of 90 kN deliver only 15 kN, assuming in both cases a degree of curvature of 6 degrees. (i.e. curves with a radius of approximately 300 m.) The number of repetitions of excessive stresses occurring at the lower edges of the rail head are small compared with the foot of the rail, and therefore a fatigue failure is not expected to occur at the lower edge of the rail head. The tensile bending stress  $\sigma_{\kappa}$  occurring at the lower edge of the rail head in the case of German rail profiles S54, UIC 54 and UIC 60 under the simultaneous application of vertical and horizontal loads has been calculated using the above analysis. Determining the rail bending stress at the centre of the rail flange  $\sigma_{\rm b}$  (Equation 3.33) calculated for various centric wheel loads and values of track modulus, a ratio  $\sigma_{\kappa}/\sigma_{\rm b}$  can be plotted against various values of the vertical wheel load, the lateral guiding force and track modulus and is presented in Figure 3.15. It is readily apparent that for certain combinations of high lateral guiding force and medium to high wheel loads, the magnitude of the tensile bending stress occurring at the lower edge of the railhead can be one to two times greater than the rail bending stress calculated at the centre of the rail flange.

#### CALCULATION OF THE VERTICAL DEFLECTION OF THE RAIL

Having determined that for the design lateral loading the rail section under consideration satisfies the permissible limit of stress at the lower edge of the rail head, the rail is further analysed for its deflection performance under load. Using the beam on an elastic foundation analysis under the action of a single wheel load the rail deflection y (mm) at a distance of x from the load point is



## Figure 3.15

Ratio between the calculated bending stresses in rail head and rail flange of  $O_K/O_b$  (ORE 1966)

$$y_{x} = \frac{P_{\cdot\beta} \cdot e^{-\beta}}{2k} (\cos\beta x + \sin\beta x), \qquad (3.52)$$

where P = single wheel load (kN),  $\beta = \left[\frac{k}{4EI}\right]^{0.25},$  k = track modulus (MPa), E = Young's Modulus of the rail (MPa), and  $I = \text{rail moment of inertia (mm}^4).$ 

Similar to the case of the calculation of the rail bending moment due to a single point load, a master diagram (Hay, 1953) has been developed which relates the vertical deflection of the rail under the load point  $y_0(mm)$  to the vertical deflection of the rail at any other location (Figure 3.12). For a single isolated load the maximum vertical rail deflection occurs directly beneath the load point and can be expressed by

$$y_0 = \frac{P_{\cdot\beta}}{2.k}$$
 (3.53)

The distance  $x_2(m)$  is that distance to the position of rail contraflexure from the point of load application and is given by

$$x_2 = 3x_1,$$
 (3.54)

where  $x_1$  is defined in Equation 3.34.

For the actual track loading condition the vertical rail deflection at any position along the rail is the sum of the vertical rail deflections caused by the interaction of wheel loads about this point. The total length of this zone of interaction is approximately a distance  $6x_1$  to the left and right of the load point, for the case of a single point load (Figure 3.12). Using the principle of superposition the maximum vertical deflection of the rail  $y_m$  (mm) under a particular wheel load, including the vertical deflections due to the interaction of adjacent wheels is

$$y_{\rm m} = \sum_{i=0}^{i=n} P_{\rm x_i} B_{\rm x_i'}$$
(3.55)

where

í

= 0, 1, 2 ..., is the number of adjacent wheels in the interaction length commencing at the reference wheel (Figure 3.12),

X<sub>i</sub> = distance to the adjacent wheels from the reference
 wheel<sup>(1)</sup>,

 $B_{X_{i}}$  = expression for calculating the vertical deflection of the rail coefficient for any location using the beam on an elastic foundation analysis<sup>(2)</sup>.

#### Allowable vertical deflection of the rail

With most typical track structures, the rail acts as a continuous beam on an elastic foundation. The resulting deflection of the rail caused by the rolling wheel creates a "wave motion" in front and behind the wheel. The rail being fastened to the sleepers which are embedded in ballast, and restrained longitudinally either by rail anchors or by elastic fasteners, is thereby prevented from sliding about the track. The track therefore, resists the motions of the wave, and although vertical motion is observed, the longitudinal movement is counteracted by resistive forces of the track itself.

The value of the theoretical rail deflection is, for a given rail size and axle loading, dependent upon the assumed track modulus k under one rail and the spacing between adjacent axles. It can clearly be seen that care must be exercised by the designer in the selection of the assumed value of the track modulus if realistic values of rail deflection are to be determined.

(1) Wheels at the distances 
$$X_i < 6X_1$$
 are not included, for the reference wheel  $\chi_0 = 0$ .  
(2)  $B_{X_i} = \frac{\beta \cdot e^{-\beta X_i}}{2k}$  (cos  $\beta \cdot X_i + \sin \beta \cdot X_i$ ).

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For typical track structures designed with light to medium rails, the AREA has recommended that the value of track modulus to be used when determining the rail deflection, be 13.8 MPa (2000 psi).

Theoretically the value of the track modulus cannot be assumed to be proportional to the rail size because it only relates to the support given to the rail by the sleepers (including rail pads and fasteners), the ballast and the subgrade.

Larger rails are designed predominantly to carry heavier loads, which consequently demand the adoption of a heavier track structure (i.e. a reduced sleeper spacing in conjunction with a probable increase in the ballast depth). It is the combination of these factors in conjunction with a reduction in the load distribution due to a stiffer beam, that lead to larger values of track modulus when the rail size is increased.

It would appear that the AREA (1973) manual recommendation 1975.22.3.15 of a maximum allowable vertical deflection of 6.35 mm (0.25 in) is based on a track modulus value of 13.8 MPa. Lundgren et al. (1970) has incorporated this recommendation in his rail deflection limits which are based upon the capability of the track to carry out its design task (Figure 3.16).

#### LONGITUDINAL TEMPERATURE STRESSES INDUCED IN THE RAIL

In addition to longitudinal stresses created by the wave action, there are two other longitudinal stresses of equal or greater magnitude which affect the rail design. The first of these is due to the forces exerted by the wheel of the locomotive when either accelerating or braking. This is a complex condition which is theoretically highly indeterminate and has not been measured in-track successfully. The second of these is due to thermal expansion and contraction caused by changes in the rail temperature from that experienced when initially laying the



Legend: A Deflection range for track which will last indefinitely.

1

- B Normal maximum desirable deflection for heavy track to give requisite combination of flexibility and stiffness.
- C Limit of desirable deflection for track of light construction ( $\leq 50 \text{ kg/m}$ ).
- D Weak or poorly maintained track which will deteriorate quickly.
- Note: Values of deflection are exclusive of any looseness or play between rail and plate or plate and sleeper and represent deflections under load.

**Figure 3.16** Track deflection criteria for durability (Lundgren et al 1970) track. The importance of knowing the magnitude of the longitudinal temperature stresses in the rail is that these stresses significantly alter the allowable rail bending stress.

Current American practice is to construct track made from flash butt welded rail of lengths up to 490 m. This is laid on timber sleepers fitted with steel bearing plates. The lengths of rail are joined either by thermit type welds or by adhesive bonding and are held down to the sleeper by the use of dog The longitudinal action of the track is resisted by the spikes. rail anchors which are boxed on either side of the rail. Patterns of anchorage vary, but usually every other sleeper is boxed and in long stretches between rail joints every third sleeper is box anchored. At breaks in the rail such as insulated joints all sleepers are box anchored from three to six rail lengths both sides of the break. The structure works on the basis that the longitudinal force is transferred from the rail to the anchor and the ballast resists the load distributed from the sleeper. Vertical rail uplift caused by wave motion is not constrained, the dog spike being incompletely driven home so as to allow the rail to "breathe" upward before and after the wheel. This leads to a rather loose arrangement where all visible movement is done by the rail.

#### The AREA method for calculating the longitudinal stresses

The general approach suggested by the AREA (Robnett et al. 1975) and as outlined in Table 3.11 is to adopt the value of the increase in the rail stress due to temperature changes as one of the following values:

σt	=	138 MPa :	CWR track
σt	=	69 MPa :	jointed mainline track
σ+	=	34.5 MPa:	jointed branch line-track.

The maximum end movement of the rail  $\delta_r(mm)$  can be calculated from the formula (Eisenmann 1970a)

$$\delta_{r} = \frac{(F - J_{r})^{2} s \, 10^{3}}{2 R_{o} A_{r} E}, \qquad (3.57)$$

where F

e F = total force required to fully restrain the rail against any rail movement due to a temperature variation from the rail temperature at laying

$$= A_r(t_1 - t_0) \alpha_t E.10^{-3} (kN),$$

 $A_{r} = \text{cross sectional area of the rail (mm<sup>2</sup>),}$   $\alpha_{t} = \text{coefficient of thermal expansion of rail steel}$   $= 1.15 \times 10^{-5} (^{\circ}\text{C}^{-1}),$ E = Young's Modulus of the rail steel (MPa),

 $t_{o}$  = base laying temperature of the rail (<sup>O</sup>C),

 $J_r = rail joint restraint force (assume <math>J_r = 0$  for track with dog spike rail fasteners),

R<sub>o</sub> = average sleeper resistance to rail longitudinal movement (kN/Sleeper/Rail).

The value of the maximum end movement of the rail r(max) is usually assumed as 9.5(mm) and Equation (3.56) is rearranged to solve for the minimum longitudinal rail restraint  $R_{o(min)}$  (kN/ sleeper/rail) required by the rail fastener, i.e.,

$$R_{o(\min)} = \frac{F^2 S. 10^3}{2^{\delta} r (\max)^A r^E}.$$
 (3.58)

The restraint provided between the rail and the sleeper, however, need not exceed the ability of the ballast section to restrain movement of sleepers longitudinally in the ballast. For gravel ballast, the restraining force provided by the ballast to the sleeper does not exceed 5.4 kN (AREA 1975).

#### The German Method for calculating the longitudinal stresses

When considering jointed rail track, Schramm (1961) makes a distinction between the occurrence of a "short rail" and a "long rail". The former being defined for the conditions where the rail joint gaps are not closed when the upper temperature limits are reached and the maximum gap width is not achieved at the lower temperature limit. Whereas the latter is defined for the conditions where the rail joint gaps are closed when reaching the upper temperature limit, or the maximum gap width is reached at the lower temperature limit. If the rails are prevented from moving lengthwise when they approach the limits of the temperature range then a condition similar to continuously welded rail is reached.

The following formulae have been suggested by Schramm for use in the calculation of the increase in longitudinal rail stress caused by temperature changes:

(a) <u>CWR Track</u>: The increase in the rail stress  $\sigma_t$  (MPa) due to a temperature variation from the temperature at laying can be calculated by

$$\sigma_{t} = (t_{1} - t_{0}) \alpha_{t} E, \qquad (3.59)$$

where	tı	=	the upper temperature limit of the rail ( $^{\circ}$ C),
	t	=	the base temperature of the rail ( $^{ m O}$ C),
	α+	=	coefficient of thermal expansion $(^{\circ}C^{-1})$ , and
	E	=	Young's Modulus of the rail steel (MPa).

This maximum rail stress occurs in the central portion of CWR and is constant for any particular temperature differential.

(b) Jointed Rail Track: For the case of jointed rail the longitudinal rail force can be transferred from rail to rail via friction between the fish plates. According to Schramm the mean force which can be developed at the fishplates is of the order of 50 kN. The length of rail over which the temperature stress increases to a maximum value is also called the breathing length. Over this length the cumulative value of rail end expansion or contraction is said to occur.

(c) "Short Rail" Track: Such track is characterised by the fact that, within the maximum temperature range the rail temperature at which the rail gap closes  $t_g$  exceeds the upper operating rail temperature  $t_1$ ,  $(t_g > t_1)$ . For this condition the maximum longitudinal rail stress  $\sigma_t$  (MPa) occurs at the centre of the rail length, i.e.,

$$\sigma_{t} = \frac{2 F_{0} + W_{0} \ell_{r}}{2.10^{3} A_{r}}, \qquad (3.60)$$

- where
- $F_0 =$ force transmitted from rail to rail via friction between fishplates (kN),
  - W<sub>o</sub> = track resistance to longitudinal movement of the rail per length of rail (kN/m), (depends upon the fastener rigidity),
  - $l_r = length of jointed rail between adjacent fishplates (m), and$

 $A_r$  = cross sectional area of the rail (mm<sup>2</sup>).

Schramm states that under German conditions the case of "short rail" track is only academic as in actual practice the jointed rail track always behaves as a "long rail" track.

(d) "Long Rail" Track: This track is characterised by the fact that within the upper temperature range the rail temperature at which the rail gap closes  $t_g$  is less than or equal to the upper operating rail temperature  $t_1$ ,  $(t_g \leq t_1)$ . For this condition the maximum compressive longitudinal rail stress  $\sigma_t$  (MPa) occurs at the centre of the rail length and is independent of the fishplate friction  $F_0$ . Consequently,

$$\sigma_{t} = \frac{\ell_{r} W_{o}}{4 \cdot 10^{3} A_{r}} + (t_{1} - t_{o}) \alpha_{t} E - \frac{\ell_{g} E}{\ell_{r}}, \qquad (3.61)$$

where 
$$\ell_g^{\cdot}$$
 = rail joint gap (mm), and  
 $\alpha_t$ ,  $t_0$ ,  $t_1$ , E,  $W_0$  and  $\ell_r$  are as previously defined in  
Equations 3.59 and 3.60.

Values of  $l_g$  and  $W_o$  for German operating conditions are presented in Tables 3.14 and 3.15.

## TABLE 3.14 - RAIL JOINT GAP LENGTHS FOR PARTICULAR LENGTHS OF RAIL, GERMAN CONDITIONS (SCHRAMM 1961)

Rail Length	Rail Joint Gap Length				
<sup>ℓ</sup> r <sup>(m)</sup>	l (mm)				
15	5				
30	9				
45	11				
60	14				

## TABLE 3.15 - TYPICAL VALUES FOR THE TRACK RESISTANCE TO LONGITUDINAL <u>MOVEMENT OF THE RAIL</u>, W<sub>O</sub> <u>PER (METRE) OF RAIL LENGTH</u> FOR VARIOUS SLEEPER TYPES (SCRAMM 1961)

Sleeper Type	W_ (kN/m)	
	0	
Timber	3.9	
Concrete	4.9	
Steel	5.9	

For "long rail" track the "breathing length",  $\imath_{\rm b}^{}({\rm m})$  can be calculated from

$$k_{\rm b} = \left[\frac{{\rm E} {\rm A}_{\rm r} {\rm g}}{2.10^6 {\rm W}_{\rm Q}}\right]^{0.5} .$$
(3.62)

If the length of the breathing  $\ell_{\rm b}$  becomes smaller than  $\ell_{\rm r}/2$  then no longitudinal movement can take place at the centre of the rail; and the characteristics of CWR are exhibited. This occurs only if the length of the joint gap  $\ell_{\rm cr}$  is less than

$$\frac{k_{\rm r}^{2} W_{\rm o} \cdot 10^{3}}{2 E A_{\rm r}}$$
 (3.63)

Therefore Equation 3.61 is only valid for rail joint gap lengths, at the time of laying where  $\ell_{g}$  is greater than

$$\frac{{}^{2}r_{0}^{2W_{0}.10^{3}}}{{}^{2EA}r}$$
(3.64)

#### WHEEL TO RAIL CONTACT STRESS CONSIDERATIONS

High values of contact stress and shear stress are generated at the wheel/rail contact zone (Figure 3.8, point C), due to heavy vehicle wheel loads. Because of the ever changing demands of railway freight operations, it is important to determine the maximum wheel loads that a particular rail (and or track) can carry before irreversible rail head damage occurs. Therefore the calculation of the applied force level at which rail head shelling and problems of a similar nature will occur is of fundamental importance. The following will review methods that have been developed to place limits upon the vehicle wheel loads. These include the simple P/D ratio, applied Hertzian theory and shakedown considerations.

#### The P/D ratio

The ratio of the static gross wheel load P to the wheel diameter D, i.e. P/D, can be used as an indication of the magnitude of the contact pressure occurring between the rail head and the wheel. With current railway operations certain P/D limits have proven satisfactory.

In 1958 British Rail limited the ratio of the wheel load P (tonnes) to wheel diameter D(m) to 9.2 t/m (90.3 kN/m). In 1963 this value was increased to 12.5 t/m (122.6 kN/m) for trailing wheels and 10.8 t/m (105.9 kN/m) for driving wheels (Koffmann and Fairweather 1975).

Birman, quoted in the ISCOR<sup>(1)</sup> Report (Taute et al. 1971), states that for a rail steel with an ultimate tensile strength  $\sigma_{ult} = 90 \text{ kg/mm}^2$  883 MPa wheels with P/D ratios of ll-l2(t/m) (107.9-l17 kN/m) caused no contact pressure problems between the rail and the wheel. Whereas wheels with P/D ratios of l4-l7 t/m, (137.3-l66.8 kN) caused considerable damage to the rail head and spalling occurred in the wheels. American practice is to establish general limits for the maximum wheel load P(t). Given (Koffmann and Fairweather 1975)

$$P = 33(D)^{0.5} - 18.4 \tag{3.65}$$

the equation for the P/D ratio becomes,

$$P/D = \frac{33}{D^{0.5}} - \frac{18.4}{D}$$
 (3.66)

P/D ratios of 16 - 18.5 t/m (157 - 181.5 kNm) are attained in service. The P/D ratios currently used by various railway operators and the P/D ratios recommended by various railway organisations are presented in Table 3.16. It can clearly be seen that the Australian mining company railway operators base their design wheel load upon North American practice.

Although the simple P/D ratio provides a convenient indicator of whether contact stress problems are likely to occur, it oversimplifies the situation and should only be used to determine whether more detailed analysis is required. The limits set by British Rail (Koffmann and Fairweather 1975) or by Birmann (Taute et al. 1971) provide conservative levels for this purpose.

(1) ISCOR: South African Iron & Steel Corporation.

TABLE	3.16	-	P/D	RATIOS	CUR	RENTLY	USED	BY	RAILWAYS	OPERA	FORS	AND
			RECO	OMMENDEI	) BY	RAILWA	AY OR	GAN:	ISATIONS	(TAUTE	et	al.
			1971	L, READ	197	2)						

Railway Organisation or Operator	P/D Value			
	(tonnes/m)	(kN/m)		
British Railways (BR)	10.8 - 12.5	105.9 - 122.6		
French Railways (SNCF)	9.0	88.3		
Japanese Railways (JNR)	9.2 - 10.0	90.3 - 98.1		
South African Railways (SAR)	11.4	111.8		
Australian State Railways	9.2 - 10.8	90.3 - 105.9		
AAR Recommendation	14.7	144.2		
Hamersly Iron Railways	16.3	159.9		
Quebec Cartier	17.1	167.8		
Lamco JV	18.3	179.5		
Quebec-North Shore-Labrador	16.9	165.8		

# The calculation of the wheel/rail contact stresses and the maximum allowable shear stress in the rail head

An examination of the stress components in the rail head shows that in the immediate vicinity of the contact point between the wheel and the rail the principal stresses are very high (Eisenmann 1970) (Figure 3.17), with the following assumed stress condition occurring:

 $\sigma_1 \simeq \sigma_2 \simeq \sigma_3 \tag{3.67}$ 

Although the extreme values of the compressive stresses exceed the ultimate tensile stress of the steel, no failure occurs because the shear stresses vanish at the surface, i.e.,

$$\tau_1 = \tau_2 = \tau_3 = 0.0 \tag{3.68}$$

The compressive stresses lead to plastification and thus to hardening of the steel in the top zones of the rail head. With increasing depth below the rail surface the major principal stress  $\sigma_1$  in the direction of the applied load decreases slowly whilst the minor principal stresses  $\sigma_2$ ,  $\sigma_3$  decrease very rapidly. As a result of this principal stress differential with increasing rail head depth a maximum value of shear stress, amounting to approximately 30 per cent of the specific compressive contact stress  $\sigma_c$  at the surface, occurs at a depth corresponding to half the contact length (Figure 3.17).

Eisenmann (1970a) in an analysis of contact and shear stress levels assumes that the contact pressure is uniformly distributed over the contact area and that the rail and the wheel can be represented by a plane and a cylinder. The contact length 2a(mm) is derived from Hertz's formula and can be calculated by

$$2a = 3.04 \left[ \frac{PR.10}{2bE}^3 \right]^{0.5}, \qquad (3.69)$$

where P = vertical wheel load (kN), R = wheel radius (mm), E = Young's Modulus of rail steel (MPa), and 2b = breadth of contact area (mm); Eisenmann adopts the value of 2b = 12 mm.

Due to the compressive stress being assumed to be uniformly distributed over the contact surface, the contact pressure  $\sigma_{\alpha}$  (MPa) can be calculated by

σ

$$c = \frac{P \cdot 10^{3}}{2a \ 2b}$$
(3.70)

When the ratio of the contact pressure,  $\sigma_{\rm C}$  to the rail yield strength,  $\sigma_{\rm Y}$  is such that  $\sigma_{\rm C}/\sigma_{\rm Y}$  < 2.0 the useful life of the rail will be limited only by wear (Taute et al. 1971). Neglecting the effects of work hardening, total plasticity at the contact surface occurs when  $\sigma_{\rm C}/\sigma_{\rm Y} \ge 3.0$ . From fatigue considerations the allowable contact pressure  $\sigma_{\rm C}$  (MPa) according to Eisenmann (1970a) is



## Figure 3.17

Half space with a strip load and the resulting stress distribution with depth (Eisenmann 1970a)

$$\sigma_{all} = 0.5 \sigma_{ult'}$$
(3.71)

where  $\sigma_{ult}$  = ultimate tensile strength of rail (MPa).

As previously mentioned the maximum value of the shear stress,  $\tau$ , is about 30 per cent of the contact stress  $\sigma_{\rm C}$  at the contact surface, and this value occurs at a depth of a (half the length of the contact surface). Thus the maximum shear stress in the rail head  $\tau_{\rm max}$  (MPa) can be calculated by

$$\tau_{\rm max} = 0.3\sigma_{\rm c}$$
 (3.72)

The hypothesis of shear strain energy applied for the condition of two principal compressive stresses  $\sigma_1$  and  $\sigma_3$  gives for the relation between the allowable shear stress  $\tau_{all}$  and the allowable normal stress  $\sigma_{c_{all}}$ 

$$\tau_{all} = \frac{1}{\sqrt{3}} \sigma_{c_{all}} ,$$
 (3.73)

Substituting Equation 3.71 into Equation 3.73 yields the maximum allowable shear stress in the rail head  $\tau_{all}$  (MPa) from fatigue considerations; where

$$\tau_{all} = 0.3 \sigma_{ult}$$
 (3.74)

A special type of fatigue failure, called shelling, may occur if the rail head shear stress exceeds its allowable value. This failure starts at rail head depths of 5 - 7.5 mm, where the shear stress reaches a maximum. The rail yeild stress can be exceeded by the occasional occurrence of extremely high wheel loads and the resulting shear stress moves the material into the elastoplastic domain.

Combining Equations 3.69, 3.70 and 3.72 and assuming that the breadth of the contact surface, 2b equals 12 mm,

$$\tau_{\max} = 410 \left[\frac{P}{R}\right]^{0.5}$$

can be derived (Eisenmann 1970a),

where R = wheel radius (mm), and P = wheel load (kN).

The results of this formula for wheel radii ranging from 300-600 mm (12-24 in) are sufficiently accurate compared with those for an elliptical contact surface region. The maximum shear stress in the rail head as a function of wheel radius and wheel load has been plotted in Figure 3.18. Allowable limits of the maximum shear stress in the rail head for various rail strengths are also shown.

The problem of excessive maximum shear stress in the rail head can be solved by:

- . reducing the wheel load
- . increasing the wheel radius
- . increasing the yield strength of the rail,  $\sigma_{\rm y}$  .

Once the shear strength of the rail steel, due to local contact stress, is exceeded the effect of an increase in the rail mass to prevent yielding from occurring will be of little value. Rail quality, axle load and wheel diameter must therefore be mutually adapted, and the rails should not just be regarded as carriers.

The results of Eisenmann's analysis may be adopted as a more sophisticated limiting criterion than the simple P/D ratio. The recommended limits draw on the experience of the D.B. with their wheel and rail profile combinations and are recommended to avoid the occurrence of shelling of the rail head. At higher vertical loads, or in curves where high shear forces occur at the wheel/ rail contact interface, gross plastic flow may be evident and further restrictions need to be placed on load levels.

(3.75)





## Figure 3.18

Maximum shear stress in the railhead as a function of wheel radius and wheel load (Eisenmann 1970a)

#### Yield and shakedown in plain-strain rolling contact

The effective loading on the interface between the wheel and the rail head is both normal, due to vertical wheel loading, and tangential, due to driving and braking tractions. Desirable wheel loadings when transmitted to the rail head through the elliptical contact area, (applicable to rails with radiused rail heads) frequently exceed the yield limit of the contacting materials. Hence, the resulting surface plastic flow coupled with wear processes acts to flatten out the contact area. The contact surface can be approximated as bounded by a rectangle of length 2a and breadth w based upon the assumption of contact between a plane (rail) and a cylinder (wheel). The length, 2a (mm) can be calculated from Hertz's formula, (Smith and Liu 1953) i.e.,

$$2a = 3.19 \frac{P(1 - v^2)R}{wE}, \qquad (3.76)$$

where P = wheel load (kN), R = wheel radius (mm) w = contact width (mm), v,E = Poisson's Ratio and Young's Modulus (0.3 and 207 GPa for steel);

with the breadth w treated as a statistical variable, assumed to be normally distributed, which depends on the conditions of wear of the wheel and rail.

The corresponding maximum normal stress  $\sigma_{app}$  (MPa) on the contact surface is (Mair 1974) (Figure 3.19)

$$\sigma_{\rm app} = \frac{2P}{\pi a \ w \ 10^3}$$
 (3.77)

It is usually assumed that the tangential traction q (MPa) is directly related to the normal Herzian pressure at all points on the contact surface, with a maximum value

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Figure 3.19(a) Schematic contact area for new wheel and rail



Figure 3.19(b) Contact stress at shakedown

Contact surface between rail and wheel (Mair 1974)

$$q_{max} = \mu \sigma_{app}$$

where  $\mu$  = coefficient of contact friction.

In the absence of residual stresses the limiting elastic loading may be associated with either the maximum principal shear stress (Tresca) or maximum octahedral shear stress (von Mises) yield criteria (Mair 1974). Mathematically, these state that yield will occur when, either

$$\tau_{\text{max}} = \frac{1}{2}(\sigma_1 - \sigma_3) = k \text{ (Tresca } k = \sigma_{\chi}/2)$$
 (3.79)

or

$$\pi_{\max} = \frac{1}{3} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{0.5} = k,$$
(von Mises  $k = \sqrt{\sqrt{3}}$ ) (3.80)

Extension of the rolling contact elastic analysis to the postyield condition has been carried out by Johnson and Jefferies (1963) for an elastic-plastic material. Beyond initial yield the maximum contact load which can be sustained by a body in rolling contact, with limited plastic deformation, is known as the shakedown limit. Adopting the von Mises yield criterion, the effect of tangential force on the limit of elastic behaviour and on the shakedown limit is shown in Figure 3.20. For a prescribed tangential force/normal force ratio the limiting value of normal force, or alternatively the material strength to sustain a known normal stress,  $\sigma_{all}$ , is obtain from the vertical axis.

Since rail defects due to gross plastic deformation are the result of cumulative damage, it is not necessary to preclude the exceedance of the shakedown limit by the applied stress, but it is necessary to ensure that the frequency with which it occurs is sufficiently small such that rail replacement takes place for other reasons prior to the development of an unacceptable degree

of deformation. Diagrammatically, this means that the overlap between the distributions of applied contact stress,  $\sigma_{app}$ , and the maximum permissible contact stress  $\sigma_{all}$ , has to be limited to a predetermined level (Figure 3.21). Expressed mathematically, it is required to determine the reliability, R, against yield such that the contact stress between the wheel and rail will not exceed the shakedown limit of the material, where (Mair and Groenhout 1975, 1978)

$$R = \text{prob} (\sigma_{\text{all}} > \sigma_{\text{app}}). \tag{3.81}$$

However, the analysis is more readily accomplished by calculating the probability of yield, Q, (represented by the region of overlap) given by

$$Q = 1 - R$$
 (3.82)

= prob  $(\sigma_{all} \leq \sigma_{app})$ 

$$= \int_{-\infty}^{\infty} \operatorname{prob} (\sigma_{all} \leq z) \operatorname{prob} (z \leq \sigma_{app} < z + dz) dz$$
$$= \int_{-\infty}^{\infty} F_{all}(z) f_{app}(z) dz, \qquad (3.83)$$

Mair and Groenhout (1975) take the reliability parameter to be a function of the following variables each of which is assumed to be a random variable with a normal distribution:

- . the rail material yield strength
- . the vertical wheel load
- the contact area
- . the tangential force on the rail head.



### Figure 3.20

Yield and shakedown limits in plane strain with combined normal and tangential loading (Johnson and Jefferies 1963)



## Figure 3.21

Schematic representation of interaction between applied and allowable contact stress values (Mair and Groenhout 1975)
To obtain an acceptable value of reliability, R<sub>o</sub>, the parameters relating to tangent track were substituted into Equation 3.83, this approach being based upon the observation that gross plastic flow (e.g. corrugations) is almost entirely limited to curved track. In particular, the tengential force in tangent track was assumed (conservatively) to be zero and the yield stress of standard carbon rail steel was used.

On the above basis it can be shown that

$$F_{all}(z) = F_{u} \left[ \frac{zm_{\mu} - m_{\gamma}}{\sqrt{s_{\gamma}^{2} + z^{2}s_{\mu}^{2}}} \right]$$
 (3.84)

and

$$f_{app}(z) = \frac{2z(\lambda^2 m_w s_p^2 + \lambda m_p s_w^2 z^2)}{\sqrt{2(\lambda^2 s_p^2 + s_w^2 z^4)^3}} \exp\left[\frac{-(\lambda m_p - m_w z^2)^2}{2(\lambda^2 s_p^2 + s_w^2 z^4)}\right]$$
(3.85)

where	Fu	=	the cumulative distribution function of the standard normal variable with mean zero and unit standard deviation,
	m	=	mean of subscripted variable,
	S	=	standard deviation of subscripted variable,
	Υľ	=	2.31 Y,
	μ'	=	1/(1-0),
	Y	=	material yield stress $(\Xi^_{\mathbf{y}})$ ,
	μ	=	wheel/rail coefficient of contact friction,
	Р	=	vertical wheel load,
	w	=	wheel/rail contact width,
	λ	=	$0.16E/(1-v^2)R$ ,
	R	=	wheel radius, and
	υ <b>,</b> Ε	=	Poisson's ratio and Young's modulus.

In most railway applications the operating parameters have been set and the available means of adjustment to overcome rail deformation problems are usually limited to the replacement of the rail steel. On this basis Figure 3.22 draws on the experience



Note: Reliability is 92.0%

### Figure 3.22

Rail strength requirements to avoid corrugation

of the Mt Newman Mining Co to establish recommended mean 0.2 per cent stress levels for systems having alternative combinations of mean wheel load distribution and wheel diameter (Mair, Jupp and Groenhout 1978). The data presented can be used to prevent gross plastic flow (leading to corrugation). However, it cannot be presumed that adoption of the above strength levels will compensate for poor standards of track maintenance nor the presence of excessive vertical joint misalignment, both of which will contribute to more severe conditions than those present on the Mt Newman System.

The correspondence between the yield stress and the ultimate tensile strength of the rail material for a range of as rolled alloy rail steels was obtained from a range of test results and can be expressed by (Figure 3.23)

$$\sigma_{\rm Y} = \frac{\frac{580 \ \sigma_{\rm ult}}{2080 \ - \ \sigma_{\rm ult}}}{\frac{1}{2080 \ - \ \sigma_{\rm ult}}} \tag{3.86}$$

or

$$\sigma_{ult} = \frac{2080 \sigma_{y}}{580 + \sigma_{y}} .$$
 (3.87)

The data of Figure 3.22 can be used as a guide to the selection of required strength levels, however, for an accurate estimate the full reliability analysis should be used with the actual operating conditions.

#### LATERAL TRACK STABILITY

The ability of the track to withstand applied loadings caused by traffic and the environment is important if the track is to fulfill its purpose. Apart from vertical geometry problems which are due to voiding and deformation of the ballast and the formation, the stability of the track is also affected by lateral geometry problems caused by excessive lateral wheel/rail loadings and buckling due to high track temperatures. In the following sections the critical lateral force to shift the track and the buckling stability of CWR will be discussed in detail.



0.2% proof stress,  $\sigma_{\rm V}$  (MPa)

## Figure 3.23

Variation of  $O_Y/O_{ult}$  with  $O_Y$  for a range of AS rolled alloy rail steels (Mair and Groenhout 1978)

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#### The critical lateral force required to shift the track

High lateral forces between the wheel flange and the rail head occur when vehicles travel around curves, and these forces are redistributed to the track. Amans & Sauvage (1969) have developed a semi empirical relationship to determine the critical lateral force that is required to shift the track. The reference track used in these experiments was a track that had recently been resurfaced and tamped and was constructed with 46 kg/m rails on timber sleepers at 600 mm spacings. The lateral resistance of the reference track,  $H_o(kN)$  was found to be

$$H_{0} = 10 + \frac{2P}{3} , \qquad (3.88)$$

The lateral resistance of any track  $H_t$  (kN) can be expressed in terms of the reference track, i.e.,

$$H_{+} = \eta_{+} H_{0'}$$
 (3.89)

where  $n_t$  = dimensionless coefficient relating to lateral strength of track.

The lowest value of  $n_t$  obtained from field measurements was  $n_t = 0.85$ , this being obtained from timber sleepered curved track at very high track temperatures. For the case of concrete sleepered track values of  $n_t = 1.10$  were common. The critical lateral force  $H_c$  (kN) that is required to shift the track can be expressed as

$$H_{c} = \Gamma H_{+}$$
 (3.90)

where  $\Gamma$  is a dimensionless factor defined by

$$\Gamma = 1 - \rho_0 A_r \Delta_t (1 + \frac{R_0}{R}) (\frac{k}{k_0}) (\frac{1}{E_1}) (\frac{\epsilon_0 (E_1 y)}{(E_1 x)^{0.125}} (3.91)$$

The terms  $\rho_0$ ,  $R_0$ ,  $k_0$  and  $\varepsilon_0$  are parameters relating to the reference track, Equation 3.85; the values of which are as follows:

٩	=	0.125 m <sup>-2</sup> / <sup>o</sup> C
Ro	=	800 m (curve radius)
k <sub>o</sub>	==	20 MPa (track modulus)
с о <sup>3</sup>	= .	0.225 N <sup>-0.125</sup> m <sup>-0.25</sup>

The other terms are defined as:

Ar	==	cross sectional area of rail $(m^2)$
∆t	=	temperature difference from base temperature ( $^{\rm O}{\rm C})$
R	=	curve radius of track in question (m)
ΊY	=	moment of inertia of rail in the transverse direction $(\ensuremath{\mathfrak{m}}^4)$
IX	=	moment of inertia of rail in vertical direction $(m^4)$
E	=	Young's Modulus of rail steel (N/m <sup>2</sup> )
k	=	track modulus of track.

#### CWR track buckling considerations

Detailed experimental research has been undertaken by Bartlett (1960) to determine the longitudinal force required to buckle CWR track. Bartlett states that the longitudinal force  $P_B$  required to buckle CWR track must overcome the buckling resistances of the rail, rail fastener and the ballast, i.e.,

 $P_B = \frac{rail}{resistance} + \frac{rail fastener}{resistance} + \frac{ballast}{resistance}$ 

The formula developed to calculate the longitudinal force  $P_{B}$  (kN) required to buckle the track is

$$P_{B} = \frac{\pi^{2} E I_{s}}{\ell_{b}^{2} \cdot 10^{3}} + \frac{\pi^{2} C_{t}}{16s} \left[ \frac{\pi \ell_{b}}{q_{b}} \right]^{0.5} + \frac{W_{\ell} \cdot \frac{\ell_{\ell}}{b}}{\pi^{2} q_{b}}$$
(3.92)

where I <sub>s</sub>	=	horizontal moment of inertia of two rails $(mm^4)$ ,
$\mathbf{E}$		Young's modulus of rail steel (MPa),
<sup>l</sup> b	=	length over which buckling is likely to occur (m) - about 6 m,
S	=	sleeper spacing (m)
$\mathbf{q}_{\mathbf{b}}$	=	maximum misalignment occurring over length $b^{(m)}$ - say 7.5 mm,
° <sub>t</sub>	=	torsional coefficient of the rail fastener (kNm rad $^{-0.5}$ ), and
W <sub>r</sub>		maximum lateral ballast resistance per sleeper $(kN/m)$ .

Srinivasan (1969, Chapter 9) has outlined in detail the development of this forumla.

The expression

 $\pi^{2}E^{1}s$  $\frac{1}{\ell_{h}^{2}10^{3}}$ 

represents that part of the track panel resistance which relates to track acting as a composite strut.

The expression

$$\frac{{}^{2}C_{t}}{16S} \left[ \frac{\pi l}{q_{b}} \right]^{0.5}$$

represents that part of the track panel resistance relating to the torsional resistance of the rail fastener.

The expression

$$\frac{W_{\ell}\ell_{b}^{2}}{\pi^{2}q_{b}}$$

represents that part of the track panel resistance relating to the ballast resistance of the track.

A comparison of Bartlett's experimental and theoretical values of the longitudinal force required to buckle the track is presented in Table 3.17. Upon closer examination of Table 3.17, and ignoring the case of the sleepers on rollers, it is evident that the rail resistance is of the order of 11-16 per cent of the total buckling resistance, whereas the rail fastener resistance and the ballast resistance account for 13-37 per cent and 50-70 per cent respectively.

Unfortunately Bartlett has not indicated what the fastener system codes used in Table 3.17 represent. New Zealand Railways (Vink 1978) have carried out tests, for the range of fastener systems shown in Figure 3.24, in order to determine a representative value of the torsional coefficient,  $C_t$ , to be used with a particular fastener system. The results of these tests are presented in Figure 3.25. It is apparent that of the tests conducted with timber sleepers, only new sleepers were used, and that the variation of torsional fastener resistance with sleeper age was not tested.

Sinha (1967) has assumed that the torsional resistance of dogspike rail fasteners can be taken effectively as zero. It would appear that the torsional resistance of both dogspikes and screwspikes is highly dependent upon the age and condition of the timber sleepers.

The factor which governs the ability of track to withstand thermal buckling is the lateral resistance of sleepers, W , which consists of three separate components:

- the resistance offered by the base of the sleeper (this depends upon the weight of the sleeper and the load on the sleeper)
- . the resistance between the sides of the sleeper and the ballast in the cribs







Type 'P' Pandrol Bedplate Assembly



Ribbed Canted Bedplate Assembly



Type 'P' (concrete) Pandrol on Concrete Sleeper



Type 'RR' Ribbed Bedplate & Rubber Pad



Type 'A' Spring Clip Assembly





Note: N, P, R, RR and A refer to the fastening types illustrated in Figure 3.24

**Figure 3.25** Torsional resistance of fastenings (Vink 1978)

C,	Fastening	<sup>l</sup> h	q <sub>b</sub>	Nature of Ballast	w ,	Experimental	Th	eoretic	cal Calculations	(a)
(kNm rad <sup>-0.5</sup> )	Туре	(m)	(mm)		(kN/m)	P <sub>B</sub> (kN)	P <sub>B</sub> (kN)	$\frac{\pi^2 EI_s}{\frac{l^2 I0^3}{b^{(per})}}$	$\frac{\pi^2 C_t}{16S} \sqrt{\frac{\pi i_b}{q_b}}^{0.5}$	$\frac{W_{\ell}^{\ell}b}{\frac{2}{\pi}q}b$
	4/4	8.8	9.5	38(mm) uncompacted	1.27	1 710	1 748	11.7	27.7	60.6
11.39		8.8	9.5	only	1.27	1 710	1 748	11.6	28.2	60.2
11 77	4	7.1	6.4	Ditto	1.27	1 899	1 937	16.5	28.2	55.2
	4	7.1	6.4	DICCO	1.27	1 899	1 939	16.7	27.5	55.8
9.87	15	8.1	11.1	Ditto	1.27	1 513	1 383	17.7	36.8	55.5
13,92	10	8.1	11.1	DICO	1.27	1 513	1 535	15.9	34.1	50.0
8.25	10/2	10.2	12.7	Cleanang on wallang	-	515	446	34.5	65.5	-
9.67	10/3	10.2	12.7	steepers on forfers	-	515	494	31.2	68.8	-
8.25	10/2	10.1	20.3	38(mm) uncompacted	1.27	1 030	1 061	15.0	24.2	60.8
9.62	10/3	10.1	20.3	only	1.27	1 030	1 103	14.4	27.1	58.5
4.13	10/2	9.8	17.0		1.27	998	1 032	16.3	13.4	70.3
4.81	10/3	9.8	17.0	DICCO	1.27	998	1.055	16.0	15.3	68.7
8.25	10/2	7.7	12.7	Ditto Blug full bowing	2.55	1 759	1 759	15.5	16.0	68.5
9.62	10/3	7.7	12.7	No shoulder	2.55	1 754	1 805	15.1	18.2	66.7
8.25	10/2	9.1	17.8	Ditto Blug Full boying and	2.94	1 449	1 586	12.2	16.5	71.3
9.62	10/3	9.1	17.8	230 (mm) shoulder	2.94	1 449	1 629	11.8	18.7	69.5

TABLE 3.17 - COMPARISON BETWEEN THEORETICAL & EXPERIMENTAL BUCKLING RESULTS, (BARTLETT 1960)

(a)  $I_s = 2I_y = 8.05(10^6 \text{mm}^4); E = 202 \text{ GPa}; S = 790 \text{ mm} (31").$ 

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. the resistance offered by the ballast in the track shoulder to the sleeper end.

British Rail have carried out the most detailed investigation to determine the value of the lateral sleeper resistance for a variety of conditions (Shenton & Powell 1973). In this investigation a summary of all previous tests was carried out and a comparison of the lateral sleeper resistance was made for a range of ballast types, ballast conditions, shoulder sizes and sleeper material types. The main results of these tests are presented in Tables 3.18 to 3.23. The results of laboratory tests presented in Tables 3.20 and 3.21 clearly show that the total lateral resistance of the sleepers depends very much on the packing condition of the ballast.

Field experiments have shown a large variability in the measured value of lateral resistance of sleepers. It was found on one site that there was a variation of 22 per cent along a length of 110 m, where all ballast conditions appeared to be identical. Other field measurements of the lateral resistance of concrete sleepers have shown a range of 3.5 kN/sleeper on a nominal value of 10.4 kN/sleeper. In these experiments the effects of track maintenance and consolidation operations were also quantified, and these are presented in Table 3.23 (Shenton et al. 1973).

The lateral resistance of sleepers drops immediately following tamping of the track, and is gradually regained after subsequent traffic. This is clearly illustrated in Figure 3.26. It can be seen that 93 per cent of the original value of sleeper resistance is regained after 1.3 million tonnes of traffic; (this was equivalent to 34 days of traffic under the test conditions).

Test	Sleeper and		Resistance - (kN)/sleeper									
Series	ballast type	Base	Sides	Base & Sides	Shoulder	Total						
1	Wood Concrete	1.00 - 1.30 1.80 - 2.70	0.90 - 3.90 1.10 - 4.60	-	0.10 - 0.88 0.13 - 1.65	2.10 - 4.45 3.25 - 8.80						
2	Wood: Granite Slag Ash	1.00 0.85 0.85	0.65 0.70 0.30	- - -	<0.55 <0.60 <0.33	1.65 - 2.20 1.55 - 2.15 1.15 - 1.50						
	Concrete	2.60	1.10	-	<0.20	3.90						
3	Concrete	2.00	<u> </u>	2.52	<u> </u>	4.11						
4	Concrete	3.17	-	-	-	6.15						

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TABLE 3	3.18	- TYPICAL	VALUES	$\mathbf{OF}$	LATERAL	SLEEPERS	RESISTANCE	(SHENTON	$\mathbf{ET}$	AL	1973	)
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1.	Wood Concrete	29 - 47 30 - 58	37 - 65 30 - 54		5 - 25 4 - 28	100 100
2	<u>Wood</u> : Granite Slag Ash	45 40 55	30 30 20	- - -	25 30 25	100 100 100
	Concrete	65	30	-	5	100
3	Concrete	50	-	60	38	100
4	Concrete	50	-	-	-	100

TABLE 3.19 - VALUES IN TABLE 3.18 PRESENTED AS A PERCENTAGE OF THE TOTAL RESISTANCE

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It can be seen that in general the base resistance of the sleeper is the greatest proportion, this being approximately 50 per cent of the total resistance. The resistance provided by the sides of the sleeper is of the order of 20 to 30 per cent of the total.

Sleepers	Bal	last	Resistance kN/sleeper								
-	Size	State	150 mm	Shoulder	230 mm	Shoulder	300 mm	Shoulder			
			Total	Shoulder Only	Total	Shoulder Only	Total	Shoulder Only			
Wood	38 mm 38 mm	Loose Compact	2.12 3.45	0.10 0.74	2.46 3.52	0.44	2.71 3.59	0.69			
Concrete	38 mm 38 mm	Loose Compact	3.41	0.15	3.72 5.06	0.46 0.51	4.39 6.31	1.13 1.76			
Concrete	75 mm 75 mm	Loose Compact			3.24 8.50	0.13 1.30	3.48 8.80	0.10 1.65			

TABLE	3.20	-	TEST	SERIES	51,	EFFECT	OF	SHOULDER	SIZE	UPON	THE	INCREASE	$\mathbf{OF}$	LATERAL
			SLEEP	PER RES	SIST	ANCE, (	SHEI	NTON ET A	ь 197:	3)				

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These values are all slightly higher than the results from series 2. A few results available from test series 5 can be summarised as:

Shoulder size (mm)	Resistance kN/sleeper	
305	6.46	-
460	 6.82	
610	6.79	

These show that no benefit is gained by increasing the shoulder size beyond 460 mm.

	Resistan	ce kN/sle	eper		Component of		
Sleeper		Resistance					
	3	8 mm		76 mm			
	Loose	Compact	Loose	Compact			
Wood	1.01	1.01	1.03	1.28			
Concrete	2.14	2.68	1.83	2.64	Base Friction		
Wood	1.01	1.70	0.90	3.90	Cido Eriction		
Concrete	1.11	1.87	1.29	4.56	Side filction		
Wood	0.44	0.80	0.28	0.30			
Concrete	0.46	0.51	0.13	1.30	229mm Shoulder		
Wood	2.46	3.51	2.21	4.44	Total		
Concrete	3.71	5.06	3.25	8.50	Resistance		

 TABLE 3.21 - TEST SERIES 1, COMPARISON OF TIMBER AND CONCRETE

 SLEEPERS LATERAL RESISTANCE FOR VARIOUS BALLAST

 CONDITIONS (SHENTON ET AL 1973)

TABLE 3.22 - TEST SERIES 2, COMPARISON OF TIMBER AND CONCRETELATERAL SLEEPER RESISTANCE ON GRANITE, (SHENTONET AL 1973)

Sleeper		Resistanc	e kN/sleeper	
	Base	Sides	30mm Shoulder	Total
Soft Wood	1.00	0.65	0.40	2.05
Concrete	2.60	1.10	0.25	3.95

It would appear that the greatest part of the increase in total resistance comes from the increase in weight of the sleeper.

 TABLE 3.23 - EFFECT OF VARIOUS MAINTENANCE OPERATIONS ON THE

 MAGNITUDE OF THE LATERAL RESISTANCE OF CONCRETE

 SLEEPERS (SHENTON ET AL 1973)

Maintenance Operation	% Loss in Lateral Resistance
Tamping only	16 - 37
Tamping and shoulder consolidation	12
Tamping and crib and shoulder cons	olidation 15

Having calculated the longitudinal force required to buckle the track  $P_B$ , the track temperature over and above the stress free temperature  $t_B^{OC}$  at which this buckling will occur can be deduced by

$$t_{\rm B} = \frac{P_{\rm B}}{E 2A_{\rm r} \alpha_{\rm t}} , \qquad (3.93)$$

where  $P_B$  = longitudinal force required to buckle the track (kN),  $2A_r$  = cross sectional area of two rails (mm<sup>2</sup>),  $\alpha_t$  = coefficient of thermal expansion ( $^{\circ}C^{-1}$ ), and E = Young's Modulus of rail steel (MPa).

For curved track Magee (1965) has developed a simple empirical equation to determine the lateral forces per unit length of the rail  $P_f$  produced by continuously welded rail subject to a change in temperature. This equation expressed in metric units is

$$P_{f} = \frac{D_{c} \Delta t}{86.3} .$$
 (3.94)

where  $P_f$  = total lateral force per metre of track length (kN/m),  $\Delta_t$  = temperature difference from base temperature (<sup>O</sup>C),  $D_c$  = degree of track curvature (degrees).



## Figure 3.26

Relationship between sleeper lateral resistance and cumulative volume of traffic, showing the influence of the tamping operation (Shenton and Powell 1973)

This equation was originally developed for continuously welded rail track elevated on timber trestles. Prause et al. (1974) state that if reliabile data are available for the lateral resistance of sleepers, for specified ballast types and cross sectional geometry, the maximum sleeper spacing S<sub>max</sub> in curved track to prevent lateral buckling (from simple stability considerations) is given by

$$S_{\text{max}} = R_{\ell} / P_{f} , \qquad (3.95)$$

where  $R_{\ell}$  = lateral resistance of sleeper to movement in the ballast (kN).

Details of the conventional design of railway track relating to the analysis of sleepers (i.e. rail seat load determination, contact area and flexural strength assumptions) are presented in Chapter 4.

#### RAIL WEAR LIFE

The equations that have been developed for rail wear life prediction are all based upon empirical measurements and have no real theoretical backing. In the following section three major methods of estimating rail wear; the University of Illinois method, the AREA method and Couard's method, will be presented in detail. An Australian formula used in rail wear life prediction, the Westrail formula, will also be discussed. The main criticism of the University of Illinois and the AREA methods for determining the life of rails is that there are no guides for improving their wear performance (Prause et al. 1974). It is also not apparent whether the influence of other conditions that effect the life of rails, such as rail breakages, rail shelling, transverse defects etc., are included in the following methods. The effect of transposing the rails resulting in an increase in rail wear life is also not treated explicitly in any of the following methods.

#### The University of Illinois Rail Wear Formula

Field measurements of rail wear have been carried out for a number of U.S. railway systems by the University of Illinois to determine the service life of continuously welded rail (Hay et al. 1973, 1975). Rail head wear data were obtained in the field from the Burlington Northern System (BN), Norfolk and Western Railway (N&W), Illinois Central Railroad (IC) and the Atchinson, Topeka and Santa Fe Railroad (AT&SF). Observations were recorded and using a planimeter the loss of rail head area caused by the passage of traffic during a specified time period was measured.

With the advent of continuous welded rail which has virtually eliminated rail joints, the primary causes of rail head wear can be considered to be due to abrasion between the wheel and the rail, plastic flow of the rail head due to excessive wheel loads or, a combination of both. Abrasion is influenced in turn, by such factors as curvature, traffic, speed, type of rolling equipment maintenance standards and the operating environment.

The formula developed by the University of Illinois (Hay et al 1973) to calculate the anticipated abrasive rail head area wear caused by traffic has, in order to determine the annual rail head area wear, been restated as

$$W_a = W_t (1 + K_w D_C) D_A'$$
 (3.96)

where

 $D_{C}$  = degree of curve (degrees),

$$D_{A}$$
 = annual gross tonnage (Mt gross/year), and

K<sub>w</sub> = wear factor varying with the degree of curve, i.e. the amount by which the tangent wear value must be increased to equal the measured curve wear value. The average rail head wear from field data for tangent track and commonly used rail sizes are listed in Tables 3.24 and 3.25. The average head wear values are weighted to account for the volume of traffic over each test section. It should be noted that the method of weighting (outlined in the footnote to Table 3.24) assumes a linear relationship between head wear and traffic volume in order to relate different railway systems.

Some errors and inconsistencies of data presentation were discovered in the original source reports and these have been rectified.

It should be noted that the field data presented in Tables 3.24 and 3.25 define the average head area wear in terms of a 100 million gross tons of traffic carried. Here tons refers to U.S. short tons and the conversion to equivalent metric units are presented at the bottom of each table. The value of wear factor,  $K_w$  for any particular degree of curvature of the track, was calculated by comparing the average rail head wear obtained in tangent track and defining the relationship as

Average rail wear in curved track =  $W_{+}$  (1 +  $K_{U}$  D<sub>c</sub>).

Calculated values of  $K_w$  are listed for various degrees of track curvature and rail size in Tables 3.26 and 3.27. It should be noted that for 57 (kg/m) rail, field data were only available for curves of  $0^{\circ}15'$  through to  $3^{\circ}00'$  whereas for 66 kg/m rail field data were only available for low degree curves (Hay et al. 1973). Therefore the assumed and extrapolated values given in Tables 3.26 and 3.27 should be treated with reservation until more rail wear data for the higher degree curves becomes available. Specific formulae for determining the amount of rail head area wear, derived from the results of Tables 3.26 and 3.27 and written in the form of Equation 3.96 are as follows:

SYSTEMS, (HAY ET AL 1973) Weight<sup>(a)</sup> Railroad Average Head Wear Average Weighted Average per 100 MGT Tonnage Factor Head Wear(a)  $(in^2)$ (MGT)  $(in^2)$  $(mm^2)$  $(mm^2)$ 0.0671 43.29 62.55 1.00 0.0671 43.29 ΒN 0.0850 54.84 180.90 2.89 0.2457 158.52 N&W IC 0.0414 26.71 96.51 1.54 0.0638 41.16 0.0472 30.45 296.35 AT&SF 4.74 0.2237 144.32

 TABLE 3.24 - SUMMARY OF THE AVERAGE FIELD TANGENT TRACK RAIL WEAR

 DATA FOR 57(kg/m) RAILS IN VARIOUS U.S. RAILWAY

 CNEMENS
 (HAV ET AL 1072)

(a) Lowest average tonnage (62.55 for the BN) taken as base (1.00) and divided into the other tonnages to obtain a weighting factor. (eg. 180.9 ÷ 62.55 = 2.9, for the N&W values).

10.17

0.6003

387.29

NOTE: Average head wear of 57 kg/m rail for tangent track is

 $\frac{387.29}{10.17} = 38.08 \text{ mm}^2/100 \text{ MGT}$  $= 0.420 \text{ mm}^2/\text{Mt gross.}$ 

 TABLE 3.25 - SUMMARY OF THE AVERAGE FIELD TANGENT TRACK RAIL WEAR

 DATA FOR 66(kg/m) RAILS IN VARIOUS U.S. RAILWAY

 SYSTEMS, (HAY ET AL 1973)

Railroad	Average Head Wear per 100 MGT		Average Tonnage	Weight Factor	Weighted Average Head Wear	
	(in <sup>2</sup> )	(mm <sup>2</sup> )	(MGT) <sup>(a)</sup>		(in <sup>2</sup> )	(mm <sup>2</sup> )
N & W	0.0720	46.45	94.8	1.00	0.0720	46.45
IC	0.0387	24.97	245.67	2.59	0.1002	64.65
SF	0.0742	47.87	215.51	2.27	0.1684	108.64
				5.86	0.3406	219.74

NOTE: Average head wear of 66 kg/m rail for tangent track is  $\frac{219.74}{5.86} = 37.50 \text{ mm}^2/1000 \text{ MGT}^{(a)}$   $= 0.413 \text{ mm}^2/\text{Mt gross}^{(b)}.$ 

(a) U.S. Short tons.

(b) Metric tonnes.

TABLE 3.26 - SUGGESTED K VALUES FOR USE WITH 57(kg/m) RAIL, (HAY ET AL 1973)

Curvature <sup>(a)</sup>	High Rail	Low Rail
$0^{\circ}01' - 0^{\circ}59'$	0.20	0.10
1 <sup>0</sup> 00' - 1 <sup>0</sup> 59'	0.40	0.20
2 <sup>°</sup> 00' - 2 <sup>°</sup> 59'	0.60	0.30
3 <sup>0</sup> 00' - 3 <sup>0</sup> 59'	0.80	0.40
4 <sup>0</sup> 00' - 4 <sup>0</sup> 59'	1.00	0.50
5 <sup>°</sup> 00' - 5 <sup>°</sup> 59'	1.00	0.60
6 <sup>°</sup> 00' - 6 <sup>°</sup> 59'	1.00	0.70
7 <sup>0</sup> 00' - 7 <sup>0</sup> 59'	1.00	0.80
8 <sup>0</sup> 00' - 8 <sup>0</sup> 59'	1.00	0.90
9 <sup>0</sup> 00' and over	1.00	1.00

(a) Degrees of curvature are based upon the American practice of using 100 ft chord lengths.

(b) K value based on groupings around low degree curves from field data and interpolated therefrom to K = 1 for 8-degree curves.

TABLE 3.27 - SUGGESTED K VALUES FOR USE WITH 66(kg/m) RAIL, (HAY ET AL 1973)

Curvature <sup>(a)</sup>	High Rail	Low Rail
$0^{\circ}01' - 2^{\circ}29'$	0.46	0.11
2 <sup>°</sup> 30' - 2 <sup>°</sup> 59'	0.50	0.15
3 <sup>0</sup> 00' - 3 <sup>0</sup> 29'	0.54	0.20
3 <sup>°</sup> 30' - 3 <sup>°</sup> 59'	0.58	0.22
4 <sup>0</sup> 00' - 4 <sup>0</sup> 59'	0.62	0.28
5 <sup>0</sup> 00' - 5 <sup>0</sup> 59'	0.71	0.37
6 <sup>0</sup> 00' - 6 <sup>0</sup> 59'	0.78	0.46
7 <sup>0</sup> 00' - 7 <sup>0</sup> 59'	0.87	0.54
8 <sup>0</sup> 00' and over	1.00	0.63

(a) Degrees of curvature are based upon the American practice of using 100 ft chord lengths.

(b) K value based on groupings around low degree curves from field data and interpolated therefrom to K = 1 for 8-degree curves. (a) For 57 (kg/m) plain carbon CWR:

$$W_a = 0.420 (1 + K_w D_c) D_h$$
 (3.97)

(b) For 66 (kg/m) plain carbon CWR:

$$W_a = 0.413 (1 + K_w D_c) D_A',$$
 (3.98)

where  $W_a$  = annual rail head area wear (mm<sup>2</sup>/year),  $K_w$  = wear factor varying with degree of curve, (values presented in Tables 3.26 and 3.27),  $D_c$  = degree of curve (degrees), and  $D_A$  = annual gross tonnage (Mt gross/year).

Denoting the permissible limit of rail head wear by  $\theta_A \pmod{2}$ , the expected life of the rail  $T_v$  (years) can be defined by

$$T_{y} = \frac{\partial_{A}}{W_{a}} . \qquad (3.99)$$

Consequently, the expected life of rail T (Mt gross) can therefore be defined by

$$\mathbf{T} = \mathbf{T}_{\mathbf{v}} \mathbf{D}_{\mathbf{A}}$$
 (3.100)

#### The AREA Rail Wear Formula

Underlying the following analyses is the basic assumption that the track structure behaviour can be established for a set of "base" track/traffic conditions and that variations from these base conditions can be reflected by a process of factor analysis, i.e. that basic track structure performance can be "factored" to predict relative effects of differing track/traffic environments (Danzig, Hay & Reinschmidt, 1976). The method developed by the AREA (based upon available research data developed by the AREA and by several individual railroads) evaluates rail life as a function of the following track and traffic characteristics:

- . annual tonnage density (Mt gross)
- composite of rail type, weight, rail joint type, metallurgy and condition
- . actual average train speeds for a particular train service type and traffic axle load class
- . gradient of the track
- . curvature of the track and the use of lubricants in the track
- . actual traffic axle loads for a particular train services type and train speed class.

For tangent, level, fish-plated rail track under mainline speed conditions, here assumed to be 80 (km/h), the life of a rail can be estimated by

$$T = 1.839 K_T R_{Wt} (1.102 D_A)^{0.565}, \qquad (3.101)$$

where

T = rail life (Mt gross), D<sub>A</sub> = annual gross tonnage (Mt gross/year), R<sub>Wt</sub> = rail weight (kg/m), and K<sub>T</sub> = composite constant reflecting track conditions and the level of maintenance.

This formula was derived using imperial units, therefore the value 1.102 represents the conversion of tonnes to U.S. short tons, while the value 1.839 represents the product of the conversion of kg/m to lb/yd with the conversion of U.S. short tons back to tonnes.

For the case of tangent, level, fish plated plain carbon rail track, the value of  $K_{\rm T}$  is generally adopted as being 0.545. This value was developed from surveys of U.S. industry experience.

A more general formula which adjusts the rail life estimate in tangent track to allow for a wide range of track and traffic conditions has been expressed as (Danzig et al. 1976)

$$= \frac{1.839 \ \text{K}_{\text{C}} \ \text{K}_{\text{G}} \ \text{K}_{\text{R}} \ \text{R}_{\text{Wt}} \ (1.102 \ \text{D}_{\text{A}})^{1.565}}{\prod_{i}^{n} \left[ \frac{1.102 \ \text{D}_{i}}{\text{K}_{\text{V}_{i}} \ \text{K}_{\text{A}_{i}} \ \text{K}_{\text{S}_{i}}} \right]}$$
(3.102)

where

Т

T = rail life (mt gross),

 $D_{\Lambda}$  = annual gross tonnage (Mt gross/year),

 $R_{Wt}$  = rail weight (kg/m),

K<sub>R</sub> = rail factor (which includes rail weight, rail strength and joint type; preliminary values are presented in Table 3.30),

n = total number of different types of traffic in the overall annual gross tonnage,

D<sub>i</sub> = sub-tonnage (Mt gross/year), which relates to a
 particular speed class, axle load class and service
 type,

in Table 3.31),

 $K_{A} =$  wheel load class factor, that relates to a particular i sub-tonnage D<sub>i</sub> (preliminary values are presented in Table 3.32), and

K<sub>S</sub> = service type factor, this factor relates to the type of traffic that is represented by the sub-tonnage D<sub>i</sub>. (Preliminary values of K<sub>S</sub> have been suggested as; 0.91 for Unit-Train Operations and 1.0 for other traffic).

The conversion factors in Equation 3.102 are those as previously mentioned in Equation 3.101. It should be recognised that the majority of the reduction factors listed in Tables 3.28 to 3.32 are preliminary; some being entirely based upon judgement, and have as yet not been based upon adequate experimental data.

Prause et al. (1974) expresses Equation 3.102 in a different form to enable the prediction of rail life for tangent track in years to be determined, i.e.,

Curve Class <sup>(a)</sup> (degrees)	Factor	Factor (oilers)
0 - 0.5	1.00	1.00
0.5 - 1.5	0.87	1.00
1.5 - 2.5	0.74	0.88
2.5 - 3.5	0.61	0.79
3.5 - 4.5	0.49	0.70
4.5 - 5.5	0.38	0.62
5.5 - 6.5	0.30	0.55
6.5 - 7.5	0.22	0.48
7.5 - 8.5	0.16	0.44
8.5 - 9.5	0.12	0.40
<b>&gt;9.</b> 5	0.10	0.37

TABLE 3.28 - RAIL LIFE "K<sub>C</sub>" FACTOR - CURVATURE AND USE OF CURVE OILERS, (DANZIG ET AL 1976)

(a) Degrees of curvature are based upon the American practice of using 100 ft. chord lengths.

## TABLE 3.29 - RAIL LIFE "K<sub>G</sub>" FACTOR - TRACK GRADIENT, (DANZIG ET AL 1976)

Gradient Class	Factor
0 - 0.5 per cent grades	1.0
0.5 - 1.0 per cent grades	1.0
1.0 - 1.5 per cent grades	0.9
1.5 - 2.0 per cent grades	0.8
>2.0 per cent grades	0.7

Rail, Type Metallurgy, Condition		Rail Weight	
	<55 kg/m	55-61 kg/m	>6l kg/m
Fish-plated Plain (Control Cooled) New	0.545	0.545	0.545
Fish-plated Plain (Control Cooled) S.H.	0.545	0.545	0.545
CWR, Plain (Control Cooled) New	n/a	0.774	0.964
CWR, Plain (Control Cooled) S.H.	n/a	0.774	0.964
23.8m, Plain (Control Cooled) New	0.545	0.545	0.545
23.8m, Plain (Control Cooled) S.H.	0.545	0.545	0.545
23.8m, Flame-Hardened New	n/a	0.545	0.545
21.0m, Flame-Hardened S.H.	n/a	0.545	0.545
23.8m, High Silicon New	n/a	0.812	0.567

# TABLE 3.30 - RAIL LIFE "KR" FACTOR - RAIL WEIGHT<sup>(a)</sup> (DANZIG ET AL 1976

(a) These preliminary values were based upon rail research conducted at University of Illinois (Hay et al 1973) and are recommended by the AREA.

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Speed Range (km/h)	Assumed Speed (km/h)	K - Factor
108.6 - 116.7	112.7	0.80
100.6 - 108.6	104.6	0.85
92.5 - 100.6	96.6	0.90
84.5 - 92.5	88.5	0.95
76.4 - 84.5	80.5	1.00
68.4 - 76.4	72.4	1.05
60.4 - 68.4	64.4	1.10
52.3 - 60.4	56.3	1.15
44.3 - 52.3	48.3	1.20
36.2 - 44.3	40.2	1.25
28.2 - 36.2	32.2	1.30
20.1 - 28.2	24.1	1.35
12.1 - 20.1	16.1	1.40

TABLE 3.31 - RAIL LIFE "Ky" FACTOR - TRAIN SPEED (DANZIG ET AL 1976)<sup>(a)</sup>

(a) Based upon work of Prof. Talbot (AREA), this work predicted increases in dynamic impact on the track structure of 1 per cent per mph increase in vehicle speed for speeds in excess of 5 mph (Talbot 1918-1934).

## TABLE 3.32 - RAIL LIFE - "K" FACTOR - WHEEL LOAD, (DANZIG ET AL 1976)

Wheel	Load Range	(kN) Factor	
	< 67	1.153	
67 <b>-</b>	116	0.862	
116 -	133	0.841	
133 -	147	0.545	
147 -	156	0.439	
156 <del>-</del>	167	0.381	
:	>167	0.325	

$$T_{y} = \frac{T}{D_{A}} \left( \frac{\theta V}{4.76} \right), \qquad (3.103)$$

where  $T_y$  = rail life for tangent track (years),  $D_A$  = annual tonnage density (Mt gross/year), T = rail life (Mt gross), and  $\theta_x$  = permissible limit of vertical head wear (mm).

and 4.76 mm represents the commonly assumed vertical head wear limit of 3/16 inch, and as such was used as the limit of rail life in Equation 3.102.

#### The Couard Method of Calculating Rail Wear

The Couard method for calculating the annual rail wear rate is based on the following equations (Prause et al. 1974):

(a) Vertical Rail Head Wear on Tangent Track:  

$$W_v = 8.70.10^{-5} Q_1 V D_A (1 + 0.23 G^{1.7}) + 0.0635.$$
 (3.104)

(b) Vertical Rail Head Wear on the High Rail of Curved Track:

$$W_v = 9.57.10^{-5} Q_1 V D_A (1 + \frac{U_B}{254} + 0.23 G^{1.7}) + 0.0635.(3.105)$$

(c) Side Head Wear on High Rail of Curved Track:

$$W_{\rm s} = 1.39.10^{-4} Q_2 V D_A D_C (1 + \frac{U_{\rm b}}{254} + 0.23 G^{1.7}) + 0.0635.$$
(3.106)

(d) Side Head Wear on Curves with Rail Lubricators:

$$W_{sl} = 0.7 (W_s - 0.0635) + 0.0635,$$
 (3.107)

where

Wv	=	annual vertical rail head wear rate (mm/year),
Ws	=	annual side rail head wear rate (mm/year),
Wsł	=	annual side rail head wear rate for lubricated cail (mm/year),

G	-	track gradient (per cent),
D <sub>A</sub>	=	annual traffic density (Mt gross/year),
V	=	operating speed (km/h),
U <sub>b</sub>	=	unbalanced superelevation (mm),
D <sub>C</sub>	=	degree of curve (degrees) $^{(\perp)}$ ,
Q <sub>1</sub>	=	ratio of rail head width to that of the 69.5 (kg/m), rail, and
Q <sub>2</sub>	=	ratio of rail head depth to that of the 69.5 kg/m, rail.

The constant 0.0635 mm/year in the above equations represents the assumed rate of annual head loss attributed to the effects of rail corrosion.

The Couard method for determining the annual rail head wear rate was originally developed for a track with 69.5 kg/m, (140 lb/yd RE), rail. Therefore, the rail size correction factors,  $Q_1$  and  $Q_2$ , have to be applied to the equations when smaller rail sizes are used. Values of the rail size correction factors for Australian and American rails are presented in Table 3.33. It should be noted that the coefficient expressed in Equations 3.104, 3.105 and 3.106 also contain the conversion of U.S. short tons to tonnes.

By dividing the annual wear rate  $W_v$  or  $W_s$  into the corresponding permissible limit of rail head wear the rail life in years can be predicted. For the case of curved track the rail life is the less of

$$T_{y} = \frac{\theta_{v}}{W_{v}} \text{ or } T_{y} = \frac{\theta_{s}}{W_{s}}, \qquad (3.108)$$

where

 $T_{Y}$  = rail life (years),  $\theta_{v}$  = permissible limit of vertical rail head wear (mm),  $\theta_{s}$  = permissible limit of side rail head wear (mm), and  $W_{v}$ ,  $W_{s}$  are as above.

(1) Establish a maximum allowable wear (mm/year).

Rail Section (kg/m)	Wear Factor	
	Ql	Q <sub>2</sub>
31	1.20	1.49
41	1.20	1.29
47	1.09	1.21
50 <sup>(a)</sup>	1.09	1.17
50 RE	1.12	1.25
53	1.09	1.14
57 RE	1.10	1.22
59	1.13	1.10
60 <sup>(a)</sup>	1.09	1.07
66	1.09	1.07
68	1.02	1.06
69 RE	1.00	1.00

TABLE 3.33 - WEAR FACTORS FOR CALCULATING RAIL HEAD WEAR BY THE COUARD METHOD (PRAUSE ET AL 1974)

(a) New Australian rail section.

The life of the rail T (Mt gross) can be defined by Equation 3.97.

Several significant factors which affect the rate of rail wear are not considered at present by the Couard method. These factors include the rail steel composition, the track environment (dry or wet, corrosive and/or abrasive), and the extent of lubrication. Prause et al. (1974) consider that the currently available methods for estimating the rail wear on curves are not entirely satisfactory. However, an improved procedure could be developed using an empirical relation similar to the Couard formula and modifying the results with corrective factors. This empirical relation should be verified and modified and should also be based on data from wear test metallurgical experiments using typical wheel and rail steels. The following procedure was suggested:

- (1) Establish a maximum allowable wear (mm/year).
- (2) Establish a base wear value for a given traffic density, (Mt gross/year) moving at a given average velocity for a standard rail steel ( $\approx 0.5$  per cent C).
- (3) Modify these results for the degree of curvature.
- (4) Multiply by an environmental factor.
- (5) If the predicted wear is too high, estimate the amount of wear reduction that could be obtained by alloy using the relation that wear is reduced about 8 per cent for every 0.1 per cent increase in carbon or carbon equivalent.
- (6) Apply a factor for lubrication if further reduction in wear is desired or if alloy steel is undesirable. Wear life can be increased considerably by lubrication.

#### The Westrail Rail Life Formula

A formula which predicts the expected life of rails in track has been developed by the Western Australian Government Railways (Westrail 1976, Hoare and Payne 1978). This simple formula expresses the life of the rail as a function only of its weight, and incorporates AREA rail life data for rails with weights above 50 kg/m (AREA, 1951), with Westrail rail life data for rails with weights below 30 kg/m, see Figure 3.27. The formula was developed for jointed rail track and is stated as

$$T = 35.6.10^{-6} R_{Wt}^{3.98}$$
, (3.109)

where

T = rail life (Mt gross), and  $R_{W+}$  = rail weight (kg/m).



## Figure 3.27

Approximate relationship between rail weight and expected rail life (Westrail 1976) The formula does not give an indication of what annual tonnage, or what axle loading carried is inherent in the relationship. It is also not apparent whether this formula was developed for tangent or curved track, or both; and whether it incorporates other conditions that influence the life of the rail, such as rail breakages, rail shelling, transverse defects etc.

#### Permissible limits of rail head wear

The permissible limits of rail head wear can be defined in the following interrelated geometric ways:

- . alloweable vertical head wear loss
- . allowable side head wear loss
- . allowable head area loss
- . maximum angle of side wear.

The definition of these permissible limits can, at best, be regarded as subjective, the values of which largely depend upon the railway operator. In the following discussion a comparison of typical rail head wear limits is made.

Although varying with rail section, an average rail head vertical wear of 4.76 mm, (3/16 in.) is sometimes taken as limiting the useful life of rail in U.S. mainline track (Prause et al. 1974). It should be noted that the rails may be removed from the track for reasons other than abrasive wear of the rail head above. Such factors as the occurrence of an excessive number of rail breakages, or fatigue defects, per kilometre will also significantly influence the actual life of the rail.

Rails are not normally worn to the condemning limit imposed by the bending stress criteria. Before that point is reached, the wheel flanges strike the upper edges of the fish plates in the case of jointed rail track. Although the bar may then wear along with the rail, good practice warrants removal of the rail at that stage (Prause et al. 1974).

For CWR the vertical wear criteria of when the wheel strikes the fish-plate may be excessively restrictive, as much of this type of track is constructed with small numbers of joints. The long string lengths from the flash butt welding plant are thermit welded in the field into lengths of several kilometres. When strings of CWR are secured with fish plates, such as at insulated joints, the relatively few joints per kilometre would pose no excessive risk in allowing the wheel flange to make contact with the fish plate. Under those conditions, the fish plate will wear downward as the rail head wears downward. An unfavourable condition could exist, however, if a worn fish plate had to be replaced with one not worn. Milled fish plates could be provided for such situations. By the time the rail had worn to this degree, it would undoubtedly have been placed in low speed track (Hay et al. 1973). For these reasons, Hay has used a fish plate clearance of 0.8 mm (1/32 in.) as the condemning limit for the 57 and 66 kg/m rail sections analysed. This represents an equivalent head wear area of 350.3 mm<sup>2</sup> for the 57 kg/m and 518.7  $mm^2$  for the 66 kg/m rail (Table 3.34). These head wear areas correspond to 14.0 and 18.5 per cent loss respectively in the total head area of the two rail sections. The AAR have established permissible limits for rail head wear based upon the operating train speeds over a particular class of track (Railway Track and Structures 1970). These limits expressed as percentages of total head area and probably defined for CWR track are presented in Table 3.35. An envelope of allowable rail wear limits, incorporating both loss of rail height and side head wear due to flange contact, can be drawn to suit the conditions of particular railway operators; for example those based upon Canadian National recommendations for various rail section are presented in Figure 3.28 (King 1976). These limits correspond quite closely with those recommended by Hay et al. (1973), Table 3.34. All railway systems have established similar rail head wear condemning limits.
Rail Weight (kg/m)	Condemning Criteria Bending Stress Criteria Fish Plate Clearance <sup>(a)</sup>								(a)	
	Factor of Vertical Safety with Wear		Head	Head Equivalent Area of Wear		Vertical Head Wear		Equivalent Area of Wear		Loss of Rail He <b>ad</b>
	One Sleeper Missing	(in)	(mm)	(in <sup>2</sup> )	(mm <sup>2</sup> )	(in)	(mm)	(in <sup>2</sup> )	(mm <sup>2</sup> )	Area (per <b>ce</b> nt)
57	1.11	5/16	7.94	0.68	438.7	1/4	6.35	0.543	350.3	14.0
66	1.34	3/4	19.05	1.93	1 245.2	11/32	8.73	0.804	518.7	18.5

TABLE 3.34 - LIMITS OF RAIL HEAD WEAR, (HAY ET AL 1973)

(a) Recommended by Hay.

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TABLE 3.35 - AAR LIMITS FOR RAIL HEAD WEAR (RAILWAY TRACK & STRUCTURE 1970)

Class of	Train Speeds (km/h)	Maximum Permissible Reduction in Rail Head Area (per cent), for Various Rail Sections					
Rail		<35	35-42	42-49	49-56	56-64	>64
A	100 - 130	-	-	_	30	34	36
В	75 - 100	-	-	30	32	36	38
С	50 <b>- 7</b> 5	-	20	32	34	38	40
D	25 <del>-</del> 50	20	24	34	38	42	44
E	0 - 25	24	28	36	42	46	50

\_ . \_ . . . . . . .



Note: Values used in drawing the envelope of rail wear limits are:

Loss of Height (mm)				
	66 (kg/m)	57 (kg/m)	50 (kg/m)	≪42 (kg/m)
1.57	7.92	6.35	4.75	3.18
3.18	6.35	4.75	3.18	2.39
4,75	4.75	3.18	1.57	1.57

## Figure 3.28

Envelope of rail wear limits for loss of rail height and side head wear caused by flange contact; Canadian limits (King 1976) It is notable that the permissible limits on the amount of rail head wear do not adequately relfect rail strength levels, nor are they related to vehicle dynamic tracking, which apart from other factors, is influenced by changes in the track gauge and the rail cant.

Rail replacement constitutes a major track maintenance cost, consequently the rail head wear condemning limits, together with the optimum worn rail head condition suitable for transposing should be given more detailed study.

## CHAPTER 4 - SLEEPER ANALYSIS

The functions of the sleepers are to transfer the vertical, lateral and longitudinal rail seat loads to the ballast and formation, and to maintain the track gauge and alignment by providing a reliabile support for the rail fasteners. The vertical loads subject the sleeper to a bending moment which is dependent upon the condition of the ballast underneath the sleeper. The performance of a sleeper to withstand lateral and longitudinal loading is dependent upon the sleeper's size, shape, surface geometry, weight and spacing.

Before the sleeper can be analysed in terms of its capacity to withstand the bending stresses caused by the vertical rail seat loads the sleeper support condition and its effect upon the contact pressure distribution must be quantified. The contact pressure distribution between the sleeper and the ballast is mainly dependent upon the degree of voiding in the ballast under the sleeper. This voiding is caused by traffic loading and is due to the gradual change in the structure of the ballast and the subgrade.

The exact contact pressure distribution between the sleeper and the ballast and its variation with time will be of importance in the structural design of sleepers. It is practically impossible to predict the exact distribution for a sleeper in the in-track condition (ORE 1969). In order to calculate the sleeper bending stresses a uniform contact pressure distribution between the sleeper and the ballast is assumed to occur in practice; thereby enabling a visual interpretation of how the vertical force exerted by the rail on the sleeper is transmitted to the ballast. Various hypothetical contact pressure distributions between the sleeper and the ballast and their corresponding sleeper bending moment diagrams as postulated by Talbot (1918-1934) are presented in Figure 4.1.

Distribution of bearing pressure

Uniform pressure	
Flexure of sleeper produces variations from (a).	
Tamped either side of rail	
Laboratory test	
Principal bearing on rails	t t
Maximum intensity at ends	
Maximum intensity in middle	1 1
Centre-bound	1 f
End-bound	
End-bound Well-tamped	
End-bound Well-tamped Depression at rail seat	
End-bound Well-tamped Depression at rail seat	
End-bound Well-tamped Depression at rail seat	



## Figure 4.1

Hypothetical distribution of sleeper bearing pressure and corresponding sleeper bending moment diagrams for a sleeper (Talbot 1920) When the track is freshly tamped the contact area between the sleeper and the ballast occurs below each rail seat. After the track has been in service the contact pressure distribution between the sleeper and the ballast tends towards a uniform pressure distribution. This condition is associated with a gap between the sleeper and the ballast surface below the rail seat for timber sleepers (Talbot 1918-1934). The condition of centrebinding of timber and concrete sleepers tends to develop when maintenance is neglected. Hence it can be readily seen that the contact pressure distribution between the sleeper and the ballast is a time dependent, i.e. cumulative traffic tonnage, variable.

In order to prevent centre-binding of the sleepers the ballast is tamped under the rail seat. The current maintenance of concrete sleepers usually requires the provision of a non-pressure bearing centre section of the sleeper. This width is arbitrary, but is usually about 500 mm (ORE 1969). The provision of this centre gap is to ensure that the sleeper will not become centre bound, thereby preventing detrimental bending stresses from occurring.

Referring to the flowchart of the conventional railway track design procedure, presented in Figure 1.1, it is apparent that the selection of the required sleeper size and spacing is determined by an iterative trial and error solution. A sleeper size is adopted and a sleeper spacing is selected on the basis of being the maximum spacing which satisfies the sleeper design constraints. These being principally the contact pressure between the sleeper and the ballast and the sleeper flexural capacity. If the sleeper size chosen is unable to satisfy these constraints, or if the sleeper spacing thus determined is considered inadequate, the procedure is repeated for another sleeper size. Details of timber sleeper dimensions currently used by Australian railway systems are presented in Table 4.1.

Before the contact pressure between the sleeper and the ballast can be calculated the following factors must be quantified:

Railway System	Gauge		Sleeper I	Sleeper Dimensions		Sleeper Spacing		g
	(mm)	(ft.in)	(length x bread) (mm)	(ft.in)	km	( mm )	No per mile	(in)
New South Wales Public Transport Commission	1435 1435	4'85" 4'85"	2438x229x114 2440x230x130	8'0"x 9"x4½"	1641 1641	610 610	2640 2640	24 24
Victorian Railways	1600 1435	5'3" 4'8½"	2743x254x127 2591x254x127	9'0"x10"x5" 8'6"x10"x5"	1600	625	2580	24
Australian National Railways								
(a) Commonwealth Railways	1435 1067	4'8½" 3'6"	2438x229x114 1981x203x114	8'0"x 9"x4%" 6'6"x 8"x4%"	1641 1312	610 760	2640 2112	24 30
	1600 1600 1435	5'3" 5'3" 4'8½"	2591x254x127 (a) 2591x229x165 2591x254x127	8'6"10"x5" 8'6"x 9"x6½"(a) 8'6"x10"x5"	1502	665	2420	26
(b) South Australian Railways	1435 1067 1067	4'8½" 3'6" 3'6"	(a) 1981x203x114 1981x203x114 1981x203x140(a)	8'6"x 9"x6½" (a) 6'6"x 8"x4½" 6'6"x 8"x5½" (a).				-
(c) Tasmanían Railways	1067	3'6"	2134x229x127	7'0"x 9"x5"	1641	610	2640	24
Western Australian Government Railways	1435 1067	4'8½" 3'6"	2438x229x114 2134x229x114	8'0"x 9"x4날" 7'0"x 9"x4날"	1641	610	2640	24
Queensland Railways	1067 1067	3'6" 3'6"	2134x229x114 2134x229x152	7'0"x 9"x45" 7'0"x 9"x6"	1551	645	2500	25.3
New Zealand Railways	1067 1067 1067	3'6" 3'6" 3'6"	2134x263x127 2134x263x152 2134x229x114	7'0"x 8"x5" 7'0"x 8"x6" 7'0"x 9"x45"	1641	610	2640	24
Mount Newman Mining Co	1435	4'8½"	2591x229x152	8'6"x 9"x6"	1876	530	3020	21
Hamersley Iron Railway	1435	4'85"	2667x229x152	8'9"x 9"x6"	2020	495	3250	19.5
Cliffs Robe River Mining Co	1435	4'85"	2591x229x152	8'6"x 9"x6"	1790	560	2880	22

TABLE 4.1 - TIMBER SLEEPER DIMENSIONS AND SPACINGS CURRENTLY USED BY AUSTRALASIAN RAILWAY SYSTEMS (GORDON 1973)

(a) Denotes softwood sleepers (all others specified are hardwood).

....

. the effective sleeper support area beneath the rail seat

. the maximum rail seat load occurring at a sleeper.

These factors are discussed in detail in the following sections.

THE EFFECTIVE SLEEPER SUPPORT AREA BENEATH THE RAIL SEAT

The effective sleeper support area beneath the rail seat is defined as the product of the breadth of the sleeper and the assumed value of the effective length of sleeper support at the rail seat. The AREA defines the effective length of a sleeper as being the distance from the end of the sleeper to the point inside of the edge of the rail base over which tamping operations extend. Magee (1965-1971) assumes that this distance is 300 mm for 2440 x 200 x 150 mm sleepers, 380 mm for 2590 x 230 x 180 mm sleepers and 466 mm for 2740 x 250 x 200 (mm) sleepers. Resulting in assumed sleeper effective lengths of approximately 840, 990 and 1140 mm respectively. The effective length of a sleeper can also be deduced by using either of the relationships developed by Clarke (1957) or by Schramm (1961).

Clarke defines the effective length of sleeper support L (mm) under the rail seat for timber sleepers as

$$L = (l-g) \left[ 1 - \frac{(l-g)}{125 t^{0.75}} \right], \qquad (4.1)$$

where l = total sleeper length (mm), g = distance between the centre-line of the rail seats (mm), and t = sleeper thickness (mm).

For common sleeper lengths this equation can be approximated by

$$L = \ell/3. \tag{4.2}$$

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There are no comparable equations derived for concrete sleepers in which the effective length of sleeper support incorporates the effect of sleeper thickness.

Schramm defines the effective length of sleeper support L (mm) under the rail seat for both timber and concrete sleepers as

$$\mathbf{L} = \ell - \mathbf{g}, \tag{4.3}$$

where *l* and g are as previously defined.

The quantity (l-g) used by both Clarke and Schramm in Equations 4.1 and 4.3 can also be expressed as twice the overhand length of the sleeper Q (Figure 4.2).

The effective sleeper support area beneath the rail seat  $A_s (mm^2)$  can now be calculated by the following equations:

(a) According to Clarke (1957), for timber sleepers

$$A_{g} = B(\ell - g) \left[ 1 - \frac{(\ell - g)}{125 t^{0.75}} \right].$$
 (4.4)

(b) According to Schramm (1961), for both timber and concrete sleepers

$$A_{g} = B(\ell - g), \qquad (4.5)$$

where B = sleeper breadth (mm).

If the sleepers are not of a uniform breadth, for example concrete sleepers manufactured in a manner where the centre breadth is less than the end breadth; it is reasonable to assume an average sleeper breadth over the effective length of sleeper support when determining the effective sleeper support area. For steel sleepers with an inverted trough shape crosssection it is reasonable to assume that the effective sleeper breadth is the maximum distance between the sleeper flanges when



**Figure 4.2** Principal sleeper dimensions

determining the effective sleeper support area because of the ballast confinding characteristics of the sleeper section.

### DETERMINATION OF THE MAXIMUM RAIL SEAT LOAD

The exact magnitude of the load applied to each rail seat depends upon the following known and unknown parameters:

- . the rail weight
- . the sleeper spacing
- . the track modulus per rail
- . the amount of play between the rail and the sleeper
- . the amount of play between the sleeper and the ballast
- . proud sleeper plates (in the case of timber sleepers).

The influence of the last three factors vary in accordance with the standard of track maintenance, and their effect is to redistribute the applied sleeper loading to sleepers with adequate support.

Various methods have been developed to calculate the magnitude of the rail seat load and of these, the methods outlined in detail in the following section are thought to be the most frequently used.

#### The Beam on a Continuous Elastic Foundation Model

The maximum rail seat load can be determined theoretically by using the beam on a continuous elastic foundation model, and this is the approach suggested by Talbot (1918-1934) and by Clarke (1957). Using this model the maximum rail seat load  $q_r$  (kN) can in general be determined by

$$q_r = Sky_m F_1$$
 (4.6)

- where S = sleeper spacing (m),
  - k = track modulus (MPa) per rail,
  - Y<sub>m</sub> = the maximum rail deflection caused by the interaction of a number of axle loads about a given reference position (mm) (Figure 3.13), and
  - $F_1$  = factor of safety to account for variations in the track support caused by variations in the standard of track maintenance (Clarke (1957) adopts a value of  $F_1$  equal to 1).

The Special Committee on Stresses in Railraod Track (Talbot 1918-1934) came to the following conclusions about how various combinations of rails, sleepers and ballast react under load:

- tests showed that for a given modulus of rail support the maximum rail depression is not greatly affected by the rail section, although it is slightly greater for lighter rail sections (Danzig et al. 1976)
- . for a given wheel load, the product of rail depression and the modulus of rail support  $(ky_m)$  is nearly constant, consequently the upward resisting pressure is nearly constant and is approximately the same for any rail section and any support stiffness included in the tests. From this it follows that the pressure of rails (i.e. the rail seat load) is also the same regardless of the weight of rail or value of the modulus of support (Danzig et al. 1976).

O'Rourke (1978) has shown theoretically that under unit train conditions, where the adjacent wheel loads can be assumed to be the same, the product  $ky_m$ /unit load is, for a given rail section, nearly constant for any value of track modulus. It was found that this product was mainly dependent upon the axle spacing of the vehicle, (smaller axle spacings having a more significant effect, as expected, refer to Figure 4.3). For the heavy axle load conditions on the Mount Newman Mining Co. and the Hamersley Iron railways, a formula was developed to determine the rail seat load  $q_r$  (kN), which simplifies Equation 4.6 to



Note: Rail Moment of Inertia I =  $37.5 \times 10^{-6} \text{ m}^{-4}$  (90in<sup>4</sup>)

Figure 4.3

Values of the modulus of rail support (k y) for various axle spacings and assumed track moduli (O'Rourke 1978)

$$q_r = 0.56 \text{ SF}_1 P$$

where

Ρ

S = sleeper spacing (m),

= design wheel load (kN), and

F<sub>1</sub> = factor to account for variations in the track support caused by variations in the standard of track maintenance.

The coefficient 0.56 represents the average value of the product  $ky_m$ /unit load (Figure 4.3), for the smallest axle spacings of the iron ore wagons.

These axle spacings being approximately 1.8 m (for axles in the same bogie), and 2.3 m (for axles between adjacent wagons).

#### The AREA method

The AREA (1975) has developed a relationship to determine the maximum rail seat load  $q_r$  (kN) and is implicitly based upon the beam on elastic foundation model. Examination of Magee's Reports (1965-1971) led to an understanding of how this method was developed. It was recognised that none of the experiments conducted by Talbot (1918-1934) gave comparative data on the effect of sleeper spacing on the rail seat load; all of Talbots' experimental work being based on a sleeper spacing of about 510 mm. As a basis in the development of a method to calculate the maximum rail seat load, the beam on the ealstic foundation method was adopted and a track modulus per rail of 13.8 MPa for a sleeper spacing of 510 mm was assumed. Therefore using Equation 3.27 the track spring constant D(kN/mm) for the rail support was defined to be

$$D = kS = \frac{13.8 \times 510}{10^3} \simeq 7 \text{ kN/mm}.$$

It was also recognised that the larger the bearing area of the sleeper on the ballast, the more resistant the sleeper will be to depression under loading. To account for this effect the

(4.7)

effective area of sleeper support beneath the sleeper was incorporated into the expression for the track spring constant. This was achieved by assuming that the track spring constant of 7 kN/mm related specifically to a 2590 x 230 x 180 mm sleeper. As previously mentioned, Magee assumed that this sleeper has an effective length of 990 mm and consequently an effective area of  $228.10^3 \text{ mm}^2$ . The track spring constant D (kN/mm) was determined for any other sleeper size by the following assumption

$$D = 7 \cdot \frac{A_s}{228 \cdot 10^3}$$
(4.8)

where  $\mathbf{A}_{s} = \text{effective}_{s} \text{sleeper support area beneath the rail seat (mm<sup>2</sup>), of any other sleeper.}$ 

For a rail size of 57 kg/m with an adopted vehicle configuration of four 150 kN wheel loads spaced apart by 1.8, 2.0 and 1.8 m respectively, the maximum rail deflection was calculated by the beam on an elastic foundation method (using the above relationship to determine the track modulus for a variety of sleeper sizes). The maximum rail seat load was then calculated using Equation Magee (1965-1971) also found that the product of track 4.6. modulus and the maximum deflection,  $\mathrm{ky}_\mathrm{m},$  is nearly constant and consequently that the influence of the effective area, Equation 4.8, did not have a significant effect on the magnitude of the rail seat load. A simplified diagram was then constructed, based on the above vehicle loading and the beam on elastic foundation method, which shows that the rail seat load (expressed as a percentage of the wheel load) varies in proportion to the sleeper spacing (Figure 4.4, line A).

It would appear that the AREA (1975) have based their method of determining the maximum rail seat load upon this diagram. Basically the AREA method can be expressed by

$$q_r = (DF) P , \qquad (4.9)$$

- where  $q_r = maximum rail seat load (kN),$ 
  - Ρ = design wheel load (kN), and
  - distribution factor, expressed as a percentage of the wheel load. DF =

Recommended values of the distribution factor for timber, concrete and steel sleeper types and over a range of sleeper spacings are also presented in Figure 4.4; (lines B, C and D respectfully). It is clearly evident that AREA recommended distribution factors for timber and concrete sleepers (lines B and C) have identical slopes to Magee's estimate (line A). This leads to the conclusion that the AREA estimate of the distribution factor for timber sleepers, line B, is based upon Magee's estimate the line A being shifted upward on the vertical scale by 10 per cent. This would correspond to an equivalent factor of safety, F1 in Equation 4.6, of between 1.22 and 1.33.

When prestressed concrete sleepers were first introduced into the Americal railways networks in the early 1960s many inservice performance problems were encountered (Weber 1975). The AREA Special Committee on Concrete Ties (AREA 1975, Weber 1975) was established to investigate the causes of these performance problems and to recommend specifications for prestressed concrete sleepers which would reasonably guarantee reliable performance under mainline conditions. One of the Special Committee's recommendations was that the assumed distribution factor for timber sleepers, line B, should for concrete sleepers be shifted upward a further 5 per cent on the vertical scale, resulting in line C. Upon comparison with the original Magee estimate, line A, the net effect is to increase the factor of safety  $F_1$  in Equation 4.6 to between 1.33 and 1.50.

A distribution factor has been adopted for use with the design of steel sleepers and this is shown as line D in Figure 4.4 (Brown and Skinner 1978).





## Figure 4.4

# Approximate percentage of wheel load carried by an individual sleeper

#### The ORE Method

The ORE (1969) have developed a statistical method to calculate the maximum rail seat load on an individual sleeper. Based upon experimental data a formula was empirically derived for determining the maximum rail seat load  $q_r(kN)$ , i.e.,

$$q_r = \bar{\epsilon} c_1 P , \qquad (4.10)$$

where P = design wheel load, (kN) based upon the ORE formula for the impact factor,  $\varepsilon$  = dynamic mean value of the ratio  $\overline{q}_{r/\overline{p}_{g}}$  where  $\overline{q}_{r}$  and  $\overline{P}_{s}$  are the mean values of the rail seat load and the static axle load respectively,  $\varepsilon$  = the maximum value of the ratio  ${}^{q}r/P_{s}$ ,  $c_{1} = \frac{\varepsilon}{\varepsilon}$ , and  $c_{1}$  = is approximately equal to 1.35.

Measured values of  $\varepsilon$  are tabulated for various types of sleepers in Table 4.2.

## The applied wheel load on the rail considered as being distributed between three adjoining sleepers

Due to the effects of the unknown parameters it is commonly assumed that the maximum possible sleeper loading occurs when the wheel load on a rail is distributed between three adjacent sleepers. The load at the rail seat of the central sleeper is denoted by x per cent and the load at the rail seat of the two other adjacent sleepers is x/2 per cent. The value of x is commonly assumed as 50 per cent of the design wheel load (equivalent to 25 per cent of the design axle load). This simple assumption has been used by Raymond (1971), Heath and Cottram (1966), Eisenmann (1969a) and by the ORE (ORE 1968). The maximum rail seat load  $q_r(kN)$  is

	<sup>1</sup> S				
Sleeper Type	Sleeper Dimensions length x breadth x rail seat thickness (mm)	Sleeper spacing (mm)	Area of Sleeper support (10 <sup>3</sup> x mm <sup>2</sup> )	Q <sup>(a)</sup> (mm)	$\overline{\varepsilon} = \overline{q}_r / p_s$ (range)
French, type VW Prestressed Concrete	2300 x 250 x 140	600	200.0	400	0.56 - 0.59
British, type F Prestressed Concrete	2515 x 264 x 200	760	268.8	510	0.32 - 0.48
German, type 58 Prestressed Concrete	2400 x 300 x 190	600	252.0	450	0.32 - 0.44
French, Hardwood	2600 x 255 x 135	600	280.5	550	0.46 - 0.76

<u>TABLE 4.2 - VALUES OF THE RATIO</u>  $\bar{\epsilon} = \bar{q}r/\bar{p}$  <u>FOR VARIOUS SLEEPER TYPES AND SPACINGS (ORE 1969)</u>

(a) The area of sleeper support =  $2Q \times sleeper$  width at rail seat, where Q = distance of the centre line of the rail from the end of the sleeper.

$$q_{r} = 0.5P$$
,

where P = design wheel load (kN).

A comparison of the above methods used in the calculation of the maximum rail seat load is presented in Table 4.3. It is interesting to note that the value of the factor of safety  $F_1$  used by the AREA for concrete sleepers is approximately 1.33 to 1.50 whereas the ORE recommended value of  $C_1$  is 1.35. The determined value of  $C_1$  is based upon experimental data and would seem to be more justifiable for use than the AREA assumed values of  $F_1$ .

## TABLE 4.3 - COMPARISON OF THE FORMULAE USED FOR THE CALCULATION OF THE MAXIMUM RAIL SEAT LOAD

Method	Maximum Rail Seat Load (q <sub>r</sub> ) (kN)
3 Adjacent Sleepers Method	$q_r = 0.50P$
BEF Formula (O'Rourke 1978) (Mt Newman ore car axle spacing and sleepers at 760 mm centres)	$q_r = 0.43 p^{(a)}$
AREA Method (Birmann 1968) (Prestressed concrete sleepers at 760 mm centres)	$q_r = 0.60P$
ORE Method (ORE 1969) (BR Type F prestresse concrete sleepers at 760 mm centres)	$q_r = 0.65P$

(a) In BEF Formula, a factor of safety  $(F_1)$  to account for variations in the track must be applied to the value determined above. If  $F_1$  assumed to be 1.5, then  $q_r = 0.65P$ .

NOTE: Where P = design wheel load (kN), and the sleeper spacing adopted is 760 (mm) for the comparison.

In the selection of the required sleeper size and sleeper spacing, the conventional design procedure as illustrated by Figure 1.1, is essentially based upon a trial and error method. A sleeper size is adopted and a sleeper spacing is selected on the basis of

(4.11)

that which satisfied the tolerable limits of sleeper to ballast contact pressure and flexural capacity. The methods outlined previously to calculate the effective sleeper support area and the magnitude of the rail seat load are used in the following sections, which will discuss in detail:

- . the contact pressure between the sleeper and the ballast
- . the maximum allowable contact pressure
- . the maximum sleeper bending moments and bending stresses
- . the flexural requirements of various sleeper types.

Using an adopted sleeper size in conjunction with any selected design method the maximum allowable sleeper spacing is that which just satisfies the above sleeper design constraints.

THE CONTACT PRESSURE BETWEEN THE SLEEPER AND THE BALLAST

The general approach for the calculation of the contact pressure beneath the rail seat is to assume a uniform contact pressure distribution over the assumed effective area of the sleeper. This assumption is made in order to facilitate the ease of calculations. A factor of safety is then usually applied to account for variations in the sleeper support.

The average contact pressure between the sleeper and the ballast  $P_{a}$  (kPa) in all the developed formulae can be reduced to

$$P_a = (\frac{q_r}{B.L}) F_2$$
, (4.12)

where

re q<sub>r</sub> = maximum rail seat load (kN),

- B = breadth of sleeper (m),
- L = effective length of sleeper under the rail seat
   (m), and
- $F_2$  = factor depending upon the sleeper type and the standard of track maintenance.

Clarke (1957) has used this method to determine the average contact pressure between the sleeper and ballast; (L being determined by Equation 4.1).

Now, assuming that the effective length of the sleeper L, is approximately a third of the total sleeper length l(m), using Equation 4.2, it is possible to rewrite Equation 4.12 as

$$P_{a} = \begin{bmatrix} 3q_{r} \\ Bl \end{bmatrix} F_{2}$$
 (4.13)

The AREA recommends that when calculating the average contact pressure between the sleeper and the ballast the maximum rail seat load should be doubled and the average contact pressure calculated for the full length of the sleeper. The AREA has adopted this value of two for  $F_2$  to account for possible excessive contact stresses due to non uniform sleeper support caused by the lack of track maintenance. The result of this procedure is to effectively determine a maximum contact pressure between the sleeper and the ballast for the condition of non uniform sleeper support. This contradicts the initial assumption of uniform support over the entire sleeper length.

The AREA formula for calculating the average contact pressure  $P_a(kPa)$  between the sleeper and the ballast is

$$P_{a} = \begin{bmatrix} 2q_{r} \\ Bl \end{bmatrix} F_{2} , \qquad (4.14)$$

where  $\ell$  = total length of the sleeper (m)  $q_r$ , B and F<sub>2</sub> are as above.

Raymond (1977) has suggested that the value of  $F_2$  equal to 2 has been adopted by the AREA in order to account for the worst possible conditions of contact pressure between the sleeper and the ballast. This occurs when lateral forces are applied at the rail seat and are caused by flanging of the wheels against the rail head whilst travelling through curves. In Figure 4.5 it is suggested that extremely high lateral forces act simultaneously



## Figure 4.5

Effect of lateral loads on sleeper contact pressure and the A.R.E.A. recommended design assumption (Raymond 1977) on both rails in the same direction. In travelling through curves contact of the wheel on the rail takes place at two points on one rail, (at the rail head and at the side of the rail head). This loading condition results in a high lateral force on one rail caused by the contact of the wheel flange with the rail, but a smaller lateral force on the other rail caused by the component of the wheel load acting at the top of the rail head. Consequently, it would appear that the value of the safety factor has been based entirely upon the judgement of the railway authorities and researchers. In fact Clark (1957) has not recommended a value of  $F_2$  in his paper.

The assumed contact pressure distribution at the rail suggested for use by the AREA and referred to by Kerr (1976) and Schramm (1961) is shown in Figure 4.6. Both authorities assume that the contact pressure is uniform over the effective length L(m) given by

 $L = \ell - g, \qquad (4.15)$ 

where l = total length of the sleeper (m), andg = distance between rail centres (m).

This assumed presure distribution forms the basis of the sleeper moment calculations at the rail seat used by both authorities. According to Kerr (1976) the AREA has also assumed a pressure distribution under the middle portion of the sleeper for use in the calculation of the maximum midspan sleeper moment, and this is presented in Figure 4.6(b).

The ORE (1969) have carried out experiments to determine the contact pressure between the sleeper and the ballast due to the action of known wheel loads passing over the track. Tests were carried out for a range of:

- . sleeper types
  - timber
  - concrete

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- sleeper spacings
  - 630 mm
  - 760 mm

type of ballast packing condition under the sleeper

- manual
- portable electric hammer
- machine packed.

The experimental values of the contact pressure were reduced to a normalised form  $p_N(kPa/kN)$ , by dividing the observed contact pressure  $p_O(kPa)$  with the rail seat load  $q_O(kN)$  causing that contact pressure, i.e.,

$$p_{\rm N} = \frac{p_{\rm o}}{q_{\rm o}}$$
 (4.16)

Results indicate that the manual packing method produces contact pressure values some 40 per cent higher than the pressures observed with both the portable electric hammer and the fully mechanised packing methods.

The general trend of these results are as follows (ORE 1969):

- . the peak normalised contact pressures  $\mathbf{p}_{\mathrm{N}}$  are in the range of 5 to 7 kPa/kN
- . the maximum contact pressures occur in the vicinity of the rail seat and the minimum contact pressures occur in the middle zone of the sleeper; and this is clearly illustrated in Figure 4.7.

The ORE concluded the following from these experiments:

(a) The sleeper to ballast contact pressure distribution is very much a function of the ballast packing method used, and that it tends to be, "unpredictable and random with a high degree of scatter".



Figure 4.6(a)

Assumed distribution for the determination of the maximum sleeper bending moment at rail seat as used by Battelle (Prause et al 1974) the area (Kerr 1976) and Schramm (1961)



## Figure 4.6(b)

Assumed distribution for the determination of the maximum sleeper bending moment at centre of the sleeper, as used by Battelle (Prause et al 1974) and the area (Kerr 1976)

Assumed contact pressure distributions between sleeper and ballast for sleeper bending moment calculations



Effective support length under each rail approaches  $\ell/3$ 

Note: Thickness of construction 230 mm (9 in) Concrete sleepers at 790 mm (31 in) Portable electric hammer packing

## · · · · ·

Mean of two passes of two axles Force balance error = 9.0%

## Figure 4.7

Measured sleeper soffit to ballast contact pressure for concrete sleepers and an assumed idealised contact pressure distribution (ORE 1968)

- (b) As the contact pressures developed under timber sleepers are of the same order of magnitude as those beneath concrete sleepers, it was considered that flexural rigidity of sleepers was a secondary influence on the magnitude of the contact pressure, (i.e. sleeper size and spacing having a far more significant influence).
- (c) Sleeper spacing does not appear to influence the contact pressures when defined in a normalised form although absolute values of contact pressure vary with the sleeper spacing.

Due to the fact that the contact pressures under the sleeper are of a random nature the ORE has suggested that an "equivalent" uniformly distributed pressure distribution in the region under the rail seat be used in the calculations of the sleeper bending moment, (Figure 4.7).

Using this assumed pressure distribution it was observed with concrete sleepers, 2.510 m long and 0.260 m wide, that the maximum normalised contact pressure between the sleeper and the ballast was

 $P_{\rm N}$  = 4.75 kPa/kN ,

i.e. 4.75 kPa of contact pressure between the sleeper and the ballast for every 1 kN of rail seat load. It is evident that this observed value of  $p_N$  is dependent upon the sleeper size.

#### The maximum allowable contact pressure

The recommended limit of the maximum allowable bearing pressure between the sleeper and the ballast varies considerably with railway authorities. All the suggested limits do not make allowance for changes in sleeper size or variations in sleeper support. It also is not clearly apparent whether these suggested limits are based on experimental data of ballast crushing or on an arbitrary selection of a likely limit. Combining Equations 4.9 and 4.14 and using the recommended value of  $F_2$  equal to 2, the AREA formula used to calculate the allowable bearing pressure between the sleeper and the ballast  $p_a$  (kPa) for timber sleepers is

$$P_{a} = \frac{4 P(DF)}{Bl}$$
(4.17)

where P = design wheel load (kN) (i.e. static wheel load multiplied by the AREA impact factor),

- DF = the AREA distribution factor (Figure 4.4 line B)
  for timber sleepers,
- B = breadth of sleeper (m), and
- $\ell$  = total length of sleeper (m).

Using this equation, the AREA design manual recommends for timber sleepers a maximum allowable contact pressure between the sleeper and the ballast of 450 kPa, (65 psi) (AREA, 1973). For the case of concrete sleepers the AREA Special Committee on Concrete Ties (AREA 1975, Weber 1975) has specified that the contact pressure between the sleeper and the ballast  $p_a(kPa)$  be calculated using the formula

$$p_a = \frac{2P_s (DF) (1+\phi)}{B\ell}$$
, (4.18)

where  $P_{c}$  = static wheel load (kN),

DF = the AREA distribution factor for concrete sleepers (Figure 4.4, line C),

\$\$\overline = impact factor, the AREA assumed value for all
conditions is 1.5,

B = breadth of sleeper (m), and

 $\ell$  = total length of sleeper (m).

The significance of the above AREA procedures for determining the contact pressure between the sleeper and the ballast will now be discussed. Assuming that the impact factor used in Equation 4.17 has a value of 1.5, and that both timber and concrete sleepers used have identical dimensions. For a sleeper spacing of 600 mm the AREA distribution factors obtained from Figure 4.4 for timber and concrete sleepers are 0.45 and 0.5 respectively. Upon

substitution of the above values into Equations 4.17 and 4.18 it is apparent that the magnitude of the contact pressure between the sleeper and the ballast obtained are approximately the same. Notwithstanding this observation, it should be emphasised that the AREA recommended design procedure for concrete sleepers significantly increases the design bending moment capacity of the sleeper at both the rail seat and centre region.

When using the above equation the AREA recommends that the average contact pressure between the sleeper and the ballast should for concrete sleepers not exceed 590 kPa, (85 psi). This limit was suggested for high-quality-abrasion-resistant ballast and if lower quality ballast materials are used it should be reduced accordingly<sup>(1)</sup>.

The above recommendations of the maximum allowable contact pressure between the sleeper and the ballast are all based upon factored values of the calculated maximum rail seat load. When using Equation 4.12 to calculate the contact pressure between the sleeper and the ballast; Clarke (1957) has recommended a maximum allowable limit for timber sleepers of 240 kPa (35 This limit is based on an unfactored average contact psi). pressure. Eisenmann (1969a) has also stated that special steps should be taken to ensure that the bearing pressure between the sleeper and the ballast does not exceed 300 kPa. This limit also appears to be based on the effective area of sleeper contact, Equation 4.12, and the apparent unfactoring of the rail seat load. The differences between the above recommendations are entirely due to the design assumptions used in the alternative methods. The AREA use a factored rail seat load value and calculates the uniform average contact pressure for the full sleeper length, whereas Clarke and Eisenmann use an unfactored rail seat load, but calculate the uniform average contact pressure for the effective length under the rail seat.

(1) The AREA do not suggest any method to determine this reduced contact pressure limit.

Using an adopted sleeper size in conjunction with any of the above design methods a first estimate of the maximum allowable sleeper spacing is obtained, and is that which satisfies the recommended limits of the allowable contact pressure. The preliminary sleeper size together with the trial sleeper spacing is further analysed in order to check that the sleeper flexural limits are also not exceeded.

THE CALCULATION OF THE MAXIMUM SLEEPER BENDING MOMENTS

As previously mentioned the adopted sleeper size together with the first estimate of a trial sleeper spacing is further analysed to check the sleepers flexural performance under the service load conditions. The maximum bending moments and bending stresses occur at the following locations along the sleeper length:

- . at the region of the rail seat
- . at the centre region of the sleeper.

Consequently the following sections will analyse the sleeper flexural performance and requirements at these two locations.

Currently, sleepers are manufactured from the following engineering materials: timber, (both hardwood and softwood), concrete (both prestressed and reinforced) and steel. The analysis of the maximum sleeper bending at the rail seat and at the centre of the sleeper is identical regardless of sleeper material used, but the type of flexural limits imposed upon the sleeper varies according to the sleeper material type. Thus the flexural limit for both timber and steel sleepers is essentially the allowable bending stress, whereas for concrete sleepers, notably of the prestressed type, the flexural limit is usually expressed in the form of a positive or negative bending moment capacity.

#### The maximum sleeper bending moment at the rail seat

The fundamental basis of the methods proposed by Clarke (1957), Schramm (1961) and Battelle (Prause et al. 1974) for determining the maximum sleeper bending moment at the rail seat is the assumption of a uniform effective contact pressure distribution between the sleeper and the ballast. Kerr (1976) states that calculations based upon this simple assumption yield upper bound solutions for the expected sleeper bending stresses and are therefore considered sufficient for sleeper design purposes. The following are the commonly used design methods for determining the maximum sleeper bending moment at the rail seat.

Solution according to Battelle: Batelle (Prause et al. 1974) has indicated that the equations derived by Clarke (1957) for determining the bending moment at the rail seat are dimensionally inconsistent, (the assumed uniform distributed load was inappropriate. The following formulations have been suggested by Battelle for use in the analysis of sleeper bending moments:

(a) The upper bound solution for the maximum sleeper bending moment at the rail seat can be calculated by considering the sleeper is in the "end-bound" condition. Thus the entire support offered by the ballast to the sleeper is considered as a point load, equal to the rail seat load, located at the end of the sleeper, Figure 4.8(a). The solution for the maximum bending moment at the rail seat M<sub>r</sub>(kNm) is the simple couple

$$M_r = q_r \left(\frac{\ell - g}{2}\right) ,$$

(4.19)

where

l

= overall sleeper length (m), and

 $q_r$  = maximum rail seat load (kN),

g = distance between rail centres (m).

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(b) A less conservative and probably more realistic solution is recommended by Battelle (Prause et al. 1974). This solution assumes that half the rail seat load is distributed over the sleeper overhang length (l-g)/2 Figure 4.8(b). The maximum sleeper bending moment at the rail seat  $M_r$  (kNm) is therefore

$$M_{r} = q_{r} \frac{(\ell - g)}{8}$$
(4.20)

where  $q_r$ , l and g are as previously defined.

The above solutions are independent of the type of sleeper material used in the analysis.

Solution according to Schramm: For sleepers fitted with sleeper bearing plates the effect that the plate has on the calculations of the maximum sleeper bending moment at the rail seat has been considered by Schramm (1961), Figure 4.8(c). The maximum sleeper bending moment at the rail seat  $M_r$  (kNm) for this condition can be calculated by

$$M_{r} = q_{r} \frac{(l-q-j)}{8} , \qquad (4.21)$$

where  $j = \text{length of sleeper bearing plate (m), and} q_r$ , l and g are as previously defined.

#### The maximum sleeper bending moment at the centre of the sleeper

The maximum sleeper bending moment at the centre of the sleeper occurs when the sleepers are said to be centrebound. This leads to a problem in defining what the contact pressure distribution is for the centrebound condition. Talbot (1910-1934) observed that for the centrebound condition the pressure distribution between the sleeper and the ballast approached a uniform distribution (Figure 4.1). The following are the commonly used design methods for determining the maximum sleeper bending moment at the centre of the sleeper.



Solution for sleepers fitted with bearing plates (Schramm 1961)

Estimation of maximum sleeper bending moment at the rail seat

<u>Solution according to Battelle</u>: According to Battelle (Prause et al. 1974) the bending moment at the centre of the sleeper can be calculated by assuming that the sleeper is centrebound. Battelle represents this condition by assuming the sleeper is resisted by a single point load at the centre of the sleeper, and the maximum negative bending moment at the centre of the sleeper M<sub>C</sub>(kNm) can be calculated from

 $M_{c} = q_{r} \frac{g}{2}$  , (4.22)

where q<sub>r</sub> = maximum rail seat load (kN), and g = distance between rail centres (m).

This solution proposed by Battelle can only be regarded as extremely unlikely to occur in practice. If the support condition, (and the implied contact pressure) under the sleeper reached this state due to the lack of track maintenance, the load on the sleeper would most probably be redistributed to other sleepers. Nevertheless this expression can be regarded as being the upper bound solution for the maximum sleeper bending moment at the centre of the sleeper.

<u>Solution according to Raymond</u>: Raymond's (1977) definition of the assumed centre bound condition is based upon earlier experimental work carried out by Talbot (1918-1934). The solution for the maximum bending moment at the centre of the sleeper is therefore calculated on the basis of an assumed uniform bearing pressure over the total length of the sleeper <sup>(1)</sup>. The maximum negative bending moment at the centre of the sleeper  $M_c$ (kNm) can be calculated from

(1) For this assumption the uniform pressure between the sleeper and the ballast p(kPa) is  $p = 2q_r/\ell B$ where B = sleeper breadth (m).

$$M_{c} = q_{r} \left[ \frac{g}{2} - \frac{\ell}{4} \right]$$
$$= q_{r} \left[ \frac{2g - \ell}{4} \right]$$
(4.23)

where

q<sub>r</sub> = maximum rail seat load (kN), g = distance between rail centres (m), and

l = total sleeper length (m).

## THE FLEXURAL CAPACITY AND REQUIREMENTS OF SLEEPERS

As previously mentioned the type of flexural limit imposed upon the sleeper varies according to the sleeper material type used in the track design. The flexural limits of timber and steel sleepers are based upon an allowable bending stress limit. Whereas with prestressed concrete sleepers the flexural limit is entirely based upon the designed bending moment capacity of the manufactured sleeper. In the following sections the current methods of analysis used to determine the flexural performance of timber, prestressed concrete and steel sleepers are outlined.

#### The Flexural Requirements of Timber Sleepers

The maximum sleeper bending stress at the rail seat: With timber sleepers the cross sectional shape is rectangular and uniform along the entire sleeper length consequently the section modulus of the sleeper  $z \text{ (m}^3)$  is

$$Z = \frac{Bt^2}{6}$$
, (4.24)

where B = sleeper breadth (m), and t = sleeper thickness (m).

Therefore the upper bound estimate for the sleeper bending stress at the rail seat  $\sigma_{ru}$  (MPa) can be calculated from equations 4.19 and 4.24 and is
$$\sigma_{ru} = \frac{M_r}{Z} = 3.q_r \frac{(l-g)}{10^3 \text{ Bt}^2}, \qquad (4.25)$$

Clarke (1957) uses a similar approach but states that the rail seat load determined using the beam on an elastic foundation analysis (Equation 4.6 and  $F_1 = 1$ ) should be doubled when calculating the sleeper bending stress at the rail seat in order to allow for variations in the effective sleeper support condition. Consequently if Clarke's approach was followed Equation 4.25 would be multiplied by 2.

Using Equations 4.20 and 4.24 a less conservative estimate of the sleeper bending stress at the rail seat  $\sigma_r$  (MPa) is

$$\sigma_{r} = \frac{3}{4} q_{r} \frac{(l-g)}{10^{3} Bt^{2}} , \qquad (4.26)$$

where q<sub>r</sub>, *l*, g, B and t are as previously defined.

Using Equations 4.21 and 4.24 the sleeper bending stress at the rail seat  $\sigma_r$  (MPa) for sleepers fitted with bearing plates is

$$\sigma_{r} = \frac{3}{4} q_{r} \frac{(l-g-j)}{10^{3} Bt^{2}} , \qquad (4.27)$$

where j = length of sleeper bearing plate (m), and q<sub>r</sub> l, g, B and t are as previously defined.

It is apparent that the sleeper bending stress at the rail seat can be reduced considerably when sleepers are fitted with bearing plates. For example, consider a standard gauge track comprising of 54 kg/m rails laid upon 2.440 m long timber sleepers. Using identical values of the rail seat load the sleeper bending stress at the rail seat can be reduced by roughly 30 per cent when 0.300 m wide sleeper plates are fitted between the rail and the sleeper.

The maximum sleeper bending stress at the centre of the sleeper: Depicting the sleeper centrebound conditions as being represented by an assumed uniform pressure distribution over the total sleeper length, the maximum bending stress at the centre of the sleeper  $\sigma_{c}$ '(MPa) can be calculated using Equations 4.23 and 4.24 as follows:

$$\sigma_{c}' = \frac{3}{2} q_{r} \frac{(2g-\ell)}{10^{3} \text{ Bt}^{2}} , \qquad (4.28)$$

where

 $q_r = rail seat load (kN),$ 

- l = total sleeper length (m),
- g = distance between rail centres (m),
- B = sleeper breadth (m), and
- t = sleeper thickness (m).

The maximum allowable tensile bending stress of timber sleepers: Notwithstanding the effects of non uniform properties, the maximum tensile bending stress of timber sleepers varies predominantly with the following considerations:

- . the timber type, which in general terms can be classified as either a hardwood or a softwood
- . whether the sleepers are treated or untreated with preservatives

. the timber moisture content.

Clarke (1957) recommends that the maximum tensile bending stress for timber sleepers should not exceed 5.5 MPa, whereas the limiting value recommended by the AREA (1973) is 7.6 MPa. It should be noted that both Clarke and the AREA do not make any distinction between types of timber sleepers. The American timber types used in the manufacture of sleepers are typically of

the softwood variety, and it would therefore appear reasonable to assume that the AREA recommendation applies to softwood sleepers. Battelle (Prause et al. 1974) states that the maximum allowable tensile bending stress recommended by the AREA corresponds to a value determined for the lowest grades of oak and pine sleepers which had been subjected to long duration loading and also exposed to wet conditions coupled with a moderate decay hazard. Battelle also states that the endurance limit of sleepers subjected to cyclic loading has been experimentally determined to be 28 per cent of the modulus of rupture, E<sub>rupt</sub>, (for American timber types). Experimentally determined values of the modulus of rupture for Australian timber types have been quantified by Duckworth (1973) and Reid (1973) and these are presented in Tables 4.4 and 4.5. Also in these tables is presented an estimate of the sleeper endurance limit based upon the experimental modulus of rupture values of new sleepers. The variation of sleeper strength with the condition of the new hardwood sleeper is presented in Table 4.4. Upon comparison of the experimentally determined ultimate strength of treated and untreated sleepers it is noticeable that the treated sleepers show a marked reduction in flexural strength.

TABLE	4.4	-	EXE	'ERIM	ENT	LLY	DETE	RMINE	D VA	ALUES	S OF	THE	AVI	ERAGE	MODU	JLUS
			OF	RUPT	URE	AND	ESTI	MATES	OF	THE	END	JRAN	CE ]	LIMIT	FOR	
			VAF	IOUS	NEV	HAI	RDWOOI	) SLE	EPEI	R CON	IDIT:	IONS	(DI	JCKWO	RTH	
			197	73												

Hardwood Sleeper Conditions	Average Modulus of Rupture (from 12 tests) E <sub>rupt</sub> (MPa)	Estimated Endurance Limit (28 per cent of E <sub>rupt</sub> ) (MPa)
Dry untreated	110	32
Green untreated	80	23
Green treated and incised	78	22
Dry treated	61	17
Green treated	47	13

### TABLE 4.5 - EXPERIMENTALLY DETERMINED MODULUS OF RUPTURE VALUES FOR VARIOUS TIMBER SLEEPER

TYPES AND AGE CONDITION TOGETHER WITH AN ESTIMATE OF THE ENDURANCE LIMIT OF THE

NEW SLEEPERS (REID 1973)

Sleeper Type and Condition	Sleeper Age	Sleeper Cross Section Width x Depth	Modulus of Rupture <sup>E</sup> rupt	Endurance Limit of New Sleepers (28 per cent of <sup>(E</sup> rupt)
	(years)	(mm)	(MPa)	(MPa)
Softwood Softwood	New New	233 x 173 230 x 170	34 22	10 6
Softwood Softwood	37 37	238 x 165 236 x 166	20 20	-
Hardwood	New	257 x 133	55	16
Hardwood Hardwood	37 37	255 x 128 267 x 127	23 19	-
Hardwood (one outside spike) Hardwood	17 17	235 x 117 248 x 117	46 34	-
Hardwood (two outside spikes) Hardwood	17	251 x 114	23	-
(two outside spikes	17	251 x 121	29	-

It is significant that the estimated endurance limit of an Australian softwood sleeper, (Table 4.5) is of the same order of magnitude as the AREA recommendation of maximum allowable tensile bending stress. It is also apparent that the estimated endurance limit of the hardwood sleeper, (Table 4.5) is still less than the modulus of rupture values of hardwood sleepers that have been subjected to 37 years of service life under secondary line conditions with wetting and drying and decay hazard. The reduction of the sleeper strength caused by respiking operations is also shown in Table 4.5.

It should be noted that the condition of aged timber sleepers is also dependent upon the cumulative volume of traffic carried at a given time. Under mainline conditions it is therefore anticipated that the life of the sleeper would be governed by other service factors before the sleeper strength is inadequate due to age condition alone.

The maximum sleeper bending stress determined at the rail seat and the centre of the sleeper is compared with the allowable tensile bending stress<sup>(1)</sup>. If for a particular adopted sleeper size this limiting stress is found to be exceeded, the first estimate of the sleeper spacing is further reduced until the limiting condition is satisfied. If the required sleeper spacing determined is considered impractical another sleeper size is selected and the entire sleeper design is recommenced.

#### The calculation of the minimum required timber sleeper thickness

The minimum required sleeper thickness for timber sleepers can readily be calculated by rearranging the equations which determine the maximum sleeper bending stress at the rail seat and the centre of the sleeper. Into these rearranged equations the

It is apparent that the current limits of the allowable tensile bending stress of timber sleepers have been established primarily for the area of the rail seat.

maximum allowable ballast contact pressure and the maximum allowable sleeper tensile bending stress are substituted for the effective contact pressure and the actual sleeper bending stress respectively. Thus the sleeper thickness obtained is the minimum which satisfied the strength criterian. The final sleeper thickness is determined by adding to this minimum thickness an allowance for weathering and decay deterioration.

The minimum required sleeper thickness at the rail seat for sleepers without bearing plates fitted between the rail and the sleeper: The minimum sleeper thickness  $t_{min}(m)$  at the rail seat can readily be calculated by rearranging Equation 4.26 and substituting values of the maximum allowable contact pressure between the sleeper and the ballast  $p_{all}(kPa)$  and the maximum allowable sleeper tensile bending stress  $\sigma_B(MPa)$  for the calculated values of the bearing pressure and sleeper bending stress respectively. Since the maximum allowable contact pressure between the sleeper and the rail seat can be calculated by either the AREA method or by Clark's method therefore two solutions of the minimum required sleeper thickness can be stated.

<u>The AREA solution</u>: Rearranging Equation 4.14 the equation for maximum allowable rail seat load  $q_{max}(kN)$  becomes

$$q_{max} = \frac{P_{all(1)}^{B.\ell}}{2F_2}$$
 (4.29)

where  $P_{all(l)} = AREA$  limit of allowable contact pressure (KPa) = 450 (kPa), B = sleeper breadth (m),  $\ell$  = sleeper length (m), and  $F_2$  = track maintenance factor, according to the AREA  $F_2$  = 2.

Equating  $q_{max}$  to  $q_r$  in Equation 4.26 and rearranging, the minimum required sleeper thickness  $t_{min}$  (m) can be calculated from

$$t_{\min} = \begin{bmatrix} \frac{3\ell \ (\ell-g) \ P_{all(1)}}{8.10^3 \ \sigma_B \ F_2} \end{bmatrix}^{0.5}, \qquad (4.30)$$

where

σB

q

allowable sleeper tensile bending stress (MPa), and = distance between rail centres, and  $P_{a11(1)}$ , *k* and  $F_2$  are as previously defined.

The Clarke solution: As previously mentioned, Clarke's equation for bending moment is dimensionally inconsistent. Notwithstanding this fact, the equation proposed by Clarke to estimate the maximum sleeper bending stress at the rail seat is of the same form as Equation 4.25. It should also be noted that when determining the maximum sleeper bending stress Clarke suggests that the rail seat load should be multiplied by a suitable factor to account for variations in sleeper support. Rearranging Equation 4.12 the maximum allowable rail seat load  $q_{max}(kN)$  becomes <sup>(1)</sup>

$$q_{max} = \left[\frac{P_{all(2)} B (l-g)}{F_2}\right]^{0.5}$$
, (4.31)

 $P_{all(2)} = Clarke's limit of allowable contact pressure (kPa)$ where = 240 (kPa), b = sleeper breadth (m), l = sleeper length (m), = distance between rail centres, and g track maintenance factor, according to Clarke,  $F_2$ =  $F_2 = 2.$ 

Equating q<sub>max</sub> to q<sub>r</sub> in Equation 4.25 and rearranging, the minimum required sleeper thickness t<sub>min</sub> (m) can be calculated from

(1)Assuming that the effective length L = (l-q).

$$t_{\min} = (\ell - g) \begin{bmatrix} \frac{3 P_{all(2)}}{10^{3} \sigma_{B} F_{2}} \end{bmatrix}^{0.5}, \qquad (4.32)$$

where  $\sigma_B$  = allowable sleeper tensile bending stress, and  $P_{all(2)}$ ,  $\ell$ , g and  $F_2$  are as previously defined.

The minimum required sleeper thickness at the rail seat for sleepers with bearing plates fitted between the rail and the sleeper: Using a similar approach to the AREA solution for sleepers without sleeper plates, and rearranging Equation 4.27 the minimum required sleeper thickness at the rail seat  $t_{min}(m)$  can be calculated from

$$t_{\min} = \begin{bmatrix} \frac{3\ell}{8.10^3} & (\ell - g - j) & P_{all(1)} \\ \frac{3\ell}{8.10^3} & \sigma_B F_2 & 10^3 \end{bmatrix}^{0.5},$$
(4.33)

where

j = length of sleeper bearing plate (m), and  $P_{all(1)}$ ,  $\ell$ , g,  $\sigma_B$ , and  $F_2$  are as previously defined.

The minimum required sleeper thickness at the centre of the sleeper: The minimum required sleeper thickness at the centre of the sleeper can be calculated by rearranging the solution for the maximum bending stress determined by Equation 4.28 and substituting values of the maximum allowable contact pressure between the sleeper and the ballast  $p_{all}$  (kPa) and the maximum allowable sleeper tensile bending stress  $\sigma_B$  (MPa) for the calculated values of the bearing pressure and sleeper bending stress respectively. Using a similar approach to the AREA solution above the minimum required sleeper thickness  $t_{min}$  (m) at the centre of the sleeper can be calculated from

$$t_{\min} = \begin{bmatrix} \frac{3\ell \ (2g-\ell) \ P_{all}(1)}{4 \cdot 10^3 \ \sigma_B \ F_2} \end{bmatrix}^{0.5} , \qquad (4.34)$$

where  $P_{all(1)}$ ,  $\ell$ , g,  $\sigma_B$  and  $F_2$  are as previously defined.

If the area approach to the minimum required sleeper thickness were adopted, the value used would be the larger of the solutions to Equations 4.30 or 4.33 and 4.34.

# The Calculation of the required minimum flexural capacity of prestressed concrete sleepers

Theoretically the equations developed to calculate the maximum bending moment at the rail seat and at the centre of the sleeper are equally valid for timber concrete and steel sleepers. In the following section two current methods specifically developed for the calculation of the required minimum flexural capacity of prestressed concrete sleepers will be presented. These methods being the AREA design method (AREA 1975) and the ORE design method (ORE 1969).

The AREA Design Method: At the basis of the AREA design method recommended by the Special Committee on Concrete (AREA 1975) is the assumed contact pressure distribution between the sleeper and the ballast. The AREA state that over a period of time, because of repeated loads, vibration and crushing of the ballast, the ballast will gradually compact, moving away from the areas of greater concentration. The sleeper therefore settles slightly into the ballast, allowing the centre portion of the sleeper to resist some of the applied load, thus reducing the amount of load carried by the sleeper ends. Due to this gradual redistribution of contact pressure, the AREA design method assumes a uniform contact pressure distribution occurs over the entire sleeper length. This support condition produces positive flexure at the rail seats and negative flexure at the centre of the sleeper.

The approach of determining the required flexural capacity of prestressed concrete sleepers is consistent with the AREA method of determining the rail seat load, Equation 4.9 and the AREA method of determining the contact pressure between the sleeper and the ballast, Equation 4.18. The rail seat load  $q_r(kN)$  thus determined is expressed as

$$q_r = P_s(DF) (1 + \phi)$$
,

where

Ps

ø

= static wheel load (kN),

DF = distribution factor, expressed as a percentage of the wheel load (refer to Figure 4.4, line C), and

= impact factor (the AREA assumed value for all conditions is 1.5).

Therefore the assumed uniformly distributed load W (kN/m) over the entire sleeper length l(m) is

$$W = \frac{2q_r}{\ell} . \tag{4.36}$$

It should be noted that in Equation 4.35 the effective assumed value of the impact factor is 2.5. One of the main reasons why the impact factor was significantly increased was that many early U.S. prestressed sleepers had tendon bond length failures in the region of the rail seat (Weber 1973). The Special Committee on Concrete Ties (AREA 1975) recommended that one of the measures that should be used to alleviate this performance problem was to design the prestressed concrete sleeper to resist a much higher design load.

According to the AREA Special Committee on Concrete Ties the flexural design requirements for prestressed concrete sleepers can be calculated from the following equations (AREA 1975):

(a) The maximum positive sleeper bending moment at the rail seat  $M_r(kNm)$  is given by

$$M_{r} = \frac{W (l-g)^{2}}{8}, \qquad (4.37)$$

where

g

W

 $\ell$  = total sleeper length (m),

- = distance between rail centres (m), and
- = assumed uniformly distributed load (kN/m).

(4.35)

(b) The maximum negative sleeper bending moment at the centre of the sleeper M<sub>c</sub>(kNm) is given by

$$M_{c} = \frac{Wg^{2}}{8} - M_{r}$$
 (4.38)

Substitution of Equation 4.37 reduces Equation 4.38 to Equation 4.23, i.e.,

$$M_{c} = q_{r} \frac{(2g-l)}{4}$$
, (4.39)

where  $q_r, l$  and g are as previously defined.

The bending moments calculated by this method can be regarded as being adequate to insure that the prestressed sleepers will not crack under normal service conditions. The required flexural capacity of various prestressed concrete sleepers to meet the AREA design method are presented in Table 4.6. It should be noted that the AREA method of determining the maximum rail seat load is implicit in this tabulation.

The ORE Design Method: The ORE design method (ORE 1969) is based upon recorded experimental data on the performance of various prestressed concrete sleepers under actual track and laboratory conditions. From these experiments an empirical design method was established to determine the minimum required flexural capacity of prestressed concrete sleepers.

According to the ORE, the maximum rail seat load  $q_r(kN)$  can be expressed as

$$q_r = \overline{\xi} c_1 P, \qquad (4.40)$$

where P = design wheel load, based upon the ORE formula for the impact factor,  $\bar{\xi}$  = dynamic mean value of the ratio  $\bar{q}_r/\bar{p}_s$  (Table 4.2), and  $c_1 = \frac{\xi}{\bar{\xi}} = 1.35$ .

Length (a)	Spacing <sup>(b)</sup>	Reg	uired Flexural	l Capacity (	kNm)
(m)	(mm)	Rail Seat +	Rail Seat - <sup>(C</sup>	<sup>C)</sup> Centre -	Without Cracking Centre +
2.44	533	24.9	13.0	22.6	10.2
	610	24.9	13.0	24.9	10.2
	686	24.9	13.0	27.1	10.2
	762	24.9	13.0	29.4	10.2
2.51	533	25.4	13.0	22.6	10.2
	610	26.6	13.0	23.7	10.2
	686	28.3	13.0	24.9	10.7
	762	29.4	13.0	25.0	11.3
2.59	533	25.4	13.0	22.6	10.2
	610	28.3	13.0	22.6	10.2
	686	31.1	13.0	22.6	11.3
	762	33.9	13.0	22.6	12.4
2.67	533	28.3	13.0	22.6	10.7
	610	31.1	13.0	22.6	12.4
	686	33.9	13.0	22.6	13.6
	762	36.7	13.0	22.6	13.6
2.74	533	31.1	13.0	22.6	11.3
	610	33.9	13.0	22.6	11.9
	686	36.7	13.0	22.6	13.0
	762	39.6	13.0	22.6	14.1

TABLE 4.6 - FLEXURAL PERFORMANCE REQUIREMENT FOR PRESTRESSED MONO-BLOCK CONCRETE SLEEPERS (AREA, 1975)

- (a) Constant top width across sleeper. Reduced bottom width at centre of sleeper Increase rail seat and centre positive flexural requirements by 10 per cent. Reduce centre negative flexural requirement by 10 per cent.
- (b) Interpolate for intermediate sleeper spacings.
- (c) Based on elastic fastenings with upward spring rate of 2.26 to 4.0 x  $10^{6}$  N/m.

The ORE experiments indicated that the actual stresses in the sleeper during dymanic testing are higher than the theoretically determined stresses. This dynamic stress increment also varies according to the location in the sleeper i.e., the dynamic stress increment at the rail seat differs from that at the centre of the sleeper. Due to the empirical nature of the ORE design formulae

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a sleeper parameter was chosen to adjust the theoretical bending moment calculation to the observed bending moment in the sleeper. The parameter chosen to adjust the bending moment calculation was the "lever arm": half of the sleeper end distance Q(m), reduced by half of the length b(m), (at the level of the neutral axis of the sleeper) which can be determined by assuming a  $45^{\circ}$  load spread of the rail seat load through the sleeper material (Figure 4.9).

This lever arm  $\Lambda$  (m) is therefore

$$\Lambda = \frac{Q - b}{2}. \tag{4.41}$$

The theoretical bending moment at the rail seat  $M_r$  (kNm) can readily be calculated using this lever arm (Figure 4.9), i.e.,

$$M_r = q_r \frac{\Lambda}{2} . \qquad (4.42)$$

The amount required to factorize this theoretical bending moment thereby enabling the calculation to satisfy the experimentally measured maximum bending moment, was determined empirically.

The maximum "factorized lever arm"  $\Lambda_m$  corresponding to the maximum observed sleeper bending moment at the rail seat due to dynamic effects was stated to be

 $\Lambda_{\rm m} = \Phi \Lambda \tag{4.43}$ 

where  $\phi$  = coefficient of the increment of the sleeper bending moment at the rail seat due to dymanic effects.

The maximum value of the empirical coefficient  $\phi$  was found to approximately equal 1.6 when the observed sleeper bending moments at the rail seat were analysed. Combining Equations 4.40, 4.42 and 4.43 the maximum sleeper bending moment at the rail seat  $M_{r}$  (kNm) can be empirically calculated from

$$M_{r} = \overline{\xi} c_{1} \Phi \frac{\Lambda}{2} P, \qquad (4.44)$$

Similarly an empirical coefficient  $\Xi$  relating to the increment of the bending stress in the central section of the sleeper due to dynamic effects was also determined from experimental observations. The value of  $\Xi$  was found to approximately equal 1.2.

The maximum sleeper bending moment at the centre of the sleeper  $M_{c}$  (kNm) was determined empirically to be

$$M_{c} = M_{r} \equiv \frac{EI (at the centre of the sleeper)}{EI (at the rail seat)}$$
 (4.45)

The main problem with the ORE design method is that there is no adequate theoretical backing and therefore no real way of determining improvements in the sleeper design and performance.

### The flexural requirements of steel sleepers

Several types of metal sleeper have been developed principally from steel and cast iron (e.g. the I beam and Pot types respectively). Modern steel sleepers have an inverted trough shape cross-section which permits efficient use of material and provides for excellent mating between the sleeper and ballast structure. There are two fundamentally different types of trough sleeper, namely, those of uniform wall thickness and those with crosssectional shape comprising a thick "top", or web, thin "legs" and bulbous "toes". The former is manufactured by pressing flat plate and the latter by hot rolling the final shape. The latter method permits more nearly optimum use of material and is particularly suited to heavy sleepers where the cost saving in material outweighs the saving due to simplicity of the other method of manufacture.

Rail fasteners suitable for steel sleepers comprise two categories: those with a rail base plate and those without. The former type are usually attached to the sleeper by welding and the latter



Note: Lever Arm  $\Lambda = \frac{Q-b}{2}$ 

**Figure 4.9** Determination of "lever arm"  $\Lambda$  (ORE 1969)

either by welding or by notches and tunnels pressed in the sleeper. The use of high performance bolts is not favoured due to the relatively higher cost of the bolt compared to an equivalent weld (Brown & Skinner 1978).

A tabulation of proposed BHP steel sleeper section properties suitable for heavy, medium and light axleload conditions is presented in Table 4.7.

Using the equations developed to calculate the maximum sleeper bending moments' at the rail seat and at the centre of the sleeper, the maximum tensile and compressive sleeper bending stresses at each of the locations can be determined by simple applied mechanics. At the rail seat the maximum compressive bending stress occurs at the web of the steel sleeper section. Whereas for an assumed centrebound condition the maximum compressive bending stress occurs at the toe of the steel sleeper section.

Thus at the rail seat the maximum sleeper compressive bending stress  $f_{\rm c}$  occurs at the web and is calculated by

$$f_{\rm c} = \frac{M_{\rm r} \, 10^6}{Z_{\rm w}} \, , \qquad (4.46)$$

where  $M_r = \max_{(kNm)} \text{ (from Equation 4.20), and}$  $Z_w = \text{steel sleeper section modulus about the web (mm^3);}$ 

whereas the maximum sleeper tensile bending stress  $f_t$  occurs at the toe and is calculated by

$$f_{t} = \frac{M_{r} \, 10^{6}}{Z_{t}} , \qquad (4.47)$$

where  $Z_{+}$  = sleeper section modulus about the toe (mm<sup>3</sup>).

Sleeper Application	Design Axleload	Suggested Sleeper Spacing	Mass	Thickness of Web of Trough	Toe to Toe Width B	Overall Section Depth	Depth from Web to Neutral	Moment of Inertia I	Section About Web <sup>Z</sup> W	Modulus About Toe <sup>Z</sup> T
	(kN)	(mm)	(kg/m)	(mm)	(mm)	(mm)	(mm)	10 <sup>6</sup> (mm <sup>4</sup> )	10 <sup>3</sup> (mm <sup>3</sup> )	$10^3 (mm^3)$
Heavy Haul Sleeper	300	660	35	12	300	120	40	7.00	175	85
Mainline A Sleeper	250	710	30	10	300	120	40	6.00	150	75
Mainline B Sleeper	200	730	25	8	250	105	35	3.50	100	50
Secondary Track Sleeper	150	760	15	8	200	90	30	1.30	45	21

TABLE 4.7 - SECTION PROPERTIES OF PROPOSED RANGE OF BHP STEEL SLEEPERS

For the centrebound conditions of the maximum sleeper bending stresses are also calculated at the centre of the sleeper. The maximum sleeper compressive bending stress  $f_c$  occurs at the toe and is calculated by

$$f_{\rm c} = \frac{M_{\rm c} \, 10^6}{Z_{\rm t}} \, , \qquad (4.48)$$

where

M<sub>c</sub> = maximum sleeper bending moment at the centre (kNm)
Equation 4.23;

the maximum sleeper tensile bending stress  $f_t$  occurs at the web and is calculated by

$$f_{t} = \frac{M_{c} \, 10^{6}}{Z_{w}} , \qquad (4.49)$$

where  $M_{c}$ ,  $Z_{+}$  and  $Z_{w}$  are as defined above.

In order to ascertain whether a selected steel sleeper will be adequate for the determined design stress levels, these stresses must be compared with allowable limits that are based upon fatigue considerations. The life of a steel sleeper is dependent principally on its fatigue performance and this may be characterised by the performance of that part of the sleeper which is most susceptible to fatigue damage, namely the rail fastener.

Brown and Skinner (1978) have suggested a procedure for the design of steel sleepers together with the associated design of the rail fasteners and the rail insulation requirements. With rail fasteners such as for example the welded shear connector type, they suggest that the maximum allowable bending stress (both tensile and compressive) in the region of the rail fastener should not exceed 69 MPa if the life of the fastener is to be at least the life of the sleeper. At all other locations of the parent steel an allowable fatigue stress of 160 MPa (both tensile and compressible) is suggested.

#### CHAPTER 5 - BALLAST ANALYSIS

The primary functions of the ballast layer are (Prause et al. 1974, Robnett et al. 1975):

- to provide a firm, uniform bearing surface for the sleepers and to transmit the imposed track loadings at a pressure level which can be tolerated by the subgrade material, thereby limiting excessive differential settlement and the resulting loss of vertical track geometry
- to provide the necessary lateral and longitudinal stability to the track structure thereby enabling it to resist the imposed vehicular loadings in curves and the thermal forces developed by continuous welded rail
- . to facilitate track maintenance operations, such as the correction of track surface and alignment errors
- . to provide adequate drainage of the track structure, draining water away from the loaded zone of the subgrade and also to retard the possible growth of vegetation.

As the current practice for designing conventional railway track is based upon satisfying the strength criteria of the individual track components, the design of the required ballast layer depth is therefore based upon that depth which reduces the applied subgrade loading to what is considered to be a tolerable level (Figure 1.1). Nowhere in the present design method are the ballast material properties (ballast elastic modulus, etc) and its grading considered to be significant parameters influencing the calculation of the required ballast depth. Consequently the following sections are primarily concerned with the calculation of the ballast depth.

Having established, for a particular sleeper type and size, a sleeper spacing such that the allowable limits of sleeper to

ballast contact pressure and sleeper flexural capacity are not exceeded, the next step in the conventional design procedure is to determine the required ballast depth.

#### THE DETERMINATION OF THE REQUIRED BALLAST DEPTH

As previously stated one of the main functions of the ballast layer is to transmit the imposed track loadings at a pressure tolerable to the subgrade. To calculate the vertical pressure on the formation caused by sleeper loadings it is essential to know how the vertical pressure is distributed through the ballast layer. There are two main types of solutions that can be used to calculate the vertical pressure distribution with ballast depth:

- . simplified theoretical models
- . semi-empirical and empirical solutions

These will be discussed in detail in the following sections.

# Theoretical solutions of the vertical pressure distribution with ballast depth

The following are the simple theoretical solutions that have been used to calculate the vertical pressure distribution with depth. Other complex theoretical solutions have been developed specifically for road design using either multi-layer theory or finite element techniques. As this report is concerned with the current design practice of railway tracks, these solutions are outside the scope of the report.

Boussinesq Elastic Theory: This theory is sometimes referred to as single-layer elastic theory because it assumes that the ballast and the subgrade form a semi-infinite, elastic, homogeneous (its properties are constant from point to point) and isotropic (its properties are the same in each direction through a point) half space. The theory also considers the rail seat load to be

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uniformly distributed over a circular area equivalent to the assumed contact area between the sleeper and the ballast.

Boussinesq (1885) theoretically determined the stress induced at any point within a semi-infinite elastic medium by a single load  $Q_0$  normal to the surface (Figure 5.1). Expressed in rectangular co-ordinates the following solutions were derived:

(a) The vertical stress change  $\sigma_{a}$  (kPa) beneath a point load:

$$\sigma_{z} = \frac{3Q_{o}}{2\pi} \frac{z^{3}}{(r^{2} + z^{2})^{2.5}} .$$
 (5.1)

(b) the radial shear stress change  $\tau_{r}$  (kPa) beneath a point load:

$$\tau_{r} = \frac{3Q_{o}}{2\pi} \frac{rz^{2}}{(r^{2} + z^{2})^{2.5}}, \qquad (5.2)$$

where

- $Q_0 = \text{point load (kN)},$
- z = vertical depth to any point beneath the surface (m), and
- r = horizontal radius from the vertical at the position of point load to the position of any particular point beneath the surface (m).

Integrating the Boussinesq equation for the case of a uniformly loaded circular area at the surface of a semi-infinite elastic medium the following equations, relating to stresses at any depth on the vertical axis beneath the centre of the loaded area, have been derived (Department of Scientific and Industrial Research 1961):

(a) The vertical stress  $\sigma_z$  (kPa) at any depth z:

$$\sigma_{z} = P_{a} \left[ 1 - \frac{z^{3}}{(a^{2} + z^{2})^{1.5}} \right].$$
 (5.3)

(b) The horizontal stresses  $\sigma_x$  and  $\sigma_y$  (kPa) at any depth z:

$$\sigma_{\mathbf{x}} = \sigma_{\mathbf{y}} = \frac{P_{\mathbf{a}}}{2} \left[ (1+2\upsilon) - \frac{2(1+\upsilon)z}{(a^2+z^2)^{0.5}} + \frac{z^3}{(a^2+z^2)^{1.5}} \right].$$
(5.4)

(c) The vertical and horizontal stresses on the axis are major and minor principal stresses respectively. The maximum shear stress  $\tau_{max}(kPa)$  at any depth z is half the difference between the principal stresses:

$$\tau_{\max} = \frac{\sigma_z - \sigma_x}{2} ,$$

therefore

$$\tau_{\text{max}} = P_{\text{a}} \left[ \frac{(1-2\upsilon)}{4} + \frac{(1+\upsilon)z}{(a^{2}+z^{2})^{0.5}} - \frac{3z^{3}}{4(a^{2}+z^{2})^{1.5}} \right],$$
(5.5)

where	Pa	=	average uniform pressure over the loaded area (kPa),
	а	=	radius of the circular loaded area (m), and
	υ	=	Poisson's Ratio of material.

It should be noted that these expressions are independent of the modulus of elasticity, and that the vertical stress is independent of all elastic constants. This would not be so if the material had varying elastic properties. The vertical stress change in the ballast at depth can be approximated by the integration of the Boussinesq equation over the uniformly loaded area. The calculation of the minor stress and the shear stress is inaccurate due to the assumptions of a single elastic layered foundation and that this elastic medium is isotropic.

In the case of sleepers the effective contact area is of a rectangular shape and consequently the solution of Equation 5.3 must

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ł



## **Figure 5.1** Vertical stress beneath a point load at the surface

therefore be modified to account for the change in the influence coefficient (i.e. that part of Equation 5.3 enclosed in brackets). While not being strictly accurate the effective support bearing area under the rail seat  $A_s$ , may be approximated to a circular area with a radius a (m) given by (Kurzweil 1972)

$$a = \begin{bmatrix} \frac{A_{s}}{\pi} \end{bmatrix}^{0.5}$$
$$= \begin{bmatrix} \frac{BL}{\pi} \end{bmatrix}^{0.5}, \qquad (5.6)$$

where

a = equivalent radius of the bearing area (m),
 L = effective length of sleeper support (m), and
 B = sleeper breadth (m).

In order to obtain the exact theoretical Boussinesq solutions for the vertical pressure at any depth caused by a uniformly loaded rectangular area at the surface, Equation 5.1 must be double integrated over this area.

For the general three dimensional case Equation 5.1 can be restated as

$$\sigma_{z} = \frac{3Q_{o} z^{3}}{2\pi r^{5}}, \qquad (5.7)$$

where

 $\sigma_{\rm z}$  = vertical stress at depth z beneath the load point (kPa),

 $Q_0 = \text{point load (kN)},$ 

- z = vertical depth to any point M beneath the surface (m), and
- r the distance from the origin (position of point load)
  to any point M (x, y, z) for which the vertical
  stress z is sought (m)

$$= (x^{2} + y^{2} + z^{2})^{0.5}.$$

To determine the vertical stress,  $\sigma_z$ , the summation of Boussinesq's elementary, single, vertical concentrated load  $dQ_0$  on an elementary area  $dA = d\varepsilon d\eta$  over the entire rectangular bearing area A (Figure 5.2) is required,

By substituting  $\sigma_z$  by  $d\sigma_z$ ,

and 
$$Q_0$$
 by  $dQ_0 = p_a dA = p_a de d\eta$ ,

where  $p_a = average uniform$  pressure over the loaded rectangula area,

the infinitesimal vertical stress  $d\sigma_z$ , on dA because of  $dQ_o$  then follows from Equation 5.7, i.e.,

$$d\sigma_{z} = \frac{3p_{a}z^{3}}{2\pi r^{5}} d\varepsilon d\eta, \qquad (5.8)$$

and its integral is

$$\sigma_{z} = \frac{3p_{a}}{2\pi} \qquad \begin{array}{c} \varepsilon = +A \quad \eta = +B \\ f \quad f \quad f \quad f \quad \frac{d}{d} \quad \frac{d}{d} \\ \varepsilon = -A \quad \eta = -B \left\{ (x - \varepsilon)^{2} + (y - \eta)^{2} + z^{2} \right\}^{5} \end{array}$$
(5.9)

Upon examination of this integral the closed form solution has been found to be very complex and tedius. Therefore if an accurate estimate of the vertical stress at any depth beneath a rectangular uniformly loaded area is required using the Boussinesq approach it is advised that the double integral, Equation 5.9 be solved using a numerical approximation technique.

The ORE (1968) state that single layer elastic theory used with a simplified sleeper soffit contact pressure distribution provides an adequate means of calculation of formation stresses for practical engineering purposes. In view of the degree of scatter involved in the sleeper support condition a more sophisticated approach cannot be justified. The ORE experimental data of





Note: Elementary area =  $d \epsilon d$ 

## Figure 5.2

Application of Boussinesq's elementary single vertical concentrated load over a uniformly loaded rectangular bearing area actual subgrade pressure for various ballast depths is compared with the theoretical Boussinesq prediction of subgrade pressure in Figure 5.3(ORE 1961, Heath & Cottram 1966). Other important findings of the ORE study are:

- (a) The vertical stress distribution in the subgrade became practically uniform at a thickness of construction greater than 600 mm.
- (b) Sleeper spacing in the range 630 to 790 mm had a negligible influence on the vertical stress level in the subgrade for a unit loading applied to the sleeper.

Stress Below a Uniformly Loaded Strip of Infinite Length: A theoretical method has been used by Eisenmann (1970) for determining the increase in vertical stress at any location under a sleeper (Figure 5.4). The sleeper in this analysis is considered as a uniformly loaded strip of infinite length. The analysis is based upon Mohr stress circle considerations and the following relationships have been developed:

(a) The vertical stress  $\sigma_z(kPa)$  at any location (x,z):

$$\sigma_z = \frac{p_a}{\pi} \begin{bmatrix} \theta_2 - \theta_1 - \frac{1}{2} (\sin 2\theta_2 - \sin 2\theta_1) \end{bmatrix}, \quad (5.10)$$

where  $p_a = average$  uniform contact pressure between the sleeper and the ballast (kPa), and  $\theta_1$  and  $\theta_2$  are the angles shown in Figure 5.4.

(b) The horizontal stress  $\sigma_v(kPa)$  at any location (x,z):

$$\sigma_{y} = \frac{p_{a}}{\pi} \frac{\theta}{2} - \frac{\theta}{1} + \frac{1}{2} (\sin 2\frac{\theta}{2} - \sin 2\frac{\theta}{1}) \quad . \tag{5.11}$$

(c) The principal shear stress  $\tau_{x_z}(kPa)$  at any location (x,z):

$$\tau_{xz} = \frac{P_a}{2\pi} (\cos 2^{\theta}_2 - \cos 2^{\theta}_1)$$
 (5.12)

(d) The maximum shear stress  $\tau_{max}$  (kPa):

$$\tau_{\max} = \frac{p_a}{\pi} \sin \left( \frac{\theta}{2} - \frac{\theta}{1} \right), \qquad (5.13)$$

# Semi-empirical and empirical solutions of the vertical pressure distribution with ballast depth

The following are the main semi-empirical methods that have been developed to calculate the vertical pressure distribution with ballast depth.

Load Spread Methods: Simplified methods are often employed in practice which assume that the load is distributed vertically with a load spread slope of 1 vertical to 1 horizontal or a slope of 2 vertical to 2 horizontal. The stress distribution is also assumed uniform at any given plane velow the surface (Figure These simplified methods calculate only the average 5.5). vertical pressure at depth whereas the Boussinesq method calculates the maximum vertical pressure at a depth below the loaded area. A comparison of the vertical stress distribution calculated for both the 1:1 and 2:1 load spread assumptions with the theoretical Boussinesg solution is presented in Figure 5.6; the loaded areas being circular in all cases (Department of Scientific & Industrial Research 1961). It is clearly apparent that the assumed 2:1 load spread distribution of vertical pressure more closely approximates the Boussinesq pressure distribution than the assumed 1:1 load spread distribution.

Clarke (1957) adopts a 1:1 load spread distribution of vertical pressure through the ballast when calculating the average pressure on the subgrade. Using the Boussinesq theory to calculate the

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## Figure 5.3

Comparison of experimental vertical stress distribution with depth and the theoretical Boussinesq solution (ORE 1968)



**Figure 5.4** Stress beneath a uniform load of infinite length (Eisenmann 1970b)



Figure 5.5 Vertical stress transmission by means of the 2:1 distribution





## Figure 5.6

Comparison of the vertical stress distribution under a uniformly loaded circular area based on Boussinesq equations and 1:1 and 2:1 distributions (Department of Scientific and Industrial Research 1961) maximum vertical pressure at depth beneath a uniformly loaded circular area Clarke suggests that this pressure can be estimated as being between 2 to 3 times the average subgrade pressure determined by the 1:1 load spread distribution. For ballast depths in the range of half the sleeper breadth to twice the sleeper breadth, Clarke suggests that in order to obtain an estimate of the maximum subgrade pressure the average subgrade should be doubled. Referring to Figure 5.6 and assuming that the circle diameter 2a approximates the sleeper breadth, it can be seen that Clark's estimate of the maximum subgrade pressure approximates the Boussinesq estimate for the case of a uniformly loaded circular area.

For a rectangularly shaped loaded area (i.e. the assumed effective area of contact beneath the rail seat) the maximum vertical pressure  $\sigma_z(kPa)$  at any depth beneath the surface, can be estimated as twice the average vertical pressure using the 1:1 load spread method. Consequently,

$$\sigma_{z} = 2p_{a} \left[ \frac{B.L}{(B+2z) (L+2z)} \right] , \qquad (5.1)$$

L = effective length of sleeper under the rail seat (m).

<u>Schramms Solution</u>: Schram (1961) uses a method for determining the maximum vertical pressure at a depth beneath a sleeper, based upon the angle of internal friction of the ballast, which is the actual load spread of the ballast material. Schramm states that in practice the quality of the ballast, as denoted by a high angle of internal friction, has with some reservations a greater influence on the distribution of vertical pressure at depth than the length of the sleeper. The upper limit of the friction angle '0' for coarse, rough and dry ballast is about 40<sup>°</sup>, and the lower limit for fine, smooth and moist ballast is about 30<sup>°</sup>. According to Schramm, the maximum vertical subgrade pressure  $\sigma_{\rm Z}\,({\rm kPa})$  for any ballast depth z (m) beneath the sleeper can be calculated from

$$\sigma_z = p_a \frac{1.5(l-g)B}{\{3(l-g) + B\} \text{ ztan}\theta} , \qquad (5.15)$$

where

p<sub>a</sub> = average uniform contact pressure under the rail seat (kPa)

\$ = sleeper length (m),

g = distance between rail centres (m),

B = sleeper breadth (m),

z = depth of ballast layer (m), and

 $\theta$  = angle of internal friction of the ballast (degrees).

Schramm also states that it is desirable to develop some vertical pressure between the sleepers thereby reducing the danger of large differences in the ballast/subgrade pressure which would result in the forcing of soft soil up between the sleepers and contaminating the ballast (Figure 5.7).

Therefore the minimum ballast depth z<sub>min</sub> (m) required is

$$z_{\min} = \frac{S-B}{2 \tan \theta} , \qquad (5.16)$$

where	S	=	sleeper spacing (m),
	В	=	sleeper breadth $(m)$ , and
	θ	=	angle of internal friction of the ballast.

<u>Empirical Methods</u>: All of the empirically derived methods of determining the variation of the maximum vertical pressure directly beneath the rail seat express the relationship of the vertical pressure  $\sigma_z(kPa)$  at a depth z(m) to the uniform contact pressure  $p_a(kPa)$  as (Figure 5.8)

 $\sigma_{z} = p_{a} \{f(z)\},$  (5.17)

where p<sub>a</sub> is defined in all cases as being the average uniform contact pressure over the entire sleeper length.

Consequently,

$$p_a = \frac{2q_r}{A} , \qquad (5.18)$$

where  $q_r = maximum$  rail seat load (kN), and A = entire ballast contact area of sleeper (m<sup>2</sup>).

The following are the most commonly used empirical solutions of the vertical pressure distribution with ballast depth.

<u>Talbot Equation</u>: The most notable and most widely used empirical relationship is the equation recommended by the AREA and developed by Talbot (1919). The maximum vertical pressure  $\sigma_z$  (kPa) under the rail seat for any particular ballast depth is defined as

$$\sigma_z = p_a \{ \frac{1}{5.9z^{1.25}} \} , \qquad (5.19)$$

where p<sub>a</sub> = average uniform pressure between the sleeper and the ballast (kPa) (Equation 5.18), and z = ballast depth (m).

This equation was developed for  $8'-6" \ge 8"$  (2642  $\ge 203$ mm) sleepers, and agrees reasonably with the observed field results except for ballast depths less than 0.1 m or greater than 0.76 m. Clarke (1957) states the maximum vertical pressure  $\sigma_z$  (kPa) under the rail seat as defined by the Talbot equation can be approximated by the simplified expression

$$\sigma_z = p_a \left\{ \frac{0.254}{z} \right\} .$$
 (5.20)

This assumes that the maximum intensity of pressure on the formation varies inversely with the ballast depth for any particular sleeper loading.



Note: Assumes that 3 adjacent sleepers resist the design wheel load therefore  $q_r = 50\%$  design wheel load




# Figure 5.8

Maximum vertical stress Oz at depth Z below the rail seat according to the empirical methods

<u>Japanese National Railway Equations</u>: The following empirical equations have been developed by the Japanese National Railway (JNR) for determining the maximum vertical pressure  $\sigma_{z}$ (kPa) under the rail seat for any particular ballast depth (Okabe 1961):

(a) Horikoshi Equation:

$$\sigma_z = p_a \left\{ \frac{58}{10 + (100z)^{1.35}} \right\}$$
 (5.21)

where p<sub>a</sub> = average uniform pressure between the sleeper and the ballast (kPa) (Equation 5.18), and z = ballast depth (m).

(b) Okabe Equation for Broken Stone Ballast:

$$\sigma_z = p_a \left\{ \frac{350}{240 + (100z)^{1.60}} \right\} .$$
 (5.22)

(c) Okabe Equation for Gravel Ballast:

$$\sigma_z = p_a \left\{ \frac{125}{50 + (100z)^{1.50}} \right\} ,$$
 (5.23)

It should be noted that the above JNR equations have been derived for narrow gauge track conditions, where the sleeper length is 2100 mm. Therefore direct comparisons cannot be made between the Talbot equation which is relevent to standard and broad gauge track and the JNR empirical equations unless allowance is made for the difference in the average uniform pressure of the sleepers used. Comparisons of the above theoretical, semi-empirical and empirical methods of determining the change in vertical pressure with increasing ballast depth for the various gauged track are presented in Figure 5.9. The maximum vertical stress at a given ballast depth has been presented in the normalised form. Therefore in order to obtain an estimate of the maximum vertical stress at a given ballast depth for any of the design methods



# Figure 5.9

Okabe (Gravel)

8

Empirical

5.23

Comparison of the theoretical, semi-empirical, and empirical stress distributions with ballast depth

multiply the normalised vertical stress by the maximum rail seat load. In Figure 5.9 the comparison for narrow gauge track, is based on 2130 mm long x 230 mm wide sleepers, whereas the comparison for standard gauge track, is based on 2440 mm x 230 mm wide sleepers and the comparison for broad gauge track, is based upon 2590 mm x 230 mm sleepers.

Upon examination of Figure 5.9 and selecting common ballast depths of between 200 to 300 mm it can be seen that Schramm's load spread method is a close approximation to the Boussinesq method (assuming a circular loaded area). The Clarke load spread method appears to be a lower bound of the ballast depth requirement whereas the various empirical methods appear to be upper bounds. For large ballast depths the Talbot method closely approximates the Boussinesq solution.

Salem and Hay (1966) have carried out static experiments to determine the vertical pressure distribution with depth for a number of sleepers with rail seat loads of 89 kN and sleeper spacings of 530 mm. Results indicate that the pressure distribution between the sleepers can be regarded as uniform for these conditions at depths of ballast greater than the sleeper spacing minus 75 mm. This observation is not entirely conclusive because the variation along the length of sleeper of the pressure distribution at depth has to be considered in any analysis. In order to attain a more uniform pressure distribution upon the subgrade not only between sleepers but along the sleeper length, greater ballast depths may be required thereby preventing excessive differential subgrade settlement and rutting of the subgrade directly beneath the sleepers.

#### ALLOWABLE SUBGRADE BEARING PRESSURE

As previously mentioned one of the main functions of the ballast layer is to transmit the imposed track loadings at a pressure tolerable to the subgrade. At present there have been two major methods developed to determine the allowable subgrade bearing pressure:

- (a) Safe Average Bearing Pressure Methods,
  - (i) Soil Classification, and
  - (ii) Static Load tests.
- (b) The British Rail Formation Design Method (Repeated Load Tests).

### Safe Average Bearing Pressure Methods

The "safe average bearing pressure" of a subgrade is defined as the value of the "ultimate subgrade bearing capacity" (pressure at which a plastic shear failure occurs) reduced by a load factor or factor of safety. The safe average bearing pressure of a subgrade is the value used in designs where the effect of settlement is considered negligible. The "allowable bearing pressure" of a subgrade is the value used in designs which take into account the danger of both shear failure and settlement. Consequently the allowable subgrade bearing pressure is the value of the safe average bearing pressure that is further reduced by another safety factor, to account for possible settlement.

The values of allowable subgrade bearing pressure determined by these methods are all based upon various static testing techniques applied to saturated subgrades. The assumption of the saturated subgrade condition is realistic in that this condition is, due to the open texture of the ballast, more likely to be realised than under sealed roads and airfields; and it also takes into account the worse possible condition of the subgrade. The current design practice for determining the required ballast depth is to limit the subgrade pressure to some percentage, x per cent, of what is considered to be the "safe average bearing pressure" (Prause et al. 1974). This reduction factor x per cent, is adopted to prevent excessive track subgrade settlement and consequently excessive deterioration of the track geometry under service conditions.

Clarke (1957) recommends that this reduction factor be equal to 60 per cent of the safe average bearing pressure of a particular soil. The safe average bearing pressures of various soils are presented in Table 5.1. Clarke also recommends as a general rule that the maximum subgrade pressure for uncompacted formations should not exceed 83 kPa and 139 kPa for compacted formations. The 60 per cent reduction factor used by Clarke is based upon AREA data (Talbot 1934, p. 209) which indicate that due to the variations in sleeper support and track maintenance the load on an individual sleeper could be as high as 2.7 times the nominal value computed from the beam on an elastic foundation model. Talbot also reported that the observed maximum value of the rail seat load frequently attained a value roughly 66 per cent greater than the nominal mean. Consequently it therefore follows that this is the main reason why Clarke has suggested the reduction factor of 60 per cent. The reduction factor of 60 per cent corresponds exactly to a subgrade safety factor of 1.67.

# TABLE 5.1 - SAFE AVERAGE BEARING PRESSURES OF SUBGRADES (CLARKE

1957)

Subgrade Description	Safe Average Bearing Pressure (kPa)
Alluvial soil	< 70
Made ground not compacted	75 <b>-</b> 105
Soft clay, wet or loose sand	110 - 140
Dry clay, firm sand, sandy clay	145 - 210
Dry gravel soils	215 - 275
Compacted soils	> 280

The AREA (1973) recommends that the calculations of the allowable subgrade pressure be based upon laboratory tests of saturated and remoulded samples (static triaxial tests). The AREA also recommends that when estimating the bearing pressure on the subgrade the design wheel load, and therefore its corresponding maximum rail seat load, should be doubled and the calculated bearing pressure



# Figure 5.10

Approximate correlation of the Casagrande, PR and CAA classification on the basis of bearing capacity (Department of Scientific and Industrial Research 1961) for this condition be compared directly with the determined safe bearing pressure. This method results in an equivalent reduction factor of 50 per cent.

The California Bearing Ratio test, commonly called the CBR test, has been used extensively in pavement material and subgrade soil evaluation. It basically consists of a penetration test in which a standardised piston of head area  $3 \text{ in}^2$ , 19.4 cm<sup>2</sup>, is forced into a prepared specimen (field testing can also be carried out). The load required to cause 0.1 in, 2.5 mm, penetration is compared to the load required to cause the same penetration into a high quality crushed stone which is considered to have a CBR equal to 100. The ratio of these loads is used to calculate the CBR of the material. Due to the ease of field measurement, and the good correlation between the estimated design values, the CBR method as used in road design could also be considered as being adequate for estimating the safe bearing pressure of the subgrade.

A very useful approximate correlation between the CBR value, various soil classification systems and what are considered safe bearing pressures for compacted subgrades is presented in Figure 5.10. The CBR method and the current design method assumes that there is a unique relationship between the compressive strength of the material and its behaviour under repeated loads (Waters & Shenton 1968). Laboratory work with clay has shown that lower values of failure loads occur with repeated loading than with a single load application to failure. Results published by Waters and Shenton (1969) show that in the case of clay subgrades, failure under repeated loading in a drained triaxial cell occurs at approximately 50 per cent of the rapidly applied undrained failure stress. Therefore to some extent these results confirm the AREA recommendation of using a 50 per cent reduction factor to calculate the safe average bearing pressure.

### British Rail Foundation Design Method

British Rail have developed a rational method of track substructure design which relates substructure requirements to the intended traffic loading on a quantitative basis (Heath, Shenton, Sparrow & Waters 1972). The developed design method is based upon extensive laboratory tests of subgrade samples and because of the large number of samples tested the triaxial compression test was In this laboratory test two variable stresses, a major chosen. vertical principal stress  $\sigma_1$ , and two equal radial minor stresses  $\sigma_2$ , can be applied to the soil sample. The design method used a repeated loading triaxial apparatus which pulsed the vertical principal stress at a cycle frequency of 30 cycles per minute while maintaining the radial principal stresses at a constant confinement pressure. A square loading pulse was also chosen and the above conditions were regarded as typical of the pressure conditions experiences by the subgrade.

Preliminary tests were carried out on samples of London clay and the results of these tests are shown as plots of cumulative strain versus the number of loading cycles Figure 5.11. As can be seen the results fall into two distinct groups; those in which the deformation is progressive until complete failure of the sample is reached and those in which the rate of deformation is reducing and a stable condition is being attained. Plotting elastic strain against the number of loading cycles to reach failure (defined by 10 per cent cumulative strain) a curve, similar to S-N fatigue curves in metals, is obtained, Figure 5.12. On this basis a limiting repeated elastic strain can be defined, above which the deformation is continuous and below which it is terminating. Results with three different effective confining pressures  $\sigma_3$ , indicate that the threshold stress, corresponding to this limiting elastic strain, is approximately a This discovered relationlinear function of confining pressure. ship forms an important part in the design method as it allows increases in the threshold stress with depth of construction. The main assumptions inherent in the proposed BR foundation design methods are (Heath et al. 1972):

- . that the threshold stress parameters for the subgrade soil may be obtained using the standard repeated loading test described
- . that simple elastic theory can be used to predict the stresses in the subgrade from traffic loading
- that the significant traffic stresses are those produced only by the static effects of the heaviest commonly occurring axle load
- . that the water table is at the top of the subgrade.

The design method is based on the achievement of a "balanced" design which is obtained when the ballast is sufficiently deep so that the calculated maximum deviator stress  $(\sigma_1 - \sigma_3)$  difference induced in the subgrade by the heaviest commonly occurring axle load is equal to the average threshold principal stress difference established by laboratory repeated triaxial tests. The theoretical subgrade maximum deviator stress difference for various axle loads are shown in Figure 5.13. The threshold stress/depth relationship is superimposed on these curves and hence it is possible to obtain the depth of ballast at which the threshold stress is equal to the stress induced by a given axle load. Ву taking these intersection points the design depth can be obtained. The design depth is that from sleeper bottom to the top of the subgrade indicated for the heaviest commonly occurring axle load. Using these results a design chart can be constructed, Figure 5.14.

This design method was assessed for ballast track by performing a series of laboratory tests and track measurements. Reduced settlement rates were achieved rather consistently when the ballast depth equaled or exceeded the design depth, and the settlement rates for ballast depths less than the desired depth were significantly higher.



# Figure 5.11

Cumulative strain resulting from repeated loading tests of London clay (Heath et al 1972)



Figure 5.12 Modified S-N curve (Heath et al 1972)







**Figure 5.14** Design of conventional rail track foundations (Heath et al 1972)

# CHAPTER 6 - PARAMETRIC EVALUATION OF IMPORTANT TRACK DESIGN CRITERIA

As previously outlined, current practice used in the design of rail track requires that the following design criteria are satisfied:

- . allowable rail bending stress
- . allowable rail deflection
- . allowable sleeper to ballast contact pressure
- . allowable sleeper bending stress
- . allowable subgrade bearing pressure.

The influence of various design parameters to these criteria will be discussed in detail in this section.

Two standard guage mainline locomotives currently used by the state railway systems have been selected in order to undertake the parametric analyses. The first of the two locomotives selected was the Victorian Railways (VicRail) X-Class, and is illustrated in Figure 6.1. This locomotive was selected because it was considered representative of the current maximum axleload commonly permitted by state railway systems on mainline tracks. The second of the two locomotives selected was the Western Australian Government Railways (Westrail) L-Class and is illustrated in Figure 6.2. This locomotive was selected because it is representative of the axleload currently being considered by many state railway systems for future mainline freight traffic.

#### RAIL BENDING STRESS AND DEFLECTION ESTIMATES

The maximum rail bending stress and deflection is estimated using the beam on an elastic foundation analysis. The design parameters



**Figure 6.1** Victorian Railways main line diesel electric X class locomotive

24722/80-30



# Figure 6.2 West Australian Government Railways main line diesel electric L class locomotive

that influence the magnitude of the maximum rail bending stress and deflection include:

- . the rail section used in the design
- . the assumed value of the track modulus per rail
- . the axle load of the vehicle
- . the axle spacings of the vehicle
- . the speed of the vehicle.

The design vehicle combinations used in the parametric analysis of the rail bending stress and deflection were two coupled Vicrail X-Class locomotives and two coupled Westrail L-Class locomotives. The above vehicle combinations were selected in order to obtain the maximum rail bending stress and deflection caused by the interaction of adjacent axle loads. Three vehicle speeds were selected, viz. 80, 100 and 120 km/h and the variation of the maximum rail bending stress and deflection with changes in the assumed track moduli values were plotted for each of the two above vehicle combinations and for 47 and 53 kg/m rail sections. These plots are presented in Figures 6.3, 6.4, 6.5 and 6.6. These Figures also outline the A.R.E.A. limits of the maximum allowable rail bending stress and deflection.

### RAIL BENDING STRESS AND DEFLECTION ESTIMATES

The maximum allowable rail bending stress for continuously welded rail, as determined by the simple A.R.E.A. factor of safety method, was found to equal 138 MPa, (Equation 3.38) and estimating the yield strength of standard carbon rail steel equal to 410 MPa. The maximum allowable rail deflection as recommended by the A.R.E.A. is equal to 6.35 mm.

Westrail have published values of the tack modulus per rail for standard gauge mainline track, (see Table 3.5). From this table



Variation of the maximum rail bending stress limit assumed track moduli values for various locomotives at speed, using 47(kg/m) continuously welded rails



Variation of the maximum rail bending stress limit with assumed track moduli values for various locomotives at speed, using 53(kg/m) continuously welded rails



Variation of the maximum rail deflection with assumed moduli for various locomotives at speed using 47(kg/m) rails



Variation of the maximum rail deflection with assumed track moduli for various locomotives at speed, using 53(kg/m) rails

it is apparent that an estimate of the track modulus per rail of 15 MPa is typical of standard guage timber sleepered mainline track of normal construction. It is apparent in Figure 6.3 than when 47 kg/m rail sections are used in conjunction with the typical track modulus estimate, the combinations of two coupled Westrail L-Class locomotives exceed the allowable rail bending stress criterion for speeds greater than 80 km/h. In fact for speeds of 120 km/h this rail design criterion is not satisfied unless the assumed track modules per rail is estimated as 33 MPa. By comparison with Figure 6.4 it is clearly evident that the 53 kg/m rail section in conjunction with the typical track modulus estimate is adequate for all the locomotive combinations and speeds considered.

Examining Figure 6.5 and 6.6 it is apparent that when 47 and 53 kg/m rail sections are used in conjunction with the typical track modulus estimate that the combination of two coupled Westrail L-Class locomotives exceeds the allowable rail deflection criterion for speeds greater than 80 km/h. For speeds of 120 km/h the rail deflection criterion is satisfied if the assumed track modulus per rail for both 47 and 53 kg/m rail sections is estimated as being greater than 17.5 MPa.

The significance of the assumed track modulus per rail upon the resulting maximum rail bending stress and deflection estimates is summarised in Table 6.1 for the case of a 47 kg/m rail section. It is readily apparent that the assumed track modulus value is more influential upon the value of the maximum rail deflection than the maximum rail bending stress. If the value of the track modulus were varied 100 per cent from 10 to 20 MPa the maximum rail deflection is reduced by 49 per cent compared with a 10 per cent reduction in the maximum rail bending stress.

Adopting the typical track modulus value of 15 MPa, estimates of the maximum rail bending stress and deflection were calculated for the two design vehicle combinations at various speeds and for the 47, 50, 53 and 60 kg/m rail sections. These estimates are

### TABLE 6.1 - EFFECTS THAT THE VARIATION OF TRACK MODULUS HAS ON THE MAXIMUM RAIL BENDING STRESS AND DEFLECTION FOR

Track Modulus	Increase in Track	Maximum Rail Stress at 10 (MPa)	. Bending 10 km/h	Reduction in Rail Bending (Per ce	Maximum Stress nt)	Maximum Rail at 100 km/h (mm)	Deflection	Reduction in maximum Rail Deflection (Per cent)			
Rail	(Per cent)	Two VicRail X-Class locomotives	Two Westrail L-Class locomotives	Two Vicrail X-Class locomotives	Two Westrail L-CLass locomotives	Two VicRail X-Class locomotives	Two Westrail L-Class Locomotives	Two VicRail X-Class Locomotive s	Two Westrail L-Class locomotives		
10	-	132.6	156.6		_	8.0	10.2	_	-		
20	+100	119.0	142.3	10	9	4.1	5.2	49	49		
30	+200	110.5	132.2	17	16	2.9	3.6	64	65		
40	+300	105.6	124.0	20	21	2.3	2.8	71	73		

VARIOUS LOCOMOTIVES AT 100 km/h AND USING 47 kg/m RAILS

(January 1997)

presented in detail in Table 6.2. Using this data as a basis the influence on the maximum rail bending stress and rail deflection to variations in the design speed, vehicle combination and rail section were appraised. The effect of increasing the speed and the axleload of the design vehicle combination upon the maximum rail bending stress and deflection is summarized in Table 6.3. It was found that all the rail sections considered gave very similar results to the percentage variation. Although the absolute magnitude of the maximum rail bending stress and deflection were naturally at variance. The important aspect illustrated in Table 5 is that a 50 per cent increase in the speed, i.e. 80 to 120 km/h for both vehicle combinations resulted in an increase in the maximum rail bending stress and deflection of 15 per cent in both cases. Whereas increasing the axleload by 20 per cent, i.e. replacing the X-Class locomotives with L-Class locomotives, at identical speeds resulted in an increase in the maximum rail bending stress and deflection of 18 and 27 per cent respectively. It should however be emphasised that part of the increase in rail bending stress and deflection is due to the altered axle spacings in conjunction with the higher axle loads.

The effect of increasing the rail section size upon the maximum rail bending stress and deflection is summarised in Table 6.4. When the typical value of the track modulus is used in conjunction with the rail sections considered, it was discovered that increasing the rail size alone only influenced the rail bending stress to any marked degree. This is illustrated by the effect of increasing the rail sectional strength by 85 per cent, i.e. increasing the rail section from 47 to 60 kg/m, is to reduce the maximum rail bending stress by 31 per cent, but only reduces the maximum rail deflection by less than 0.02 per cent. This further reinforces the observation made in Table 6.1 that the maximum rail deflection is strongly dependent upon the assumed track modulus value.

### TABLE 6.2 - ESTIMATES OF THE MAXIMUM RAIL BENDING STRESS AND DEFLECTION

CALCULATED FOR VARIOUS RAIL SECTIONS, SPEEDS AND AXLELOADS AND

USING AN ASSUMED VALUE OF THE TRACK MODULUS OF 15 MPa

D - 11		Maxim Bendi	um Rail ng Stress (MPa)	Maximum Rail Deflection (mm)				
Rall Section (kg/m)	Speed (km/H)	2 VicRail X-Class locomotives	2 Westrail L-Class locomotives	2 VicRail V-Class locomotives	2 Westrail 1-Class locomotives			
47	80	117.1	138.0	5.0	6.4			
	100	125.6	148.1	5.4	6.8			
	120	134.1	158.1	5.7	7.3			
50	80	104.9	123.4	5.0	6.3			
	100	112.5	132.3	5.3	6.8			
	120	120.2	141.3	5.7	7.3			
53	80	92.2	108.7	4.9	6.3			
	100	98.9	116.6	5.3	6.8			
	120	105.6	124.5	5.7	7.2			
60	80	80.1	94.8	4.9	6.3			
	100	85.9	101.7	5.3	6.8			
	120	91.7	108.5	5.6	7.2			

i.

### TABLE 6.3 - EFFECT OF INCREASING SPEED AND AXLELOAD ON THE MAXIMUM RAIL BENDING STRESS AND

DEFLECTION FOR THE RANGE OF RAIL SECTIONS CONSIDERED AND USING AN ASSUMED TRACK

MODULUS VALUE OF 15 PMa (PER CENT)

-----

For all rail section	Increasing speed	Increasing axleload from				
considered	Speed range 80 to 100 km/h (25 per cent speed increase)	Speed range 80 to 120 km/h (50 per cent speed increase)	identical speeds (20 per cent axleload increase)			
Increase in maximum rail bending stress	7	- 15 .	18			
Increase in maximum rail deflection	7	15	27			

TABLE 6.4 -	EFFECT OF	INCREASING TH	E RAIL	SECTION	ON TH	E MAXIMUM	RAIL	BENDING	STRESS	AND	DEFLECTION	FOR AN	ASSUMED
1,10,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,													

Rail Section (kg/m)	Increase of rail sectional weight (Per cent)	Increase of rail sectional second moment of inertia	Reduction in t bending stress speed of 100 k (Pe	the maximum rail at a constant m/h er cent)	Reduction in the deflection at a speed of 100 km/ (Per ce	maximum rail constant h nt)
(Kg/m)		(Per cent)	2 VicRail X-Class locomotives	2 Westrail L-Class locomotives	2 VicRail X-Class locomotivess	2 Westrail L-Class locomotives
47	-	_	reference stress	reference stress	reference deflection	reference deflection
			125.6 MPa	148.1 MPa	5.4 mm	6.8 mm
50	6	26	10	11	0.01	0 <b>.</b> 0 J.
53	13	43	21	21	0.01	0.01
60	28	85	32	31	0.02	0.01

VALUE OF TRACK MODULUS OF 15 MPa

### SLEEPER TO BALLAST CONTACT PRESSURE ESTIMATES

Before the sleeper to ballast contact pressure can be estimated the sleeper rail seat load must be evaluated. Two methods of determining the rail seat load, the beam on an elastic foundation method and the A.R.E.A. method, (Equations 4.6 and 4.9) were evaluated for timber sleepered track for the two design vehicle combinations outlined in the previous section. The variation of the maximum rail seat load with sleeper spacing for the two design vehicle combinations and using the two alternate rail seat load formulae is presented in Figure 6.7 and 6.8. Under existing state railway conditions the typical sleeper spacing of standard guage mainline timber sleepers is commonly 610 mm. Using this sleeper spacing in conjunction with Figure 6.7 and 6.8 it is evident that the maximum rail seat load, as determined by the beam on an elastic foundation method, is nearly constant for all rail sections considered and for assumed track moduli values in the range of 10 to 20 MPa. The effect of varying the rail section and track moduli on the maximum rail seat load for a sleeper spacing of 610 mm and the beam on an elastic foundation analysis is evaluated for the two vehicle combinations travelling at speeds of 100 km/h. The results of this evaluation for the two coupled Vicrail X-Class locomotives and the two coupled Westrail L-Class locomotives are presented in Tables 6.5 and 6.6 respectively.

From a comparison of Tables 6.5 and 6.6 it is evident that increasing the rail section from 47 to 60 kg/m, i.e., an 85 per cent increase in rail second moment of inertia, results in a decrease of the estimated rail seat load of between 1 and 2 per cent using the beam on an elastic foundation method and assumed track moduli values of between 10 and 20 MPa respectively. The effect of increasing the track moduli values from 10 to 40 MPa results in an increase of the rail seat load of between 11 and 15 per cent for 47 kg/m rail sections and between 4 and 5 per cent for 60 kg/m rail sections. From the above observations it is clearly apparent that for tracks of normal construction and



Estimates of the variation of rail seat load with sleeper spacing for Vicrail X-class locomotives



Estimates of the variation of rail seat load with sleeper spacing for Westrail L-class locomotives

Effect on rail seat load of increasing Beam on an elastic foundation Effect on rail seat load of Track modulus per estimate of rail seat load increasing rail from 47 to the track moduli values rail (kN) 60 kg/m (Per cent) (MPa) 47 Rail 60 Rail (Per cent) 47 Rail Section 60 Rail Section Section (kg/m) Section (kg/m) (kg/m) (kg/m) 10 48.7 48.2 1 -\_ 20 50.2 49.0 2 3 2 40 56.0 50.8 9 15 5

TABLE	6.	5 – EFFECT	$\mathbf{OF}$	VARYING	RAIL	SECTION	AND	TRACK	MODULI	ON	THE	BEAM	ON .	AN	ELASTIC	FOUNDATION	ESTIMATE	$\mathbf{OF}$	THE	RAI

SEAT LOAD, FOR VICRAIL X-CLASS LOCOMOTIVES AND SLEEPER SPACING OF 610 mm

r		)
2		2
ί	5	7

TABLE 6.6 - EFFECT ON VARYING RAIL SECTION AND TRACK MODULI ON THE BEAM ON AN ELASTIC FOUNDATION ESTIMATE OF THE RAIL SEAT LOAD, FOR WESTRAIL L-CLASS LOCOMOTIVES AND A SLEEPER SPACING OF 610 mm

Track modulus per per rail	Beam on an elast estimate of rail (kN)	ic foundation seat load	Effect on rail seat load of increasing rail section from 47 to 60 kg/m	Effect on rail seat load of increasing tack moduli values (Per cent)				
(MPa)	47 Rail Section (kg/m)	60 Rail Section (kg/m)	(Per cent)	47 Rail Section (kg/m)	60 Rail Section (kg/m)	-		
10	62.3	61.5	-1	~	-			
20	63.8	62.5	-2	+2	+2			
40	69.0	64.0	- 7	+11	+ 4			

with assumed track moduli values of between 10 and 20 MPa the beam on an elastic foundation prediction of the rail seat load can be regarded as being nearly constant for all rail sections and only dependent upon the sleeper spacing. Due to this evident lack of influence on the rail seat load of both the rail section size and track moduli value in the ranges considered, it was decided to base the parametric study of the sleeper to ballast interaction on the A.R.E.A. design approach.

The A.R.E.A. method of determining the rail seat load results in an estimate which is only dependent upon the sleeper spacing; the rail section size is considered to have no significant influence. Upon closer examination of Figure 6.7 and 6.8 it is also evident that the slope of the beam on an elastic foundation estimates of the rail seat load for both locomotives on track with a moduli of between 10 and 20 (MPa) closely approximated the slope of the A.R.E.A. rail seat load estimate. In Chapter 2 it was noted in the discussion of the development of the A.R.E.A. rail seat load estimate that it was based upon the beam as an elastic foundation estimate using a 57 kg/m rail section, a track modulus per rail of 13.8 MPa and a configuration of four 150 kN wheel loads spaced at 1.800, 2.000 and 1.800 m centres. Since the influence of the rail section size and typical tack moduli values has been found to be negligible upon the beam on an elastic foundation estimate of the rail seat load it is considered that the slope of the estimate is mainly governed by the axle spacings of the design vehicle group.

As already mentioned above, the parametric study of the sleeper to ballast contact pressure was undertaken using the A.R.E.A. sleeper design method. The variation of the maximum sleeper to ballast contact pressure with sleeper spacing and for various sleeper lengths for the design vehicles travelling at 100 km/h is presented in Figure 6.9. The recommended limit of the sleeper to ballast contact pressure, 450 kPa is also indicated in the diagrams. Clearly the A.R.E.A. recommended limit of the allowable sleeper to ballast contact pressure for timber sleepers is



Variation of the sleeper to ballast contact pressure with sleeper spacing

exceeded by nearly all the sleeper lengths considered under the Westrail L-Class locomotive loading at 100 km/h. This recommended limit is also exceeded under the Vicrail X-Class locomotive loading even for sleepers with surface dimensions of 2.440 x 0.230 m spaced 610 mm apart, (i.e., the typical track configuration of the majority of standard guage mainline timbered sleepered track used by the state railway systems). Consequently, the following two general conclusions can be made, viz. either the existing tracks are underdesigned and require larger sleeper dimensions and closer sleeper spacings for the axleload and speed conditions; or, that the A.R.E.A. recommended design criterion of the allowable sleeper to ballast contact pressure has been empirically derived for a lighter axle load or smaller sleeper spacing. The track configuration mentioned above has been the standard guage track standard used by the majority of state railway systems for many years, and it is most probable that this standard was developed for a lighter axleload. Even though tracks with this configuration have exceeded this contact pressure criterion, they have performed adequately for the past twenty years under axleloadings similar to that of the Vicrail X-Class locomotive and, more recently under axleloading similar to the Westrail locomotive, (although probably at operating speeds of less than 100 km/h). This fact raises the question of the validity of this contact pressure criterion. It is therefore considered that until the allowable sleeper to ballast contact pressure criterion for specific axle load conditions are related to the standard of track maintenance, no reliable estimate of the actual contact pressure safety factor, which is inherent in the current A.R.E.A. track design procedure, can be made.

The effect of varying the sleeper length upon the sleeper to ballast contact pressure is outlined in Table 6.7 for the commonly used sleeper spacing of 610 mm. As expected the reduction in the contact pressure is linearly proportional to the increase in the sleeper to ballast surface contact area, (Equation 4.17). The values of the sleeper to ballast contact pressure are quoted in

Sleeper Length* (m)	Change in Sleeper Length	Sleeper to h pressure est A.R.E.A. des (KPa	allast contact imated by the ign method	Change in sleeper to ballast contact pressure (Per cent)				
	(Per cent)	VicRail X-Class	Westrail L-Class	Due to increase of sleeper length	Due to increase of axleload			
2.440		470	554	-	+1.8			
2.540	+4	452	534	-4	+18			
2.640	+8	436	514	-8	+18			

TABLE 6.7 - EFFECT OF VARYING SLEEPER LENGTH AND THE DESIGN LOADING CONDITION UPON THE

A.R.E.A. ESTIMATE OF THE SLEEPER TO BALLAST CONTACT PRESSURE, FOR A SLEEPER

- - - -

SPACING OF 610 mm

Note: Other sleeper dimensions are breadth 0.230 m, and thickness (irrelevant to results).
Table 6.7 in order to illustrate to the designer the influence of variations of axleload and sleeper length.

#### SLEEPER BENDING STRESS ESTIMATES

The two preselected design vehicles; viz. the Vicrail X-Class and the Westrail L-Class locomotives, travelling at speeds of 100 km/h were used to evaluate the variation of the sleeper bending stress with sleeper spacing. The variation of the sleeper bending stress with sleeper spacing was also investigated for a range of sleeper lengths and thicknesses, the reference sleeper dimensions being in both instances 2.440 x 0.230 x 0.115 m. It was considered that, due to the form of the design equations for determining the sleeper bending stress, the effect of varying the sleeper length and thickness would be more dominant than that of the sleeper breadth. In the following analyses the A.R.E.A. method calculating the rail seat load and the Schramm method of calculating the sleeper effective length were used. The sleepers were assumed to be fitted with rail bearing plates in the analysis.

# Effect of varying the sleeper length upon the maximum sleeper bending stress

Two methods of calculating the maximum sleeper bending moment at the rail seat were evaluated, there being the Battelle/Schramm method and the A.R.E.A. method (Equations 4.21 and 4.38).

Sleeper lengths of 2.440, 2.540 and 2.640 m were used in the analysis to ascertain the influence of varying the sleeper length alone upon the sleeper bending stress at the rail. While the sleeper length was varied the other reference sleeper dimensions were held constant.

Using the Battelle/Schramm bending stress method the variation of the sleeper bending stress at the rail seat with the sleeper spacing for the range of sleeper lengths and axleloads considered is presented in Figure 6.10.



# Figure 6.10

Variation of the sleeper bending stress at the rail seat with sleeper spacing using the Battelle/Schramm method. (For variable sleeper lengths) It is clearly apparent that the A.R.E.A. recommended design limit of 7.6 MPa, is exceeded for all the sleeper dimensions and axleloads considered. For sleeper spacing of 610 mm, commonly used by the state railway systems the effect of varying the sleeper length upon the sleeper bending stress at the rail seat is presented in Table 6.8. It can be seen that at this sleeper spacing an 8 per cent increase in the sleeper length causes a 33 per cent increase in the sleeper bending stress at the rail seat, whereas increasing the axleload from that of a Vicrail X-Class to that of a Westrail L-Class locomotive causes an increase of between 18 to 20 per cent.

The above analysis was repeated using the A.R.E.A. bending stress method and the results were presented in Figure 6.11. Of interest is the observation that when this method is used in conjunction with the Westrail L-Class locomotive the 2.540 m longer sleeper exceeds the A.R.E.A. recommended bending stress limit at a sleeper spacing of 560 mm. The variation of the sleeper bending stress at the rail seat with the sleeper length for a sleeper spacing of 160 mm is presented in Table 6.9. It is evident that increasing the sleeper length by 8 per cent in conjunction with the A.R.E.A. bending stress method causes a 62 per cent increase in the sleeper bending stress at the rail seat. The effect of increasing the axleload from that of a Vicrail X-Class to that of a Westrail L-Class locomotive increases the sleeper bending stress at the rail seat by between 19 and 22 per cent; which is similar to the Battelle/Schramm method result. Of interest is the observation that although the 8 per cent increase in sleeper length results in a 62 per cent increase in the bending stress at the rail seat for the A.R.E.A. method, the maximum level of stress predicted is still roughly 60 per cent of that predicted by the Battelle/Schramm method. The Battelle/Schramm method of estimating the maximum sleeper bending stress at the rail seat could therefore be regarded as being a more conservative estimate than that of the A.R.E.A. method.

Sleeper Length* (m)	Change in Sleeper Length	Sleeper bending stress at the rail seat estimated by the Schramm/Battelle method+ (MPa)		Change in sleeper bending <b>st</b> ress (Per <b>cen</b> t)		
	(Per cent)	VicRail X-Class	Westrail L-Class	Due to increase of sleeper length	Due to increase of axleload	
2.440	_	10.0	12.0		+20	
2.540	+4	11.6	13.8	+1.6	+19	
2.640	+8	13.4	15.8	+33	+18	

#### TABLE 6.8 - EFFECT OF VARYING THE SLEEPER LENGTH UPON THE SLEEPER BENDING STRESS AT THE RAIL

SEAT CALCULATED BY THE SCHRAMM/BATTELLE METHOD, FOR A SLEEPER SPACING OF 610 mm

Note: \* Other sleeper dimensions: breadth 0.230 m, thickness 0.115m + Sleeper bearing plates fitted.

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# TABLE 6.9 - EFFECT OF VARYING THE SLEEPER LENGTH UPON THE SLEEPER BENDING STRESS AT THE RAIL SEAT CALCULATED BY THE A.R.E.A. METHOD, FOR A SLEEPER SPACING OF 610 mm

Sleeper Length* (m)	Change in Sleeper Length (Per Cent)	Sleeper bending s seat estimated by (MPa	tress at the rail the A.R.E.A. method )	+ Changes in Sleeper bending stress (Per cent)		
		VicRail X-Class	Westrail L-Class	Due to increase of sleeper length	Due to increase of axleload	
2.440		5.2	6.2	_	+19	
2.540	+4	6.7	8.1	+30	+21	
2.640	+8	8.3	10.1	+62	+22	

Note: \* Other sleeper dimensions are breadth 0.230 m, thickness 0.115 m. + Sleeper bearing plates fitted.



Standard gauge track Speed 100 km/h A.R.E.A. Impact Factor A.R.E.A. Bending Stress Method Sleeper plates fitted to sleeper

Breadth: 0.230 m Thickness: 0.115 m

# Figure 6.11

Variation of the sleeper bending stress at the rail seat with sleeper spacing using the A.R.E.A. method. (For variable sleeper lengths)

In Chapter 4 it was suggested that the maximum bending moment at the centre of the sleeper should be calculated on the basis of an assumed uniform bearing pressure over the overall sleeper length, (Equations 4.23 and 4.39). Using this method the variation of the sleeper bending stress at the centre of the sleeper with sleeper spacing for the range of sleeper lengths and axleloads considered is presented in Figure 6.12. It is clearly apparent that the A.R.E.A. recommended sleeper bending stress limit of 7.6 MPa is exceeded by all the sleeper sizes considered. The variation of the sleeper bending stress at the centre of the sleeper with sleeper length for a sleeper spacing of 610 mm is presented in Table 6.10. It is evident that a 8 per cent increase in the sleeper length results in a 36 per cent reduction in the maximum bending stress at the centre of the sleeper. Whereas increasing the axleload from that of a Vicrail X-Class to that of a Westrail L-Class increases the sleeper bending stress at the centre of the sleeper by between 17 and 21 per cent. It can generally be stated that increasing the sleeper length results in an increase of the sleeper bending stress at the rail seat and a reduction of the sleeper bending stress at the centre of the sleeper.

# Effect of varying the sleeper thickness upon the maximum sleeper bending stress

The above comparison of the Battelle/Schramm and A.R.E.A. methods of determining the bending stess at the rail seat was repeated but for this case the influence of varying the sleeper thickness alone was investigated. The three sleeper thicknesses slected were 0.115, 0.135 and 0.155 m and the other reference sleeper dimensions were held constant. Using the Batelle/Schramm bending stress method the variation of the sleeper bending stress at the rail seat with the sleeper spacing, for the range of sleeper thicknesses and axeloads considered, is presented in Figure 6.13. It is significant that the A.R.E.A. criterion of the maximum allowable sleeper bending stress is only satisfied for relatively large sleeper thicknesses. For the commonly used sleeper spacing of 610 mm, the effect of varying the sleeper thickness upon the

Sleeper Length (m)	Change in sleeper length	Sleeper bending (MPa	stress at centre	Change in sleeper bending stress (Per cent)		
		VicRail X-Class	Westrail L-Class	Due to an increase of sleeper length	Due to an increase in axleload	
2.440	-	1.8.6	22.5		+21	
2.540	+4	.1.5 . 2	18.1	-19	+19	
2.650	+8	12.1	14.2	-36	+17	

## TABLE 6.10 - EFFECT OF VARYING THE SLEEPER LENGTH UPON THE SLEEPER BENDING STRESS AT THE

CENTRE FOR A SLEEPER SPACING OF 610 mm

Note: Other sleeper dimensions are breadth 0.230 m, thickness 0.115 m.



# Figure 6.12

Variation of the sleeper bending stress at the centre with sleeper spacing. (For variable sleeper lengths)



# Figure 6.13

Variation of the sleeper bending stress at the rail seat with sleeper spacing, using the Battelle/Schramm method. (For variable sleeper thickness) sleeper bending stress at the rail seat is presented in Table 6.11. It can be seen that at this sleeper spacing a 35 per cent increase in the sleeper thickness causes a 45 per cent reduction in the sleeper bending stress; whereas the increase in axleload from that of a Vicrail X-Class to that of a Westrail L-Class results in an 18 per cent increase of the sleeper bending stress.

Similarly for the A.R.E.A. bending stress method, the variation of the sleeper bending stress at the rail seat with the sleeper spacing, for the range of sleeper thickness and axleloads considered, is presented in Figure 6.14. It is clearly evident that the sleeper sizes considered satisfy the A.R.E.A. criterion of the maximum allowable sleeper bending stress for the range of sleeper spacings that would be used in track design. For the commonly used sleeper spacing of 610 mm, the effect of varying the sleeper thickness upon the sleeper bending stress at the rail seat is presented in Table 6.12. As with the Battelle/Schramm method detailed above the effect of increasing the sleeper thickness by 35 per cent in conjunction with the A.R.E.A. method results in a 46 per cent reduction of the sleeper bending stress at the rail seat. What is significant is that the sleeper bending stresses calculated by the A.R.E.A. method are roughly 50 per cent of those calculated by the Battelle/Schramm method.

The variation of the sleeper bending stress at the centre of the sleeper with the sleeper spacing, for the range of sleeper thicknesses and axeloads considered, is presented in Figure 6.15. As observed in Figure 6.12 the A.R.E.A. recommended bending stress limit of 7.6 MPa is exceeded by all sleeper sizes considered. For the commonly used sleeper spacing of 610 mm, the effect of varying the sleeper thickness upon the sleeper bending stress at the centre of the sleeper is presented in Table 6.13. As to be expected, the 35 per cent increase in the sleeper thickness reduces the sleeper bending stress at the centre of the sleeper by 45 per cent. The maximum sleeper bending stress at the centre of the sleeper for the centrebound condition is approximately

TABLE 6.11 - EFFEC	T OF	VARYING	THE	SLEEPER	THICKNESS	UPON	THE	SLEEPER	BENDING	STRESS	ΑT	THE

# RAIL SEAT CALCULATED BY THE SCHRAMM/BATTELLE METHOD, FOR A SLEEPER SPACING OF 610 mm

Sleeper Thickness* (m)	Change in sleeper	Sleeper bending str estimated by the Sc	ress at rail seat hramm/Battelle method <sup>+</sup>	Change in sleeper bending stress		
	(Per cent)	VicRail X-Class	Westrail L-Class	Due to increase of sleeper thickness	Due to increase of axleload	
0.115	-	10.1	11.9	-	+18	
0.135	+17	7.4	8.6	-27	+18	
0.155	+35	5.6	6.6	-45	+18	

Note: \* Other sleeper dimensions: length 2.440 m, breadth 0.230 m + Sleeper bearing plates fitted

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		RAIL SEA	T CALCULATED	BY THE A.R.E.A. METHOD,	FOR A SLEEPER S	PACING OF 610 mm	
	Sleeper Thickness*	Change in sleeper	Sleeper bend seat estimat	ing stress at the rail ed by the AREA method+ (MPa)	Change in sleeper bending stre (Per cent)		
2	(m)	thickness (Per cent)	VicRail X-Class	Westrail L-Class	Due to increas sleeper thickn	se Due to increase less in axleload	
,	0.115	-	5.2	6.3	<b>-</b> ,	+21	
	0.135	+17	3.7	4.5	-29	+21	
	0.155	+35	2.8	3.4	-46	+21	

TABLE 6.12 - EFFECT OF VARYING THE SLEEPER THICKNESS UPON THE SLEEPER BENDING STRESS AT THE

Note: \* Other sleeper dimensions: length 2.440 m, breadth 0.230 m + Sleeper bearing plates fitted.

Sleeper thickness (m)	Change in sleeper	Sleeper bending stress at centre (MPa)		Change in sleeper bending stress (Per cent)		
	(Per cent)	VicRail X-Class	Westrail L-Class	Due to an increase of sleeper thickness	Due to an increase in axleload	
0.115	<u> </u>	18.6	22.5	-	+21	
0.135	+17	13.5	16.3	-27	+21	
0.155	+35	10.2	12.4	-45	+21	

# TABLE 6.13 - EFFECT OF VARYING THE SLEEPER THICKNESS UPON THE SLEEPER BENDING STRESS AT THE

## CENTRE FOR A SLEEPER SPACING OF 610 mm

Note: Other sleeper dimensions: length 2.440 m, breadth 0.230 m.

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# Figure 6.14

Variation of the sleeper bending stress at the rail seat with sleeper spacing, using the A.R.E.A. method. (For variable sleeper thickness)



Note: Design Conditions – Standard gauge track Speed 100 km/h A.R.E.A. Impact Factor Other Sleeper Dimensions --Length 2.440 m Breadth 0.230 m

# Figure 6.15

Variation of the sleeper bending stress at the centre with sleeper spacing. (For variable sleeper thickness)

50 to 100 per cent greater than the maximum sleeper bending stress calculated at the rail seat. It can clearly be seen in Figures 6.12 and 6.15 the sleeper bending stress at the centre of the sleeper also exceeds the A.R.E.A. recommended sleeper bending stress limit of 7.6 MPa for the range of sleeper lengths and thickness considered. It was decided to determine what the required sleeper dimensions would have to be in order that the sleeper bending stress at the centre would satisfy or approach the A.R.E.A. design limit. In the parametric analyses above the reference sleeper dimensions used are 2.440 x 0.230 x 0.115 m. Referring to Figure 6.15 it can be seen that the maximum sleeper bending stress at the centre of the sleeper for a sleeper spacing of 610 mm is 18.5 MPa for the Vicrail X-class locomotive, whereas for the Westrail L-Class locomotive this stress is 22.5 MPa. Noting in Table 6.12 that increasing the sleeper length by 8 per cent, viz from 2.440 to 2.640 m, that the reduction in the bending stress at the sleeper centre is 36 per cent, and that in Table 6.13 that increasing the sleeper thickness by 35 per cent, viz from 0.115 to 0.155 m, and the reduction in the bending stress at the sleeper centre is 45 per cent, the following estimate of the sleeper stress at the centre of a 2.640 x 0.230 x 0.155 m sleeper can be made:

- (a) For the Vicrail X-Class locomotive: the maximum sleeper bending stress at the centre is approximately (1 0.36). (1 0.45) . 18.5 = 6.5 MPa.
- (b) For the Westrail L-Class locomotive: the maximum sleeper bending stress at the centre is approximately (1 - 0.36). (1 - 0.45). 22.5 = 7.9 MPa.

It is evident that the sleeper dimensions necessary to satisfy the A.R.E.A. design limit for the case of the sleeper bending stress at the centre of the sleeper are far greater than those that are currently used in service by the majority of state railway systems. Since many state railway systems have also used axleloads at least equal to that of the Vicrail X-Class satisfactorily for many years in conjunction with track having sleeper dimensions smaller than those above; it can be concluded that the A.R.E.A. design limit has been defined specifically for the sleeper bending stress at the rail seat.

From comparison of Figures 6.10, 6.11 6.13 and 6.14 it is also evident that the Battelle/Schramm method of determining the maximum sleeper bending stress at the rail seat is more conservative than that of the A.R.E.A. method. It is recognised that the A.R.E.A. design limit has most probably been derived for use in conjunction with the A.R.E.A. design method and consequently this limit is unsuitable for use with the Battelle/Schramm method.

The above anomoly suggests that the maximum allowable sleeper bending stress criterion should be further examined; especially the determination of reliable estimates of the endurance limit of Australian timber species subjected to the conditions likely to be experienced in service.

#### BALLAST DEPTH REQUIREMENT

As mentioned in Chapter 5 there are various methods, that are available for use in order to determine the required ballast depth, viz: theoretical, semi-empirical and empirical. In order to simplify the parametric analysis the Schramm method was adopted (Equation 5.15). Using this method the variation of the required ballast depth with the allowable subgrade pressure and with various sleeper spacings for the Vicrail X-Class and Westrail L-Class locomotives was determined and is presented in Figures 6.16 and 6.17 respectively. This variation was determined for a 2.440 m long and 0.230 m wide sleeper only and obviously if the sleeper bearing dimensions are increased the required ballast depth would be reduced. Referring to Table 5.1 it can be seen that Clarke suggests that the safe average bearing pressure of compacted subgrade is at least 280 kPa. Applying Clarke's suggested factor of safety of 1.67 the minimum allowable subgrade pressure of compacted subgrades is determined as 168 kPa.

The effect of varying the allowable subgrade pressure upon the required ballast depth determined by the Schramm method and for a sleeper spacing of 600 mm is presented in Table 6.14. Of particular importance is the effect of compaction of the subgrade upon the ballast depth. It can be seen that increasing the allowable subgrade pressure by 100 per cent, i.e. from 150 to 300 kPa reduces the required ballast depth by 50 per cent.

The effect of varying the sleeper length from 2.440 to 2.640 m upon the required ballast depth for particular allowable subgrade pressures can be seen by comparing Figure 6.14 and 6.18. The results of this comparison for a sleeper spacing of 600 mm is presented in Table 6.15. It is evident that an 8 per cent increase in the sleeper length results in a reduction of the required ballast depth in the order of 16 per cent. It would seem that increasing the subgrade bearing capacity by means of compaction has a greater benefit in reducing the ballast depth required than by increasing the sleeper size.

## TABLE 6.14 - EFFECT OF VARYING THE ALLOWABLE SUBGRADE PRESSURE UPON THE REQUIRED BALLAST

Allowable subgrade pressure (kPa)	Change in allowable subgrade pressure	Ballast de <u>r</u> method for	oth by the Schramm sleeper spacing of 600 mm (mm)	Change in Ballast D <b>epth</b> (Per cent)		
	(Per cent)	VicRail X-Class	Westrail L-Class	Due to an increase in the allowable subgrade pressure	Due to an increase in axleload	
150	_	320	370	_	+16	
225	+50	210	250	-33	+19	
300	+100	160	190	-50	+19	

DEPTH USING THE SCHRAMM METHOD

Note: Sleeper length 2.440 m, breadth 0.230 m.

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# TABLE 6.15 - EFFECT OF VARYING THE SLEEPER LENGTH AND THE ALLOWABLE SUBGRADE BEARING PRESSURE UPON THE REQUIRED BALLAST DEPTH USING THE SCHRAMM METHOD AT A 600 mm SLEEPER SPACING

	Allowable subgrade	Change in allowable	Ballast depth for VicRail X-Class locomotive (mm)		Increase in sleeper length	Approximate reductio <b>n</b> in ballast depth	
250	(kPa)	pressure (Per cent)	2.440 m sleeper length	2.640 m sleeper length	(Per cent)	(Per cent)	
	150	-	320	260	+8	-16	
	225	+50	210	180	+8	- 16	
	300	+100	160	130	+8	-16	

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Note: Sleeper breadth 0.230 m.



Vote: Design Conditions – Standard gauge track Vicrail X-class locomotives Speed 100 km/h A.R.E.A. Impact Factor A.R.E.A. Rail Seat Load Ballast friction angle 35° Sleeper length: 2.440 m Sleeper breadth: 0.230 m

# Figure 6.16

Variation of the required ballast depth with allowable subgrade pressure for a range of sleeper spacings, using Schramm's formula, Vicrail X-class locomotives, and 2.440m long sleepers



# Figure 6.17

Variation of the required ballast depth with allowable subgrade pressure for a range of sleeper spacings, using Schramm's formula, Westrail L-class locomotives, and 2.440m long sleepers



# Figure 6.18

Variation of the required ballast depth with allowable subgrade pressure for a range of sleeper spacings using Schramm's formula, Vicrail X-class locomotives and 2.640m long sleepers

#### CHAPTER 7 - CONCLUSIONS

The current practice for the design of conventional railway track is entirely based upon satisfying several criteria for the strength of individual components, (Figure 1.1). Due to variations in the track support capacity, which is dependent upon the track maintenance, the design loadings determined by the theoretical and empirical relationships are arbitrarily increased by what are considered reasonable safety factors. Having determined the maximum stresses in the individual components of the track structure, they are then compared with the corresponding maximum allowable stresses, and if satisfied, the design is completed.

It is clearly apparent that the standard of track maintenance governs the force and stress levels that are applied to the track structure under service conditions. At present the standard of track maintenance is implied in the current track design procedure by the use of various factors of safety. These factors of safety appear to be based entirely upon the considered judgement of earlier railway researchers and are always used to predict the maximum force or stress levels likely to occur for the considered worse maintenance condition of the track. The resulting track design is obviously safe although somewhat conservative since the design approach is not related to track maintenance procedures or to the track maintenance condition. Reliable inputs of actual in-track conditions which relate to maintenance procedures are required in the design procedure, or alternatively standards of track maintenance need to be specified in the design method.

In order to introduce a proper engineering approach to railroad track design, consideration of the deformation of the track under traffic loadings is also necessary. Ballast and subgrade properties need to be fully examined and allowable limits of deformation based upon the retention of track geometry and maintenance standards need to be evolved. When these factors are included in the conventional design flowchart an interaction chart signifying

the design parameter interactions can be developed, and is presented in Figure 7.1. The major advantage of this approach is that it provides a basis for maintenance and design to be treated as related items since the time dependent (or cyclic load) behaviour of the track support condition is included.

Nevertheless, the existing design expressions permit a degree of optimisation of the track cross-section. The data contained herein can be used for that purpose and an Interactive Track Design Model established.



**Figure 7.1** Preliminary track design interaction diagram

# ANNEX A THE THEORETICAL DETERMINATION OF THE TRACK SPRING RATE

The track modulus value used in the beam on an elastic foundation analysis and presented in Tables 3.4, 3.5 and 3.6 has been determined for the field condition, i.e., only after the track has been constructed. This modulus value lumps the effects of a great number of parameters together and therefore can only be considered as an average value for the design situation. In order to predict values of the track modulus before the track is constructed a ballast pyramid model has been developed at Battelle Colombus Laboratories (Prause et al. 1974). This model makes use of realistic track structure parameters in its analysis. Talbot (1918-1934) has shown that the effective ballast friction angle, (i.e. the measure of the degree of load spread of the ballast), changes considerably at a depth of about 150 mm below the sleeper (Figure A.1). Therefore the ballast layer has been divided into two sections, each having a different friction angle in order to effectively model this observed condition (Figure A.2).

Schramm (1961) also stated that the load spread angle of ballast (i.e. the ballast friction angle) is also dependent upon the condition of the ballast. For ballast in an excellent and poor condition the ballast friction angle is about  $40^{\circ}$  and  $30^{\circ}$  respectively, with average values of  $35^{\circ}$  commonly occurring in track. The expressions which define the effective area  $A(z) (mm^2)$  at ballast depths of z, for the two sections in Figure A.2 are:

(a) Upper Section:

 $A(z) = (c_1 z + B) (c_1 z + L)$ for depths  $0 \leq z \leq z_1$ . (A.1)

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# Figure A.1

Lines of equal vertical pressure (kPa) in the ballast for a single loaded sleeper (Talbot 1920)



# Figure A.2

The ballast pyramid model

(b) Lower Section:

$$A(z) = (c_2 z' + c_1 z_1 + B) (c_2 z' + c_1 z_1 + L)$$
(A.2)  
for depths  $z_1 \le z \le z_+$ ,

where

lere	<sup>c</sup> 1	_	2 cano 1'
	°2	=	$2 \tan_{\theta_2}$
	В	=	sleeper breadth (mm),
	L	=	effective sleeper length under the rail seat (mm),
ι	θι	=	angle of internal friction, upper section (degrees),
θ	2(1)	=	angle of internal friction, lower section (degrees),
	z <sub>1</sub>	=	depth of upper section of ballast (mm),
	<sup>z</sup> 2	=	depth of lower section of ballast (mm),
	zt	=	total depth of ballast layer (mm), and
	z	=	$z - z_1$ .

According to the Ballast pyramid model as developed by Prause and Meacham (1974) the stiffness of the upper section of the Ballast  $D_1$  (kN/mm) is

$$D_{1} = \frac{c_{1} (L - B) E_{b}}{\ln \left\{ \frac{L(c_{1}z_{1} + B)}{B(c_{1}z_{1} + L)} \right\}}, \quad (A.3)$$

and the stiffness of the lower section of the ballast  $\mathrm{D}_{2}\left(kN/mm\right)$  is

$$D_{2} = \frac{c_{2} (L - B) E_{b}}{\ln\{\frac{(c_{1}z_{1} + L) (c_{2}z' + c_{1}z_{1} + B)}{(c_{1}z_{1} + B) (c_{2}z' + c_{1}z_{1} + L)}\}}$$
(A.4)

where  $E_b = modulus$  of elasticity of the ballast (MPa); typical values of  $E_b$  are of the order of 200(MPa).

(1)  $z_1 = 150$  (mm).



Progressive steps in the development of a static model of a conventional track structure (Robnett et al 1975) The stiffness of the total ballast pyramid  $D_b(kN/mm)$  can be assumed as the series equivalent of effective stiffnesses of the two springs,  $D_1$  and  $D_2$ , in series, i.e.,

$$D_{b} = \frac{D_{1}D_{2}}{D_{1} + D_{2}} .$$
 (A.5)

The assumed mechanical model of the pad, sleeper, ballast and subgrade is illustrated in Figure A.3. The ballast - subgrade spring system  $D_{bs}(kN/mm)$  is denoted by

$$\frac{1}{D_{bs}} = \frac{1}{D_{b}} + \frac{1}{D_{mod}} , \qquad (A.6)$$

where  $D_{mod}$  = the modified soil stiffness (kN/mm),

$$D_{\text{mod}} = C_s A_{(z)} , \qquad (A.7)$$

$$C_s$$
 = the coefficient of subgrade reaction of the soil  
(kN/mm<sup>3</sup>), and  
 $A_{(z)}$  = effective area of the lower section of the ballast  
(mm<sup>2</sup>) Equation A.2.

Therefore,

$$D_{mod} = C_{s}(c_{2}z' + c_{1}z_{1} + L)(c_{2}z' + c_{1}z_{1} + B).$$
(A.8)

The spring constant of each sleeper  $D_{s}(kN/mm)$  is the series equivalent of the spring constant of the resilient pad (below the rail seat)  $D_{p}(kN/mm)$ , plus half (due to continuity of the ballast and the subgrade) of the spring constant of the effective ballast - subgrade foundation  $D_{bs}$  beneath the sleeper, i.e.,

$$\frac{1}{D_{s}} = \frac{1}{D_{p}} + \frac{1}{D_{bs/2}} .$$
 (A.9)

Actual measured total track depressions and the breakdown of the amount of depression caused by the ballast and the subgrade for various subgrade types and due to an axle load of 190 (kN) are presented in Figure A.4 (Birman 1968). It is clearly apparent that the overall track depression and therefore the spring rate is principally governed by the subgrade type.

As the subgrade becomes stiffer the ballast contribution to the total depression becomes more significant.

The overall foundation modulus k(MPa) is then defined as

$$k = \frac{D_s}{S} , \qquad (A.10)$$

where S = sleeper spacing (mm).

From the beam on an elastic foundation model, the maximum rail seat load  ${\bf q}_{\bf r}\,(kN)$  is defined as

$$q_r = S.ky_m , \qquad (A.11)$$

where  $y_m = maximum rail depression (mm)$ .

Combining Equations A.6 and A.7

$$q_r = D_s Y_m$$
 (A.12)

Therefore the effective contact pressure between the sleeper and the ballast  $p_a(kPa)$  is

$$p_{a} = \frac{D_{s} Y_{m}}{B.L} , \qquad (A.13)$$

where

B = sleeper breadth (mm), and

L = effective length of the sleeper under the rail seat (mm).

From this analysis the average pressure  $\sigma_{\rm Za}^{\rm \ (kPa)}$  on the subgrade is defined as

$$\sigma_{za} = \begin{bmatrix} \underline{B.L} \\ \overline{A}(z) \end{bmatrix} P_{a} , \qquad (A.14)$$

where

 $A_{(z)} =$ effective area of the lower section of the ballast  $(mm^2)$ , Equation A.2.



# Figure A.4

Track depression for different subgrade types for an axle load of 190 (kN) (Birman 1968)

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## ANNEX B THE ESTIMATION OF THE COEFFICIENT OF SUBGRADE REACTION

Terzaghi (1955) introduced the concept of the "bulb of pressure" to enable the coefficient of subgrade reaction determined from an insitu plate test to be factored according to the actual foundation dimensions.

The coefficient of subgrade reaction for a soil  $C_{s}(kN/mm^{3})$  is defined as the ratio

$$C_{s} = \frac{p}{y} , \qquad (B.1)$$

where p = pressure per unit of the surface of contact between a loaded beam or slab and the subgrade on which it rests and on which it transfers the load (kPa), and y = settlement produced by this load application (mm).

As a basis for estimating the coefficient of subgrade reaction for a loaded area of the subgrade, the coefficient of subgrade reaction  $C_o (kN/mm^3)$  for a square plate with a width of 1 (ft), 300 (mm), has been selected, because this value can readily be calculated in the field if necessary.

Terzaghi (1955) suggests that if the subgrade consists of cohesionless or slightly cohesive sand, the value of  $C_{o}$  can be estimated from Table B.1. The density-category of the sand can be ascertained by means of a standard penetration test.

These values are valid for contact pressures which are smaller than one half the estimated bearing capacity of the clay; the latter being independent of the dimensions of the loaded area.

For a subgrade of heavily pre-compressed clay the magnitude of  $C_0$  increases in simple proportion to the unconfined compressive strength of the clay  $q_u$  (kPa). Terzaghi (1955) has proposed

numerical values of C  $_{\rm O}$  for various clay types and these are presented in Table B.2.

TABLE B.1 - VALUES OF C<sub>0</sub> 10<sup>-6</sup> (kN/mm<sup>3</sup>) FOR SQUARE PLATES 1 FT X 1 FT, (300 x 300 mm) RESTING ON SAND (TERZAGHI 1955)

Relative Density of Sand	Loose	Medium	Dense
Dry or moist sand, limiting values	7-21	21-105	105-350
Dry or moist sand, proposed values	14	45	175
Submerged sand, proposed values	8	28	105

TABLE B.2 - VALUES OF C<sub>0</sub> 10<sup>-6</sup> (kN/mm<sup>2</sup>) FOR SQUARE PLATES 1 FT X 1 FT (300 x 300mm) RESTING ON PRECOMPRESSED CLAY (TERZAGHI 1955)

Consistency of Clay	Stiff	Very Stiff	Hard
Values of q <sub>u</sub> (kPa)	107-214	214-428	>428
Range of C <sub>o</sub>	17- 34	34- 68	> 68
Proposed values C <sub>o</sub>	25.5	51	102

For sands, experimental investigations have shown that the coefficient of subgrade reaction  $C_s$  for the effective stressed area of the subgrade,  $A_{(z)}$ , Equation 5.21, can be related to the measured coefficient of subgrade reaction of the plate test  $C_o$ using the concept of the "bulb of pressure" by

$$C_{s} = C_{o} \frac{W' + 1}{2W'}^{2};$$
 (B.2)

therefore,

$$W' = c_2 z' + c_1 z_1 + B . (B.3)$$

For clays, if the stressed area is of rectangular shape, as is the case with the effective subgrade stressed area, the coefficient of subgrade reaction  $C_{c}$  for this area can be determined by

$$C_{\rm g} = \frac{{\rm x} + 0.5}{1.5 {\rm x}} , \qquad (B.4)$$

where x = ratio of the width to the length of the effecitve subgrade stressed area, and

$$\mathbf{x} = \frac{c_2 z_1 + c_1 z_1 + L}{c_2 z' + c_1 z_1 + B} . \tag{B.5}$$

The main problem with this method as applied to railway conditions is that the determination of the coefficient of subgrade reaction  $C_0$ , from the field plate bearing test, is based on rather arbitrary static loading conditions (Prause et al 1974). Designs should be based on dynamic properties measured under loading conditions which better approximate the subgrade loading under the track.

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## ABBREVIATIONS

The following principal abbreviations occur throughout the report.

AAR:	Association of American Railroads
AREA:	American Railroad Engineering Association
Batelle:	Battelle Columbus Laboratories (USA)
BR:	British Railways
DB:	Deutsche Bundesbahn (German Federal Railways)
JNR:	Japanese National Railways
ORE :	Office for Research and Experiments of the International Union of Railways
SNCF :	Societe Nationale de <b>E</b> hemins de Fer Francaise (French National Railways)
CWR:	Continuously Welded Rail.

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